Supplementary material - ISSR-DIL: Image Specific Super-Resolution Using Deep Identity Learning

In this supplementary material we provide the discussion on i) Effect of gaussian blur on variance of an image ii) Quantitative study of standard ISR dataset statistics iii) Study on background noise distribution in LR images.

1. Effect of gaussian blur on variance of image

Statement: When an image with standard deviation (SD) σ_i is convolved with a gaussian filter having SD σ_f, then SD of the resulting image σ_o is approximately given as,

$$\sigma_0 \approx \frac{\sigma_i}{\sigma_f.2\sqrt{\pi}} \tag{1}$$

Proof:
Let the convolution of gaussian kernel with the image $I(x,y)$ results in image $\hat{I}(x,y)$. The relation between the image $I(x,y)$ and $\hat{I}(x,y)$ is represented as

$$\hat{I}(x,y) = \sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty} \frac{1}{2\pi{\sigma_f}^2} \exp - \frac{i^2+j^2}{2{\sigma_f}^2} I(x+i,y+j) \tag{2}$$

The variance of the output image $Var[\hat{I}(x,y)]$ is given as,

$$Var[\hat{I}(x,y)] = Var[\sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty} \frac{1}{2\pi{\sigma_f}^2} \exp - \frac{i^2+j^2}{2{\sigma_f}^2} I(x+i,y+j)] \tag{3}$$

Based on the linear combination property of variance of independent random variables, it is given that,

$$Var[\sum_i c_i X_i] = \sum_i {c_i}^2 Var[X_i] \tag{4}$$

where c_i are constants and X_i are independent random variables. Therefore, assuming the pixel intensity values in $I(x,y)$ are independent,

$${\sigma_o}^2 = {\sigma_i}^2 (\sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty} \frac{1}{2\pi{\sigma_f}^2} \exp - \frac{i^2+j^2}{2{\sigma_f}^2})^2 \tag{5}$$

where ${\sigma_o}^2$ and ${\sigma_i}^2$ are the variances of output and input images respectively. In the above eq. 5, using the definite integral operations,the sum $\sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty} \frac{1}{2\pi{\sigma_f}^2} \exp - \frac{i^2+j^2}{2{\sigma_f}^2}$ can be approximated as $\frac{1}{4\pi{\sigma_f}^2}$.

$$\text{ie., } \sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty} \frac{1}{2\pi{\sigma_f}^2} \exp - \frac{i^2+j^2}{2{\sigma_f}^2} \approx \frac{1}{4\pi{\sigma_f}^2}$$

Therefore, the standard deviation of the gaussian convolved image is given as,

$$\sigma_0 \approx \frac{\sigma_i}{\sigma_f.2\sqrt{\pi}} \tag{6}$$

In this work, using the eq. 6, the standard deviation of the degradation kernel K (i.e, gaussian kernel) was computed by the following equation,

$$\sigma_f \approx \frac{\sigma_i}{\sigma_o.2\sqrt{\pi}} \tag{7}$$

2. Quantitative study of standard ISR dataset statistics

The average mean and, average standard deviation of the High-Resolution (HR), Low-Resolution (LR) images present in RealSR [4], DIV2KRK [1], BSD100 [5], Set14 [6], Set5 [2] datasets were tabulated and given below.

Table 1. The average mean, standard deviation of the HR and LR images in the benchmark image super-resolution datasets.

S.No.	Dataset	Degradation type	HR images		Scale factor	LR images	
			Average Mean	Average Standard deviation		Average Mean	Average Standard deviation
1.	RealSR	Real dataset (Varying focal lengths of the device)	109.49211	48.4647	X2	109.5045	48.3168
					X3	109.5016	48.1367
					X4	109.5495	47.9235
2.	DIV2KRK	Synthetic dataset (Gaussian blur + downsampling)	107.6317	59.4098	X2	106.7495	56.3091
					X4	107.1264	54.3588
3.	BSD100	Synthetic dataset (MATLAB Bicubic downsampling)	104.5112	51.7480	X2	104.5113	49.9954
					X3	104.4576	49.0626
					X4	104.5058	48.3274
4.	Set14	Synthetic dataset (MATLAB Bicubic downsampling)	117.1667	58.4791	X2	117.1819	57.0439
					X3	117.2613	56.1221
					X4	117.1848	55.3365
5.	Set5	Synthetic dataset (MATLAB Bicubic downsampling)	102.8140	64.5898	X2	102.8199	63.8425
					X3	102.7931	63.0911
					X4	102.8129	62.3910

3. Study on background noise distribution in the LR images.

In our experimental analysis, we computed skewness of the uniform regions in the LR images, to measure the non-Gaussianity/ asymmetry of the (background) noise through these regions. We selected random uniform regions of LR images from i) the synthetic dataset [1], ii) a real dataset [3], and iii) from real captured LR images using an old smartphone camera (refer to Fig.6 in the manuscript). This experimental analysis results do confirm that i) the (background of the) LR images are degraded not purely by a Gaussian distribution ii) the proposed approach is robust to perform the ISR task for more complex and real degradations than Gaussian. The skewness measure along with the histogram depicting the pixel distribution in the randomly selected uniform regions of LR images using real captured LR image, RealSR and DIV2KRK datasets are depicted below in Fig. 1, Fig. 2, and Fig. 3 respectively.

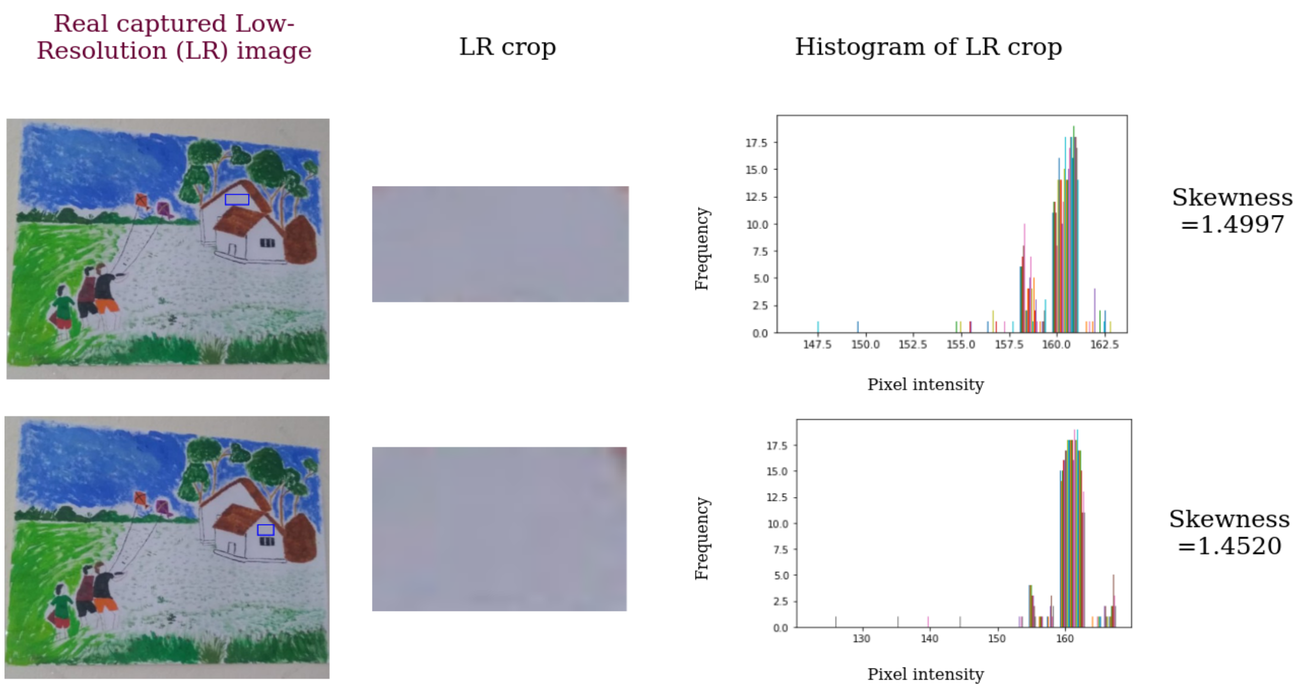

Figure 1. The skewness measure along with the histogram depicting the pixel distribution in the randomly selected uniform regions of real captured LR image.

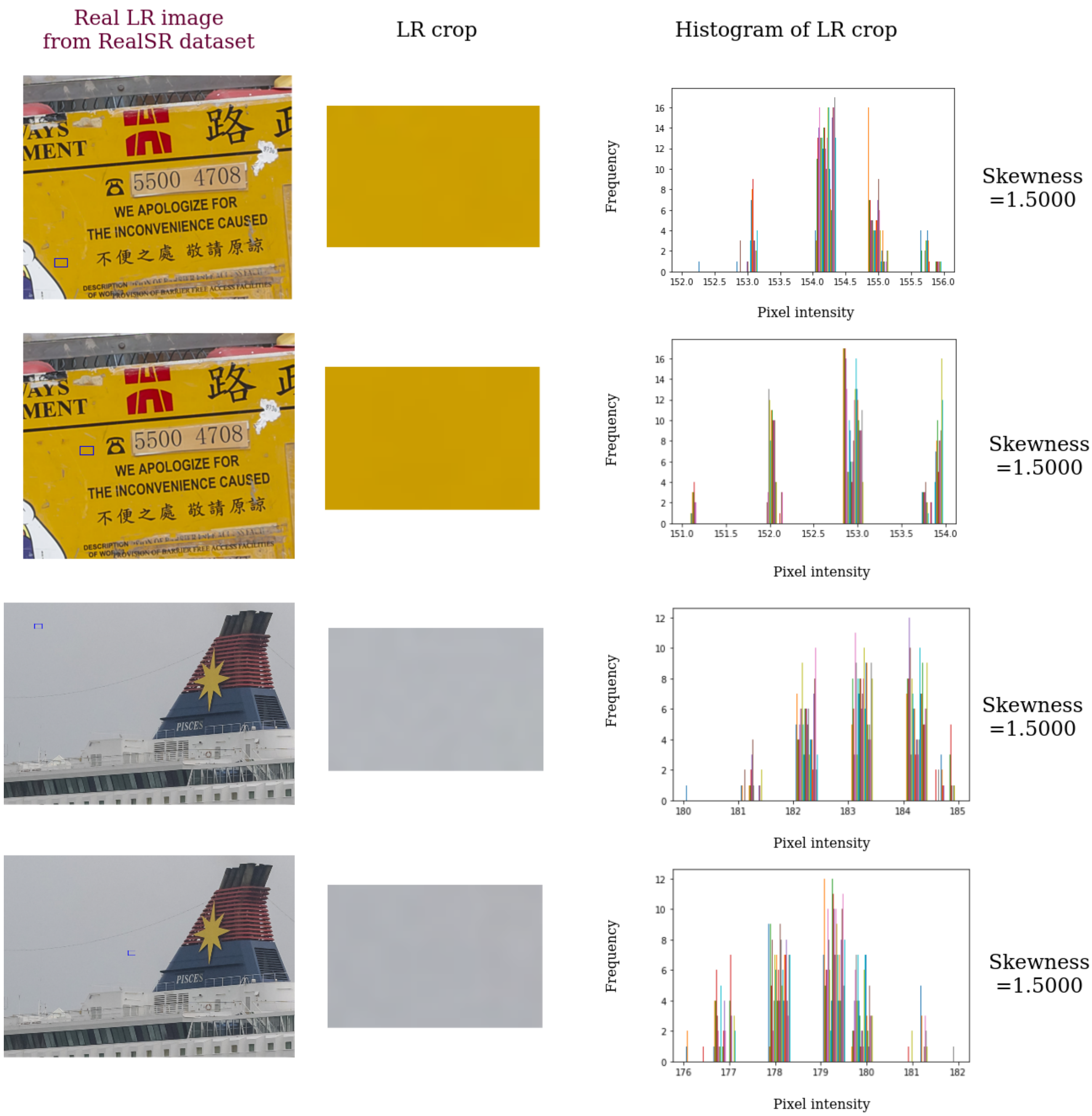

Figure 2. The skewness measure along with the histogram depicting the pixel distribution in the randomly selected uniform regions of real LR images from RealSR dataset.

References

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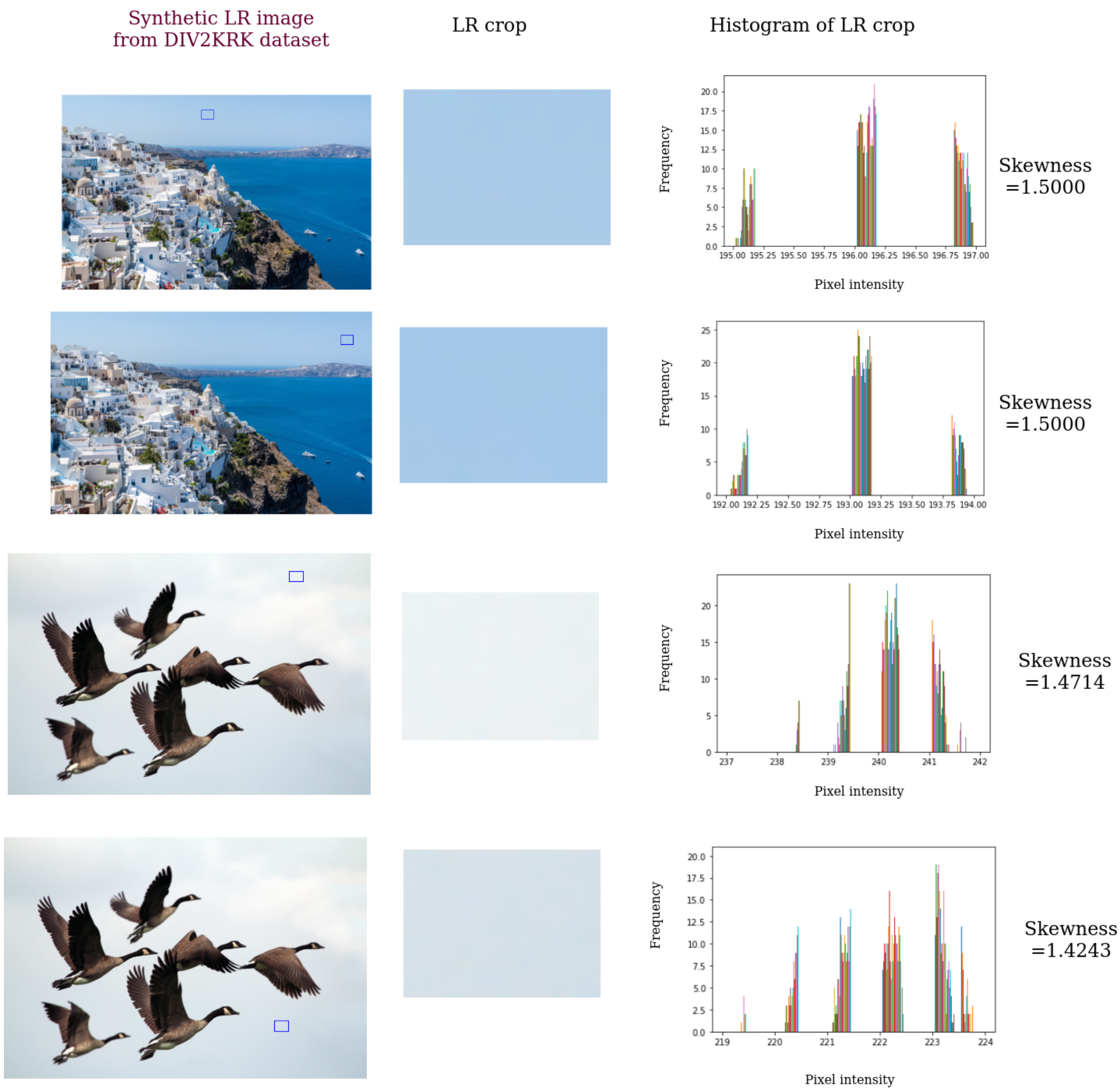

Figure 3. The skewness measure along with the histogram depicting the pixel distribution in the randomly selected uniform regions of synthetic LR images from DIV2KRK dataset.

2019.

[4] Xiaozhong Ji, Yun Cao, Ying Tai, Chengjie Wang, Jilin Li, and Feiyue Huang. Real-world super-resolution via kernel estimation and noise injection. In *2020 IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops (CVPRW)*, pages 1914–1923, 2020.

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[6] Roman Zeyde, Michael Elad, and Matan Protter. On single image scale-up using sparse-representations. In *Curves and Surfaces*, 2010.