Distribution Prototype Diffusion Learning for Open-set Supervised Anomaly Detection

Supplementary Material

8. Dataset Statistics

Extensive experiments are conducted on nine real-world anomaly detection (AD) datasets. Tab. 4 provides key statistics for all datasets used in this study. We follow the exact same settings as in previous open-set supervised anomaly detection (OSAD) studies. Specifically, for the MVTec AD dataset, we adhere to the original split, dividing the normal samples into training and test sets. For the other eight datasets, normal samples are randomly partitioned into training and test sets at a 3:1 ratio.

Table 4. The statistical information for nine real-world anomaly detection (AD) datasets, with the first 15 rows detailing the subsets of the MVTec AD dataset.

Dataset			Original Training	Original Test	
	$\|\mathcal{C}\|$	Type	Normal	Normal	Anomaly
Carpet	5	Textture	280	28	89
Grid	5	Textture	264	21	57
Leather	5	Textture	245	32	92
Tile	5	Textture	230	33	83
Wood	5	Textture	247	19	60
Bottle	3	Object	209	20	63
Capsule	5	Object	219	23	109
Pill	7	Object	267	26	141
Transistor	4	Object	213	60	40
Zipper	7	Object	240	32	119
Cable	8	Object	224	58	92
Hazelnut	4	Object	391	40	70
Metal_nut	4	Object	220	22	93
Screw	5	Object	320	41	119
Toothbrush	1	Object	60	12	30
MVTecAD	73	-	3629	467	1258
Optical	1	Object	10500	3500	2100
SDD	1	Textture	594	286	54
AITEX	12	Textture	1692	564	183
ELPV	2	Textture	1131	377	715
Mastcam	11	Object	9302	426	451
Hyper-Kvasir	4	Medical	2021	674	757
BrainMRI	1	Medical	73	25	155
HeadCT	1	Medical	75	25	100

- MVTec AD [3] is a widely-used benchmark for defect detection, comprising 15 distinct categories, each of which includes one or several subcategories. The dataset contains a total of 73 fine-grained anomaly classes at either the texture or object level.
- Optical [41] is a synthetic dataset designed for industrial optical inspection and defect detection. The artificially generated data mimics real-world tasks.
- SDD [39] is a defect product image detection dataset with pixel-level defect annotations. The original images, which have a resolution of 500 × 1250, are vertically divided into three segments. Each segment is then annotated at the pixel level.
- AITEX [38] is a fabric defect detection dataset that includes 12 defect categories with pixel-level annotations. The original images, which have a resolution of 4096 × 256, are cropped into multiple 256 × 256 patches. Each patch is then re-annotated at the pixel level.
- ELPV [8] is a dataset for defect detection in electroluminescence (EL) images of solar cells. It includes two types of defects, corresponding to different types of solar cells: monocrystalline and polycrystalline.
- Mastcam [13] is a novelty detection dataset constructed from geological images captured by the multispectral imaging system installed on the Mars rover. The dataset includes typical images and images from 11 novel geological classes. Each image comprises both shorter-wavelength (color) channels and longer-wavelength (grayscale) channels, with this study focusing on the shorter-wavelength channels.
- Hyper-Kvasir [4] is a large-scale, open-access gastrointestinal dataset collected during real endoscopy and colonoscopy procedures. It comprises four main categories and 23 subcategories of endoscopic and colonoscopic images. This work focuses on endoscopic images, where anatomical landmark categories are considered as normal samples and pathological categories are treated as abnormal samples.
- BrainMRI [36] is a brain tumor detection dataset obtained through magnetic resonance imaging (MRI).
- HeadCT [36] is a dataset for detecting intracranial hemorrhage obtained through head computed tomography (CT) scans.

9. Full Results under General Setting

Tab. 5 presents a comprehensive comparison of the proposed DPDL method with state-of-the-art (SOTA) approaches under general settings. It reports performance metrics for each category within the MVTec AD dataset. Overall, the DPDL model consistently outperforms baseline methods across all application scenarios in both ten-shot and one-shot settings, achieving the best performance in terms of Area Under the Curve (AUC).

Table 5. AUC performance (mean ± std) across nine real-world AD datasets is reported under the general setting. **red** highlights the best results, and **blue** indicates sub-optimal outcomes. All baseline SOTA results are sourced from the original papers [9, 46].

Dataset	One Training Anomaly Example							Ten Training Anomaly Examples						
	DevNet	FLOS	SAOE	MLEP	DRA	AHL	DPDL (Ours)	DevNet	FLOS	SAOE	MLEP	DRA	AHL	DPDL (Ours)
Carpet	0.746±0.076	0.755±0.026	0.766±0.098	0.701±0.091	0.859±0.023	**0.877**±0.004	**0.914**±0.006	0.867±0.040	0.780±0.009	0.755±0.136	0.781±0.049	0.940±0.027	**0.953**±0.001	**0.988**±0.002
Grid	0.891±0.040	0.871±0.076	0.921±0.032	0.839±0.028	0.972±0.011	**0.975**±0.005	**0.999**±0.001	0.967±0.021	0.966±0.005	0.952±0.011	0.980±0.009	0.987±0.009	**0.992**±0.002	**0.999**±0.001
Leather	0.873±0.026	0.791±0.057	0.996±0.007	0.781±0.020	0.989±0.005	**0.988**±0.001	**0.996**±0.001	0.999±0.001	0.993±0.004	**1.000**±0.000	0.813±0.158	1.000±0.000	**1.000**±0.000	**1.000**±0.000
Tile	0.752±0.038	0.787±0.038	0.935±0.034	0.927±0.036	0.965±0.015	**0.968**±0.001	**0.994**±0.002	0.987±0.005	0.952±0.010	0.944±0.013	0.988±0.009	0.994±0.006	**1.000**±0.000	**0.999**±0.001
Wood	0.900±0.068	0.927±0.065	0.948±0.009	0.660±0.142	0.985±0.011	**0.987**±0.003	**0.998**±0.002	0.999±0.001	**1.000**±0.000	0.976±0.031	0.999±0.002	**0.998**±0.001	**0.998**±0.000	**0.998**±0.001
Bottle	0.976±0.006	0.975±0.023	0.989±0.019	0.927±0.090	**1.000**±0.000	**1.000**±0.000	**1.000**±0.000	0.993±0.008	0.995±0.002	0.998±0.003	0.981±0.004	**1.000**±0.000	**1.000**±0.000	**1.000**±0.000
Capsule	0.564±0.032	0.666±0.020	0.611±0.109	0.558±0.075	0.631±0.056	**0.665**±0.030	**0.757**±0.017	0.865±0.057	0.902±0.017	0.850±0.054	0.818±0.063	**0.935**±0.022	0.930±0.001	**0.976**±0.004
Pill	0.769±0.017	0.745±0.064	0.652±0.078	0.656±0.061	0.832±0.034	**0.840**±0.003	**0.842**±0.002	0.866±0.038	**0.929**±0.012	0.872±0.049	0.845±0.048	0.904±0.024	0.918±0.001	**0.923**±0.001
Transistor	0.722±0.032	0.709±0.041	0.680±0.182	0.695±0.124	0.668±0.068	**0.796**±0.003	**0.748**±0.002	0.924±0.027	0.862±0.037	0.860±0.053	**0.927**±0.043	0.915±0.025	0.926±0.009	**0.928**±0.001
Zipper	0.922±0.018	0.885±0.033	0.970±0.033	0.865±0.086	0.984±0.016	**0.986**±0.000	**0.989**±0.001	0.990±0.009	0.990±0.008	0.995±0.004	0.965±0.002	**1.000**±0.000	**1.000**±0.000	**1.000**±0.000
Cable	0.783±0.058	0.790±0.039	0.819±0.060	0.688±0.017	0.876±0.012	**0.858**±0.011	**0.935**±0.008	0.892±0.020	0.890±0.063	0.862±0.022	0.857±0.062	0.909±0.011	**0.921**±0.001	**0.929**±0.000
Hazelnut	0.979±0.010	0.976±0.021	0.961±0.042	0.704±0.090	0.977±0.030	**0.984**±0.004	**0.997**±0.002	**1.000**±0.000	**1.000**±0.000	**1.000**±0.000	**1.000**±0.000	**1.000**±0.000	**1.000**±0.000	**1.000**±0.000
Metal_nut	0.876±0.007	0.930±0.022	0.922±0.033	0.878±0.038	**0.948**±0.046	**0.952**±0.003	**0.948**±0.007	0.991±0.006	0.984±0.004	0.976±0.013	0.974±0.009	**0.997**±0.002	**0.998**±0.000	0.996±0.001
Screw	0.399±0.187	0.337±0.091	0.653±0.074	0.675±0.294	0.903±0.064	**0.927**±0.009	**0.977**±0.004	0.970±0.015	0.940±0.017	0.975±0.023	0.899±0.039	0.977±0.009	**0.985**±0.002	**0.995**±0.001
Toothbrush	0.753±0.027	0.731±0.028	0.686±0.110	0.617±0.058	0.650±0.029	**0.794**±0.016	**0.807**±0.001	0.860±0.066	0.900±0.008	0.865±0.062	0.783±0.048	0.826±0.021	**0.921**±0.007	**0.929**±0.000
MVTec AD	0.780±0.020	0.755±0.136	0.834±0.007	0.744±0.019	0.883±0.008	**0.901**±0.003	**0.927**±0.002	0.945±0.004	0.939±0.007	0.926±0.010	0.907±0.005	0.959±0.003	**0.970**±0.002	**0.977**±0.002
Optical	0.523±0.003	0.518±0.003	0.815±0.014	0.516±0.009	0.888±0.012	**0.888**±0.007	**0.915**±0.002	0.782±0.065	0.720±0.055	0.941±0.013	0.740±0.039	0.965±0.006	**0.976**±0.004	**0.983**±0.005
SDD	0.881±0.009	0.840±0.043	0.781±0.009	0.811±0.045	0.859±0.014	**0.909**±0.001	**0.917**±0.003	0.988±0.006	0.967±0.018	0.955±0.020	0.983±0.013	0.991±0.005	**0.991**±0.001	**0.996**±0.001
AITEX	0.598±0.070	0.538±0.073	0.675±0.094	0.564±0.055	0.692±0.124	**0.734**±0.008	**0.838**±0.008	0.887±0.013	0.841±0.049	0.874±0.024	0.867±0.037	0.893±0.017	**0.925**±0.013	**0.975**±0.007
ELPV	0.514±0.076	0.457±0.056	0.635±0.092	0.578±0.062	0.675±0.024	**0.828**±0.005	**0.897**±0.002	0.846±0.022	0.818±0.032	0.793±0.047	0.794±0.047	0.845±0.013	**0.850**±0.004	**0.937**±0.003
Mastcam	0.595±0.016	0.542±0.017	0.662±0.018	0.625±0.045	0.692±0.058	**0.743**±0.003	**0.838**±0.011	0.790±0.021	0.703±0.029	0.810±0.029	0.798±0.026	0.848±0.008	**0.855**±0.005	**0.934**±0.010
Hyper-Kvasir	0.653±0.037	0.668±0.004	0.498±0.100	0.445±0.040	0.690±0.017	**0.768**±0.015	**0.821**±0.007	0.829±0.018	0.773±0.029	0.666±0.050	0.600±0.069	0.834±0.004	**0.880**±0.003	**0.939**±0.005
BrainMRI	0.694±0.004	0.693±0.036	0.531±0.060	0.632±0.017	0.744±0.004	**0.866**±0.004	**0.893**±0.004	0.958±0.012	0.955±0.011	0.900±0.041	0.959±0.011	**0.970**±0.003	**0.977**±0.001	0.969±0.005
HeadCT	0.742±0.076	0.698±0.092	0.597±0.022	0.758±0.038	0.796±0.105	**0.825**±0.014	**0.865**±0.005	**0.982**±0.009	0.971±0.004	0.935±0.021	0.972±0.014	0.972±0.002	**0.999**±0.003	0.981±0.003

Table 6. Detailed class-level AUC results (mean ± std) under the hard setting. The best and second-best results are highlighted in **red** and **blue**, respectively. Carpet and Metal_nut are subsets of MVTec AD.

Dataset		One Training Anomaly Example							Ten Training Anomaly Examples						
		DevNet	FLOS	SAOE	MLEP	DRA	AHL	DPDL (Ours)	DevNet	FLOS	SAOE	MLEP	DRA	AHL	DPDL (Ours)
Carpet	Color	0.716±0.085	0.467±0.278	0.763±0.100	0.547±0.056	0.879±0.021	**0.894**±0.004	**0.909**±0.001	0.767±0.015	0.760±0.005	0.467±0.067	0.698±0.025	0.886±0.042	**0.929**±0.007	**0.933**±0.002
	Cut	0.666±0.035	0.685±0.007	0.664±0.165	0.658±0.056	0.902±0.033	**0.934**±0.003	**0.941**±0.003	0.819±0.037	0.688±0.059	0.793±0.175	0.653±0.120	0.922±0.038	**0.943**±0.002	**0.951**±0.004
	Hole	0.721±0.067	0.594±0.142	0.772±0.071	0.653±0.065	0.901±0.033	**0.935**±0.014	**0.945**±0.009	0.814±0.038	0.733±0.014	0.831±0.125	0.674±0.076	0.947±0.016	**0.960**±0.003	**0.964**±0.003
	Metal	0.819±0.032	0.701±0.028	0.780±0.172	0.706±0.047	0.871±0.037	**0.931**±0.007	**0.940**±0.001	0.863±0.022	0.678±0.083	0.883±0.043	0.764±0.061	**0.933**±0.022	0.921±0.003	**0.938**±0.005
	Thread	0.912±0.044	0.941±0.005	0.787±0.204	0.831±0.117	0.950±0.029	**0.966**±0.005	**0.970**±0.002	0.972±0.009	0.946±0.005	0.831±0.297	0.967±0.006	0.989±0.004	**0.991**±0.001	**0.993**±0.000
	Mean	0.767±0.018	0.678±0.040	0.753±0.055	0.679±0.029	0.901±0.006	**0.932**±0.003	**0.941**±0.006	0.847±0.017	0.761±0.012	0.762±0.073	0.751±0.023	0.935±0.013	**0.949**±0.002	**0.956**±0.004
Metal_nut	Bent	0.797±0.048	0.851±0.046	0.864±0.032	0.743±0.013	0.952±0.020	**0.954**±0.003	**0.958**±0.001	0.904±0.022	0.827±0.075	0.901±0.023	0.956±0.013	**0.990**±0.003	0.989±0.000	**0.991**±0.002
	Color	0.909±0.023	0.821±0.059	0.857±0.037	0.835±0.075	**0.946**±0.023	0.933±0.008	**0.938**±0.003	**0.978**±0.016	**0.978**±0.008	0.879±0.018	0.945±0.039	0.967±0.011	0.958±0.001	**0.969**±0.005
	Flip	0.764±0.014	0.799±0.058	0.751±0.090	0.813±0.031	0.921±0.029	**0.931**±0.002	**0.940**±0.004	**0.987**±0.004	0.942±0.009	0.795±0.062	0.805±0.057	0.913±0.021	0.937±0.003	**0.955**±0.003
	Scratch	0.952±0.052	**0.947**±0.027	0.792±0.075	0.907±0.085	0.909±0.023	0.934±0.005	**0.936**±0.002	0.991±0.017	0.943±0.002	0.845±0.041	0.805±0.153	0.911±0.034	**0.999**±0.000	**0.992**±0.002
	Mean	0.855±0.016	0.855±0.024	0.816±0.029	0.825±0.023	0.932±0.017	**0.939**±0.004	**0.944**±0.003	0.965±0.011	0.922±0.014	0.855±0.016	0.878±0.058	0.945±0.017	**0.972**±0.002	**0.978**±0.002
AITEX	Broken end	0.712±0.069	0.645±0.030	**0.778**±0.068	0.441±0.111	0.708±0.094	0.704±0.005	**0.761**±0.010	0.658±0.111	0.585±0.037	0.712±0.068	0.732±0.065	0.693±0.099	**0.735**±0.010	**0.796**±0.001
	Broken pick	0.552±0.003	0.598±0.023	0.644±0.039	0.476±0.070	**0.731**±0.072	0.727±0.003	**0.760**±0.014	0.585±0.028	0.548±0.054	0.629±0.012	0.555±0.027	**0.760**±0.037	0.683±0.002	**0.784**±0.011
	Cut selvage	0.689±0.016	0.694±0.036	0.681±0.077	0.434±0.149	0.739±0.101	**0.753**±0.007	**0.765**±0.007	0.709±0.039	0.745±0.035	0.770±0.014	0.682±0.025	0.777±0.036	**0.781**±0.006	**0.796**±0.005
	Fuzzyball	0.617±0.075	0.525±0.043	**0.650**±0.064	0.525±0.157	0.538±0.092	0.647±0.007	**0.715**±0.013	0.734±0.039	0.550±0.082	0.842±0.026	0.677±0.223	0.701±0.093	**0.775**±0.024	**0.808**±0.003
	Nep	0.722±0.023	**0.734**±0.038	0.710±0.044	0.517±0.059	0.717±0.052	0.703±0.005	**0.757**±0.005	**0.810**±0.042	0.746±0.060	0.771±0.032	0.740±0.052	0.750±0.038	0.792±0.007	**0.811**±0.005
	Weft crack	0.586±0.134	0.546±0.114	0.582±0.108	0.400±0.029	0.669±0.045	**0.706**±0.009	**0.758**±0.006	0.599±0.137	0.636±0.051	0.618±0.172	0.370±0.037	**0.717**±0.072	0.713±0.003	**0.790**±0.004
	Mean	0.646±0.034	0.624±0.024	0.674±0.034	0.466±0.030	0.684±0.033	**0.707**±0.007	**0.753**±0.005	0.683±0.032	0.635±0.043	0.724±0.032	0.656±0.041	0.733±0.009	**0.747**±0.002	**0.798**±0.004
ELPV	Mono	0.634±0.087	0.717±0.025	0.563±0.102	0.649±0.027	0.735±0.031	**0.774**±0.013	**0.785**±0.002	0.599±0.040	0.629±0.072	0.569±0.035	**0.756**±0.045	0.731±0.021	0.745±0.004	**0.793**±0.003
	Poly	0.662±0.050	0.665±0.021	0.665±0.173	0.483±0.247	0.671±0.051	**0.705**±0.006	**0.738**±0.008	0.804±0.022	0.662±0.042	0.796±0.084	0.734±0.078	0.800±0.064	**0.831**±0.011	**0.843**±0.004
	Mean	0.648±0.057	0.691±0.008	0.614±0.048	0.566±0.111	0.703±0.022	**0.740**±0.003	**0.762**±0.003	0.702±0.023	0.646±0.032	0.683±0.047	0.745±0.020	0.766±0.029	**0.788**±0.003	**0.818**±0.003
Mastcam	Bedrock	0.495±0.028	0.499±0.056	0.636±0.072	0.532±0.036	0.668±0.012	**0.679**±0.012	**0.732**±0.003	0.550±0.053	0.499±0.098	0.636±0.068	0.512±0.062	0.658±0.021	**0.673**±0.006	**0.757**±0.004
	Broken-rock	0.533±0.020	0.569±0.025	**0.699**±0.058	0.544±0.088	0.645±0.053	0.661±0.009	**0.738**±0.004	0.547±0.018	0.608±0.085	0.712±0.052	0.651±0.063	0.649±0.047	**0.722**±0.004	**0.783**±0.002
	Drill-hole	0.555±0.037	0.539±0.077	**0.697**±0.074	0.636±0.066	0.657±0.070	0.654±0.004	**0.738**±0.011	0.583±0.022	0.601±0.009	0.682±0.042	0.660±0.002	0.725±0.005	**0.760**±0.003	**0.797**±0.004
	Drt	0.529±0.046	0.591±0.042	**0.735**±0.020	0.624±0.042	0.713±0.053	0.724±0.006	**0.745**±0.005	0.621±0.043	0.652±0.024	0.761±0.062	0.616±0.048	0.760±0.033	**0.772**±0.004	**0.818**±0.002
	Dump-pile	0.521±0.020	0.508±0.021	0.682±0.022	0.545±0.127	**0.767**±0.043	0.756±0.011	**0.764**±0.003	0.705±0.011	0.700±0.070	0.750±0.037	0.696±0.047	0.748±0.066	**0.802**±0.005	**0.830**±0.005
	Float	0.502±0.020	0.551±0.030	**0.711**±0.041	0.530±0.075	0.670±0.065	0.702±0.005	**0.739**±0.007	0.615±0.052	0.736±0.041	0.718±0.064	0.671±0.032	0.744±0.073	**0.765**±0.002	**0.816**±0.012
	Meteorite	0.467±0.049	0.462±0.077	**0.669**±0.037	0.476±0.014	0.637±0.015	0.616±0.013	**0.704**±0.004	0.554±0.021	0.568±0.053	0.647±0.030	0.473±0.047	**0.716**±0.004	0.691±0.001	**0.785**±0.017
	Scuff	0.472±0.031	0.508±0.070	**0.679**±0.048	0.492±0.037	0.549±0.027	0.581±0.020	**0.714**±0.002	0.528±0.034	0.575±0.042	**0.676**±0.019	0.504±0.052	0.636±0.086	0.656±0.009	**0.777**±0.003
	Veins	0.527±0.023	0.493±0.052	0.688±0.069	0.489±0.028	**0.699**±0.045	0.687±0.017	**0.789**±0.003	0.589±0.072	0.608±0.044	**0.686**±0.053	0.510±0.090	0.620±0.036	0.650±0.003	**0.766**±0.008
	Mean	0.511±0.013	0.524±0.013	**0.689**±0.037	0.541±0.007	0.667±0.012	0.673±0.010	**0.733**±0.004	0.588±0.011	0.616±0.021	0.697±0.014	0.588±0.016	0.695±0.004	**0.721**±0.003	**0.778**±0.007
Hyper-Kvasir	Barretts	0.672±0.014	0.703±0.040	0.382±0.117	0.438±0.111	0.772±0.019	**0.792**±0.007	**0.793**±0.000	**0.834**±0.012	0.764±0.066	0.698±0.037	0.540±0.014	0.824±0.006	0.829±0.002	**0.832**±0.004
	Barretts-short-seg	0.604±0.048	0.538±0.033	0.367±0.050	0.532±0.075	**0.674**±0.018	0.651±0.006	**0.658**±0.003	0.799±0.036	0.810±0.034	0.661±0.034	0.480±0.107	0.835±0.021	**0.895**±0.003	**0.906**±0.002
	Esophagitis-a	0.569±0.051	0.536±0.040	0.518±0.063	0.491±0.084	**0.778**±0.020	**0.760**±0.006	0.758±0.001	0.844±0.014	0.815±0.022	0.820±0.034	0.646±0.036	**0.881**±0.035	**0.878**±0.021	**0.878**±0.003
	Esophagitis-b-d	0.536±0.033	0.505±0.039	0.358±0.039	0.457±0.086	0.577±0.025	**0.622**±0.014	**0.652**±0.002	0.810±0.015	0.754±0.073	0.611±0.017	0.621±0.042	**0.837**±0.009	0.815±0.010	**0.841**±0.002
	Mean	0.595±0.023	0.571±0.004	0.406±0.018	0.480±0.044	0.700±0.009	**0.706**±0.007	**0.715**±0.004	0.822±0.019	0.786±0.021	0.698±0.021	0.571±0.014	0.844±0.009	**0.854**±0.004	**0.864**±0.002

10. Detailed Class-level AUC Results under Hard Setting

To evaluate the performance of the DPDL framework in detecting emerging anomaly classes, we conducted experiments under challenging settings and provided detailed results on six multi-subset datasets, including per-class anomaly performance, as shown in Tab. 6. Overall, the DPDL model achieved the highest AUC scores across both $M = 1$ and $M = 10$ settings.

11. The Algorithm of DPDL

Algorithm 1 Distribution Prototype Diffusion Learning

Input: Input $\mathcal{X} = \{(\mathrm{x}_i, y_i)\}$, C, ϵ, κ
for $epoch = 1$ to n **do**
  Extract features $\mathcal{F} \overset{\text{feature}}{\longleftarrow} \mathcal{X}$
  Distribution of normal samples transform $\mathcal{P}_{\text{MGP}} \overset{\text{bridge}}{\longleftarrow} P(\mathcal{F})$
  Distribution Prototype Learning $\mathcal{L}_{\text{DPL}} = \mathcal{L}^{\mathrm{n}}_{\text{DPL}} + \mathcal{L}^{\mathrm{a}}_{\text{DPL}}$
  Dispersion Feature Learning $\mathcal{L}_{\text{DFL}}$
  Sample $\mathbf{x}_i \sim \mathcal{X}$, $\mathbf{e}_{c^*} \sim \mathcal{P}_{\text{MGP}}$
  Calculate scores $S_{\mathrm{a}} \leftarrow \mathsf{M}_{\mathrm{a}}, S_{\mathrm{n}} \leftarrow \mathsf{M}_{\mathrm{n}}, S_{\mathrm{r}} \leftarrow \mathsf{M}_{\mathrm{r}}$
end for
Output : Anomaly score $S \leftarrow S_{\mathrm{r}} + S_{\mathrm{a}} - S_{\mathrm{n}}$

12. Derivation of Eqns. (13) and (14)

Following [10, 17], we use Eqns. (8) and (12) to derive Eqn. (13) as follows:

$$
\begin{aligned}
&\pi(\psi(\mathbf{x}_i^{\mathrm{n}})|\mathbf{x}_i^{\mathrm{n}}) = \frac{1}{\varpi(\mathbf{x}_i^{\mathrm{n}})}\exp(\frac{\langle \mathbf{x}_i^{\mathrm{n}}, \psi(\mathbf{x}_i^{\mathrm{n}})\rangle}{\epsilon})\sum_{c=1}^{C}\alpha_c\mathcal{N}(\psi(\mathbf{x}_i^{\mathrm{n}});\boldsymbol{\mu}_c,\boldsymbol{\sigma}_c)\\
&= \frac{1}{\varpi(\mathbf{x}_i^{\mathrm{n}})}\sum_{c=1}^{C}\alpha_c(2\pi)^{-D/2}|\boldsymbol{\sigma}_c|^{-1/2}\exp(\frac{\langle \mathbf{x}_i^{\mathrm{n}}, \psi(\mathbf{x}_i^{\mathrm{n}})\rangle}{\epsilon})\exp(-\frac{1}{2}(\psi(\mathbf{x}_i^{\mathrm{n}})))^{\top}\boldsymbol{\sigma}_c^{-1}(\psi(\mathbf{x}_i^{\mathrm{n}})-\boldsymbol{\mu}_c))\\
&= \frac{1}{\varpi(\mathbf{x}_i^{\mathrm{n}})}\sum_{c=1}^{C}\alpha_c(2\pi)^{-D/2}|\boldsymbol{\sigma}_c|^{-1/2}\exp(\frac{1}{2\epsilon}(2\mathbf{x}_i^{\mathrm{n}\top}\psi(\mathbf{x}_i^{\mathrm{n}})-\psi(\mathbf{x}_i^{\mathrm{n}})^{\top}\epsilon\boldsymbol{\sigma}_c^{-1}\psi(\mathbf{x}_i^{\mathrm{n}})^{\top}+2\boldsymbol{\mu}_c^{\top}\epsilon\boldsymbol{\sigma}_c^{-1}\psi(\mathbf{x}_i^{\mathrm{n}})-\boldsymbol{\mu}_c^{\top}\epsilon\boldsymbol{\sigma}_c^{-1}\boldsymbol{\mu}_c))\\
&= \frac{1}{\varpi(\mathbf{x}_i^{\mathrm{n}})}\sum_{c=1}^{C}\alpha_c(2\pi)^{-D/2}|\boldsymbol{\sigma}_c|^{-1/2}\exp(\frac{1}{2\epsilon}(-\psi(\mathbf{x}_i^{\mathrm{n}})^{\top}\epsilon\boldsymbol{\sigma}_c^{-1}\psi(\mathbf{x}_i^{\mathrm{n}})^{\top}+2\underbrace{(\frac{1}{\epsilon}\boldsymbol{\sigma}_c\mathbf{x}_i^{\mathrm{n}}+\boldsymbol{\mu}_c)^{\top}}_{\widetilde{\boldsymbol{\mu}}_c(\mathbf{x}_i^{\mathrm{n}})}\epsilon\boldsymbol{\sigma}_c^{-1}\psi(\mathbf{x}_i^{\mathrm{n}})-\boldsymbol{\mu}_c^{\top}\epsilon\boldsymbol{\sigma}_c^{-1}\boldsymbol{\mu}_c))\\
&= \frac{1}{\varpi(\mathbf{x}_i^{\mathrm{n}})}\sum_{c=1}^{C}\alpha_c(2\pi)^{-D/2}|\boldsymbol{\sigma}_c|^{-1/2}\exp(-\frac{1}{2\epsilon}(\psi(\mathbf{x}_i^{\mathrm{n}})-\widetilde{\boldsymbol{\mu}}_c(\mathbf{x}_i^{\mathrm{n}})^{\top}\epsilon\boldsymbol{\sigma}_c^{-1}(\psi(\mathbf{x}_i^{\mathrm{n}})-\widetilde{\boldsymbol{\mu}}_c(\mathbf{x}_i^{\mathrm{n}}))))\\
&\exp(\frac{1}{2\epsilon}(-\boldsymbol{\mu}_c^{\top}\epsilon\boldsymbol{\sigma}_c^{-1}\boldsymbol{\mu}_c+\widetilde{\boldsymbol{\mu}}_c^{\top}(\mathbf{x}_i^{\mathrm{n}})\epsilon\boldsymbol{\sigma}_c^{-1}\widetilde{\boldsymbol{\mu}}_c^{\top}(\mathbf{x}_i^{\mathrm{n}}))\\
&= \frac{1}{\varpi(\mathbf{x}_i^{\mathrm{n}})}\sum_{c=1}^{C}\alpha_c\exp(\frac{\boldsymbol{\mu}_c^{\top}\epsilon\boldsymbol{\sigma}_c^{-1}\boldsymbol{\mu}_c+(\widetilde{\boldsymbol{\mu}}_c^{\top}\epsilon^{-1}\boldsymbol{\sigma}_c+\widetilde{\boldsymbol{\mu}}_c)^{\top}(\mathbf{x}_i^{\mathrm{n}})\epsilon\boldsymbol{\sigma}_c^{-1}(\widetilde{\boldsymbol{\mu}}_c^{\top}\epsilon^{-1}\boldsymbol{\sigma}_c+\widetilde{\boldsymbol{\mu}}_c)(\mathbf{x}_i^{\mathrm{n}})}{2\epsilon})\mathcal{N}(\psi(\mathbf{x}_i^{\mathrm{n}});\widetilde{\boldsymbol{\mu}}_c(\mathbf{x}_i^{\mathrm{n}}),\boldsymbol{\sigma}_c)\\
&= \frac{1}{\varpi(\mathbf{x}_i^{\mathrm{n}})}\sum_{c=1}^{C}\underbrace{\alpha_c\exp(\frac{1}{2\epsilon^2}(\mathbf{x}_i^{\mathrm{n}})^{\top}\boldsymbol{\sigma}_c\mathbf{x}_i^{\mathrm{n}}+\frac{1}{\epsilon}(\widetilde{\boldsymbol{\mu}}_c)^{\top}(\mathbf{x}_i^{\mathrm{n}}))}_{\widetilde{\alpha}_c(\mathbf{x}_i^{\mathrm{n}})}\mathcal{N}(\psi(\mathbf{x}_i^{\mathrm{n}});\widetilde{\boldsymbol{\mu}}_c(\mathbf{x}_i^{\mathrm{n}}),\boldsymbol{\sigma}_c)\\
&= \widetilde{\eta}(\mathbf{x}_i^{\mathrm{n}})\sum_{c=1}^{C}\widetilde{\alpha}_c(\mathbf{x}_i^{\mathrm{n}})\mathcal{N}(\psi(\mathbf{x}_i^{\mathrm{n}});\widetilde{\boldsymbol{\mu}}_c(\mathbf{x}_i^{\mathrm{n}}),\boldsymbol{\sigma}_c)
\end{aligned}
$$

where $\widetilde{\eta}(\mathbf{x}_i^{\mathrm{n}}) = \frac{1}{\varpi(\mathbf{x}_i^{\mathrm{n}})} = \frac{1}{\sum_{c=1}^{C}\widetilde{\alpha}_c(\mathbf{x}_i^{\mathrm{n}})}$.

According to Eqn. (4), we derive Eqn. (14) which is similar to LightSB [10, 17] as:

$$
\begin{aligned}
g(\mathbf{x}_i^{\text{n}}, t) &= \epsilon \nabla_{\mathbf{x}_i^{\text{n}}} \log \int_{\mathbb{R}^D} \mathcal{N}(\psi(\mathbf{x}_i^{\text{n}}) | \mathbf{x}_i^{\text{n}}, (1-t)\epsilon \mathbf{I}) \exp(\frac{\|\psi(\mathbf{x}_i^{\text{n}})\|^2}{2\epsilon}) \phi_1(\psi(\mathbf{x}_i^{\text{n}})) d\psi(\mathbf{x}_i^{\text{n}}) \\
&= \epsilon \nabla_{\mathbf{x}_i^{\text{n}}} \log \int_{\mathbb{R}^D} \mathcal{N}(\psi(\mathbf{x}_i^{\text{n}}) | \mathbf{x}_i^{\text{n}}, (1-t)\epsilon \mathbf{I}) \exp(\frac{\|\psi(\mathbf{x}_i^{\text{n}})\|^2}{2\epsilon}) \sum_{c=1}^{C} \alpha_c \mathcal{N}(\psi(\mathbf{x}_i^{\text{n}}); \widetilde{\boldsymbol{\mu}}_c, \boldsymbol{\sigma}_c) d\psi(\mathbf{x}_i^{\text{n}}) \\
&= \epsilon \nabla_{\mathbf{x}_i^{\text{n}}} \log((2\pi)^{-\frac{D}{2}} |(1-t)\epsilon \mathbf{I}|^{-\frac{1}{2}} \sum_{c=1}^{C} \{\alpha_c |\boldsymbol{\sigma}_c|^{-\frac{1}{2}} \\
&\int_{\mathbb{R}^D} \exp(-\frac{(\psi(\mathbf{x}_i^{\text{n}}) - x_i^{\text{n}})^\top (\psi(\mathbf{x}_i^{\text{n}}) - \mathbf{x}_i^{\text{n}})}{2\epsilon(1-t)} - \frac{(\psi(\mathbf{x}_i^{\text{n}}) - \widetilde{\boldsymbol{\mu}}_c) \epsilon \boldsymbol{\sigma}_c^{-1} (\psi(\mathbf{x}_i^{\text{n}}) - \widetilde{\boldsymbol{\mu}}_c)}{2\epsilon} + \frac{\psi(\mathbf{x}_i^{\text{n}})^\top \psi(\mathbf{x}_i^{\text{n}})}{2\epsilon}) \}) \\
&= \epsilon \nabla_{\mathbf{x}_i^{\text{n}}} \log(\exp(-\frac{\mathbf{x}_i^{\text{n}\top} \mathbf{x}_i^{\text{n}}}{2\epsilon(1-t)}) \sum_{c=1}^{C} \{\alpha_c |\boldsymbol{\sigma}_c|^{-\frac{1}{2}} \exp(-\frac{\widetilde{\boldsymbol{\mu}}_c^\top \epsilon \boldsymbol{\sigma}^{-1} \widetilde{\boldsymbol{\mu}}_c}{2\epsilon}) \}) \\
&\int_{\mathbb{R}^D} \exp(-\frac{1}{2} [\psi(\mathbf{x}_i^{\text{n}})^\top \underbrace{(\frac{t}{\epsilon(1-t)} \mathbf{I} + \boldsymbol{\sigma}_c^{-1})}_{\boldsymbol{\Sigma}_c^t} \psi(\mathbf{x}_i^{\text{n}})] + \underbrace{[\frac{1}{\epsilon(1-t)} \mathbf{x}_i^{\text{n}} + \boldsymbol{\sigma}_c^{-1} \widetilde{\boldsymbol{\mu}}_c]^\top}_{h_c(\mathbf{x}_i^{\text{n}}, t)} \psi(\mathbf{x}_i^{\text{n}}) d\psi(\mathbf{x}_i^{\text{n}})) \\
&= \epsilon \nabla_{\mathbf{x}_i^{\text{n}}} \log(\underbrace{(2\pi)^{-\frac{D}{2}} \exp(-\frac{\mathbf{x}_i^{\text{n}\top} \mathbf{x}_i^{\text{n}}}{2\epsilon(1-t)}) \sum_{c=1}^{C} \{\alpha_c (2\pi)^{-\frac{D}{2}} |\boldsymbol{\sigma}_c|^{-\frac{1}{2}} \exp(-\frac{\widetilde{\boldsymbol{\mu}}_c^\top \epsilon \boldsymbol{\sigma}_c^{-1} \widetilde{\boldsymbol{\mu}}_c}{2\epsilon})}_{\mathcal{N}(\mathbf{x}_i^{\text{n}} | 0, \epsilon(1-t))\mathbf{I}} \\
&\underbrace{\sum_{c=1}^{C} (\alpha_c \mathcal{N}(\widetilde{\boldsymbol{\mu}}_c(\mathbf{x}_i^{\text{n}}) | \mathbf{0}, \boldsymbol{\sigma}_c) \mathcal{N}(h_c(\mathbf{x}_i^{\text{n}}, t) | 0, \boldsymbol{\Sigma}_c^t))\}}_{\rho_{\text{MGP}}(\mathbf{x}_i^{\text{n}})}) \\
&= \epsilon \rho_{\text{MGP}}(\mathbf{x}_i^{\text{n}}) \nabla_{\mathbf{x}_i^{\text{n}}} \log(\mathcal{N}(\mathbf{x}_i^{\text{n}} | 0, \epsilon(1-t))\mathbf{I})
\end{aligned}
$$