Hybrid AI–Physical Modeling for Penetration Bias Correction in X-band InSAR DEMs: A Greenland Case Study

Supplementary Material

7. Definitions of Vertical Wavenumber and HoA

In our analysis of InSAR data, two key parameters describe how heights (i.e. depths) are calculated from the observed or modelled interferometric phase: the vertical wavenumber κ_z and the Height of Ambiguity (HoA). Physically, κ_z depends on the dielectric properties of snow and ice and has to account for the refraction and change in wave propagation speed in the medium, which leads to a vertical wavenumber in the (snow and ice) volume $\kappa_{z_{\mathrm{Vol}}}$. However, TanDEM-X DEMs are produced under the *free-space* assumption, so the penetration bias correction must also adopt the *free-space* vertical wavenumber κ_z as [28]

$$\kappa_z = \frac{4\pi}{\lambda}\frac{\Delta\theta_i}{\sin\theta_i}, \tag{S1}$$

where λ denotes the radar wavelength, θ_i is the incidence angle, and $\Delta\theta_i$ is the baseline-induced angular shift.

The Height of Ambiguity (HoA), which quantifies the phase-to-height sensitivity by representing the elevation difference corresponding to a full 2π interferometric phase cycle, is defined as

$$\mathrm{HoA} = \frac{2\pi}{\kappa_z}. \tag{S2}$$

8. Metric Definitions

We evaluate model performance using standard metrics that assess bias, error magnitude, and goodness-of-fit. The following equations define these metrics:

$$\mathrm{ME} = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i), \tag{S3}$$

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|\hat{y}_i - y_i\right|, \tag{S4}$$

$$\mathrm{MAPE} = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{\hat{y}_i - y_i}{y_i}\right|, \tag{S5}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}, \tag{S6}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}, \tag{S7}$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} e_i, \tag{S8}$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(e_i - \mu)^2}, \tag{S9}$$

where $\hat{y}_i$ represents the predicted values, y_i denotes the corresponding reference values, and $\bar{y}$ is the mean of the reference values. Additionally, $e_i = h_{\mathrm{InSAR}} - h_{\mathrm{ref}}$ denotes the DEM error before or after applying bias correction. The mean error μ provides insight into systematic bias, while the standard deviation σ captures the variability of the DEM error.

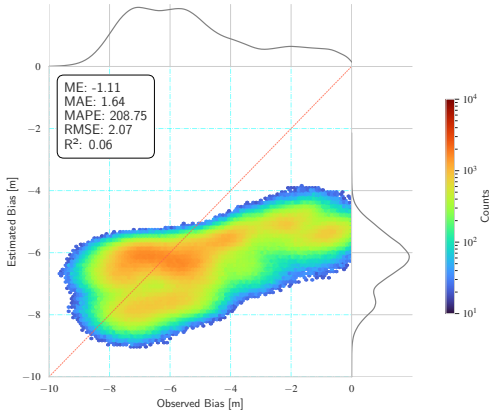


Figure 11. 2D histograms of estimated versus observed penetration bias for the Physical (UV) model

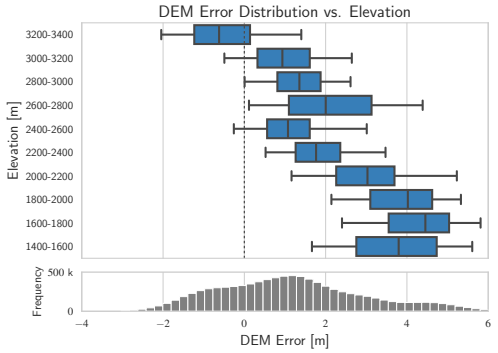
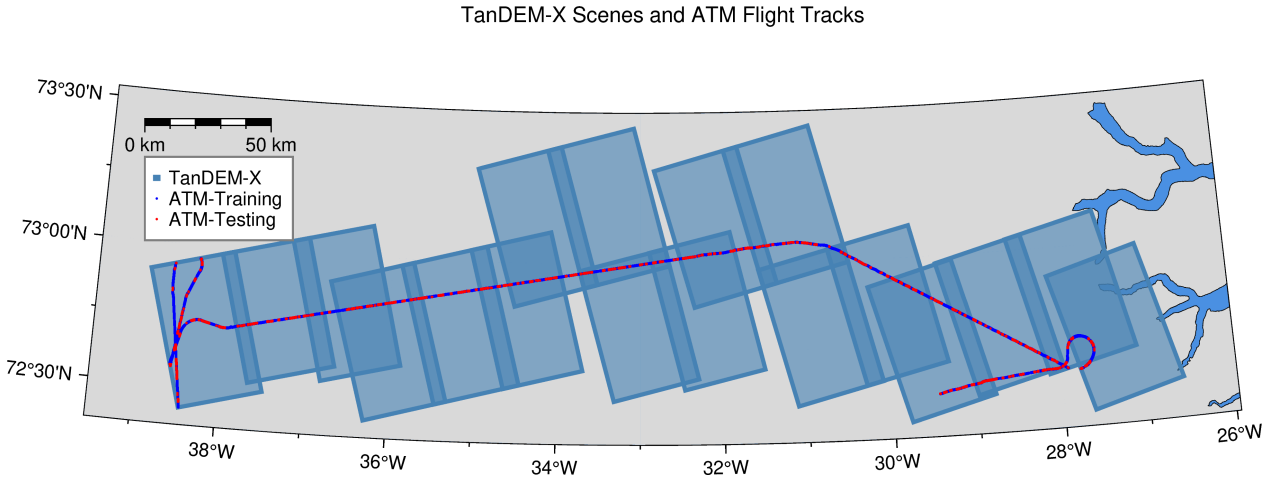
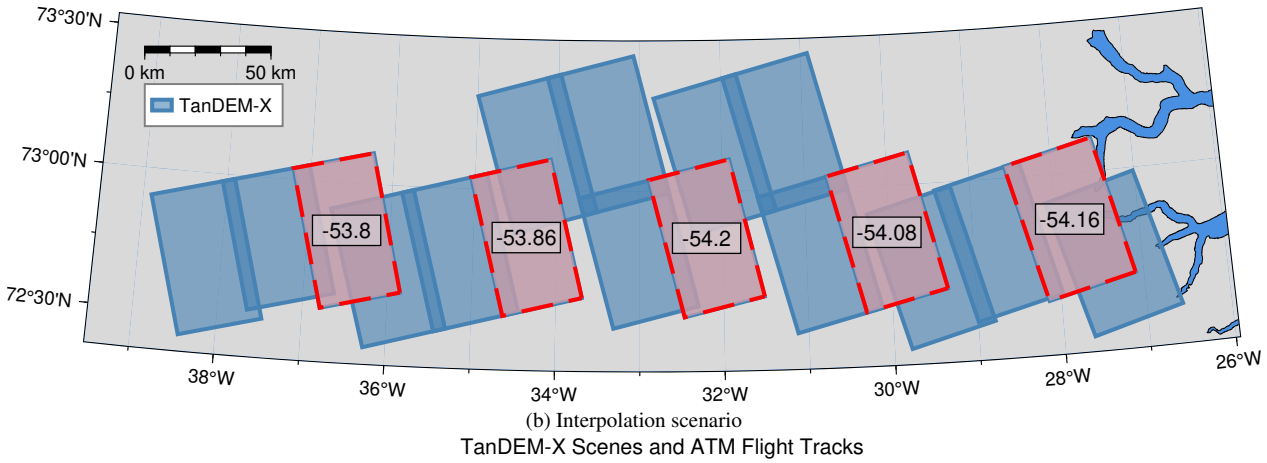


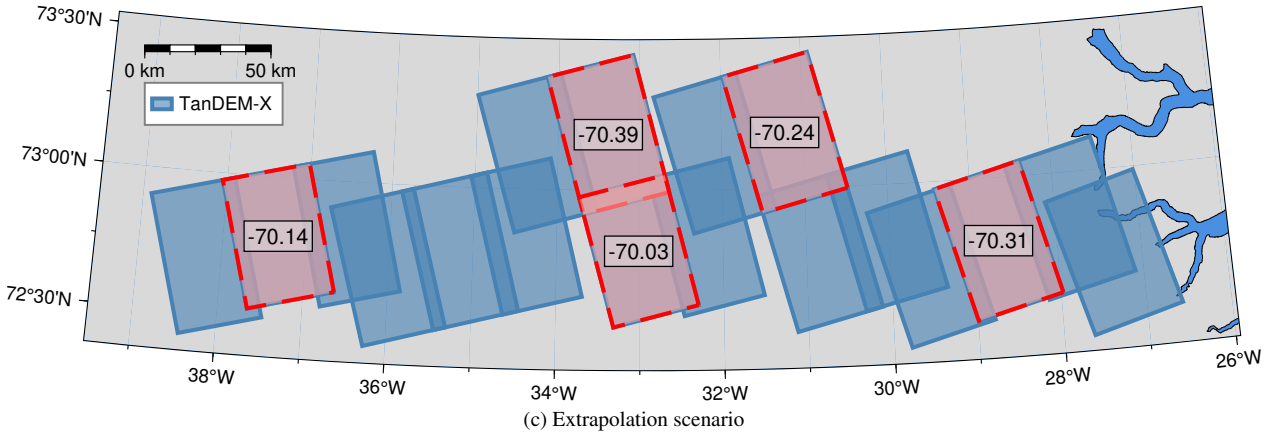
Figure 12. Corrected DEM "using physical (UV) model" error distribution across elevation bins, computed using ATM LiDAR as reference.



(a) All scenario



(b) Interpolation scenario



(c) Extrapolation scenario

Figure 13. Overview of the TanDEM-X scenes (blue) and ATM flight tracks (red/blue) used for each training scenario. The ATM flight lines are split into training (blue) and testing (red) segments, ensuring coverage of different surface conditions for model evaluation. Scenes outlined in red are excluded from training under the specified HoA range. (a) *All*: uses every scene, (b) *Interpolation*: excludes HoA in [50, 60] m, (c) *Extrapolation*: excludes HoA above 70 m.

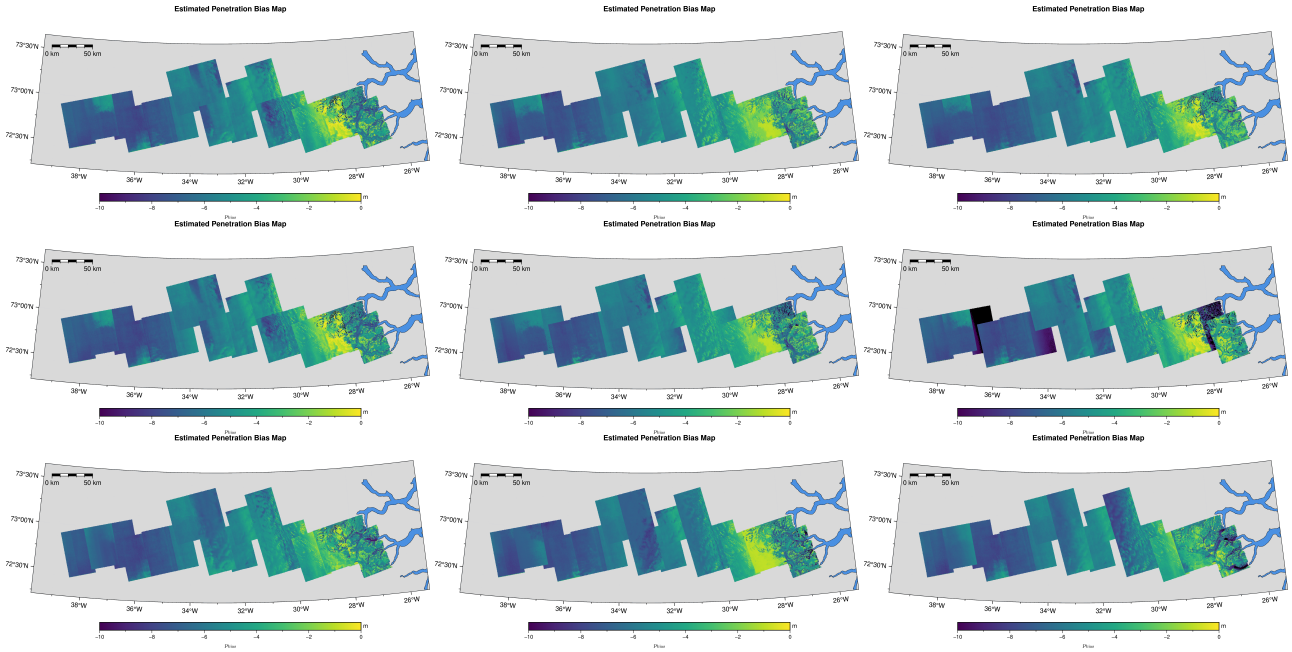


Figure 14. Estimated penetration bias maps for the study region under three HoA training scenarios (rows) and three modeling approaches (columns). *Rows* (top to bottom): *All*, *Interpolation*, and *Extrapolation* scenarios. *Columns* (left to right): *Exponential*, *Weibull*, and *MLP* models. Each panel shows the spatial distribution of the predicted bias (in meters), with blueish colors indicating deeper penetration bias.

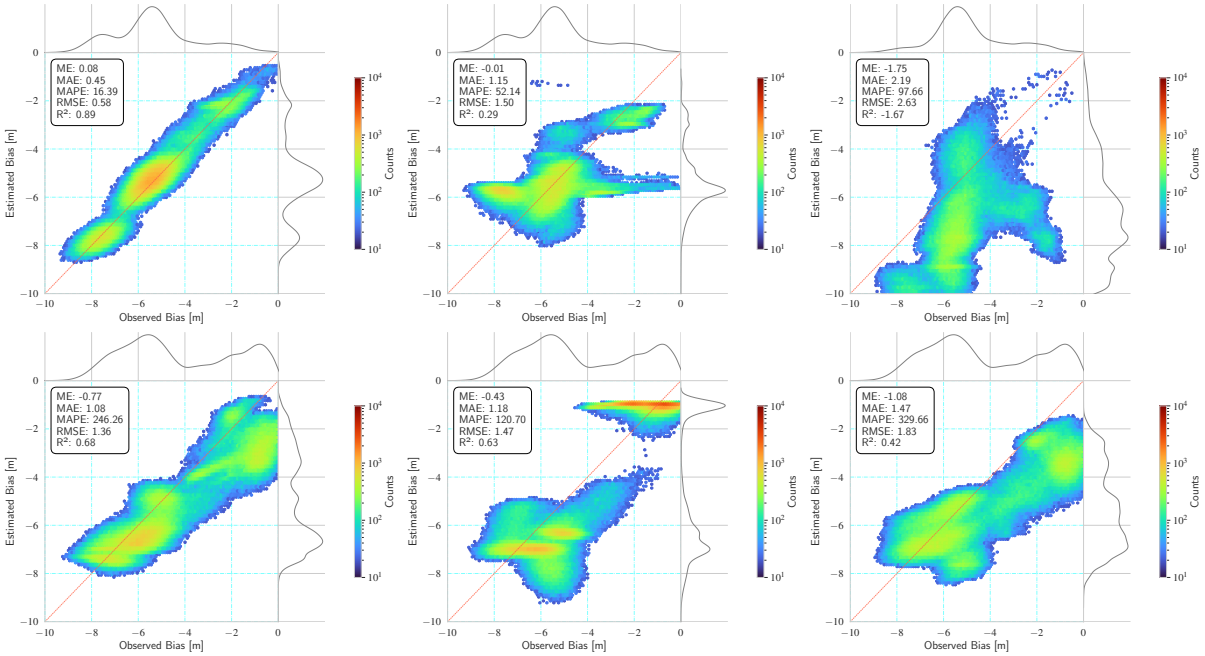


Figure 15. Comparison of model estimations under different evaluation conditions showing the results only over the excluded scenes during training (unseen HoA scenes). The **columns** represent different modeling approaches: (Left) Hybrid Model with an Exponential Profile, (Middle) Hybrid Model with a Weibull Profile, (Right) Pure Machine Learning (ML) model using MLP. The **rows** indicate different training scenarios: (Left) Hybrid Model with an Exponential Profile, (Middle) Hybrid Model with a Weibull Profile, (Right) Pure Machine Learning (ML) model using MLP. The **rows** indicate different training scenarios: (Top) *Interpolation* experiment; (Bottom) *Extrapolation* experiment.

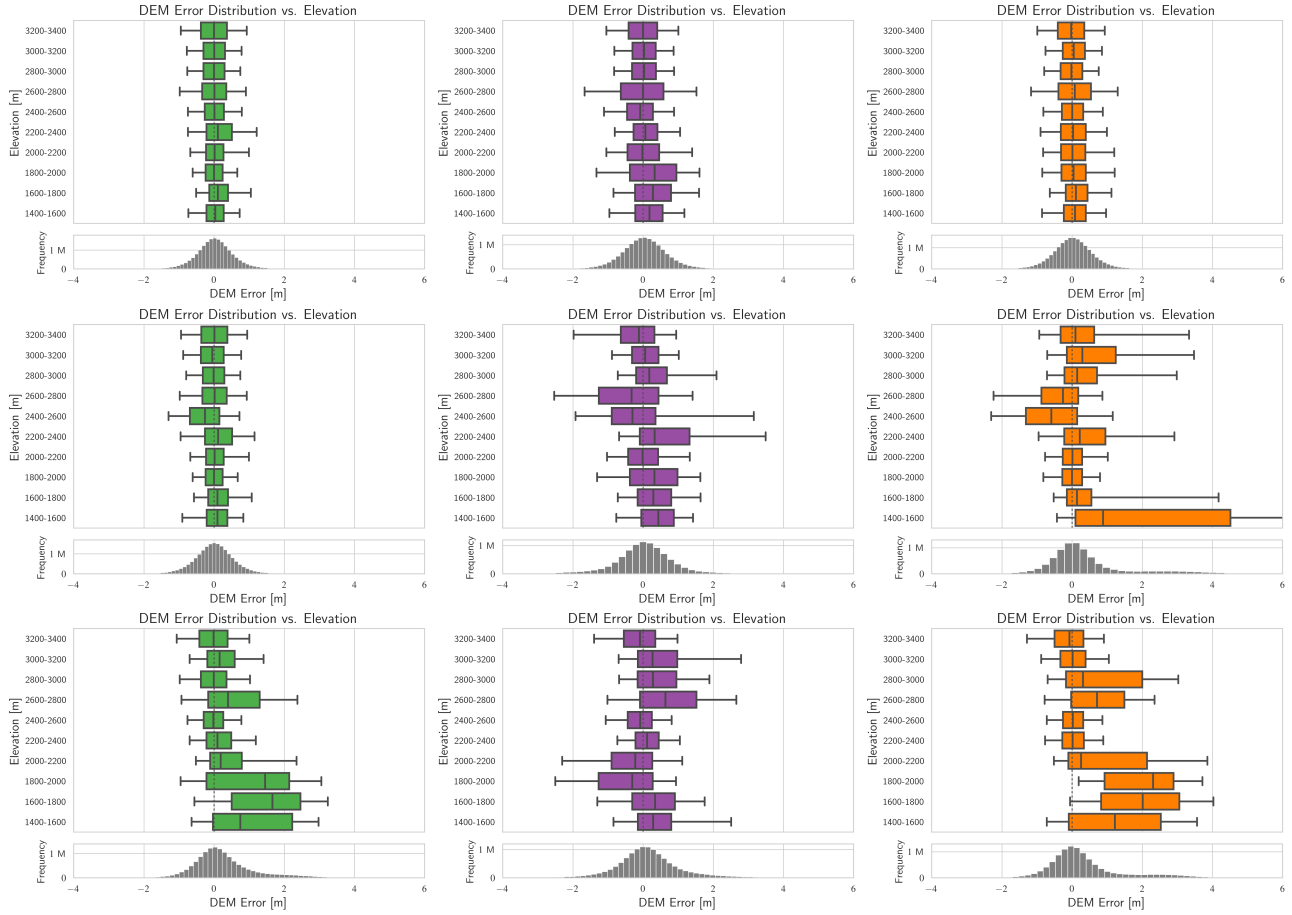


Figure 16. Errors of the corrected DEMs, by compensating the original InSAR DEMs with the estimated penetration bias using different models and training scenarios. Shown as error distribution across elevation bins, computed using ATM LiDAR as reference under three HoA training scenarios (rows) and three modeling approaches (columns). *Rows* (top to bottom): *All*, *Interpolation*, and *Extrapolation* scenarios. *Columns* (left to right): *Exponential*, *Weibull*, and *MLP* models.