Food Degradation Analysis Using Multimodal Fuzzy Clustering

Supplementary Material

6. Texture features

The Gray-Level Co-occurrence Matrix (GLCM) $p(i,j)$ is defined as:

$$p(i,j) = \frac{\text{number of occurrences of gray levels } i \text{ and } j \text{ at distance } d \text{ and angle } \theta}{\text{total number of occurrences}}$$

where i and j are gray levels, d is the distance between pixels, and θ is the angle.

As defined in [11], let $p(i,j)$ be the (i,j)-th entry in a normalized gray-tone spatial dependence matrix, $p_x(i)$ be the i-th entry in the marginal-probability matrix obtained by summing the rows of $p(i,j)$, N_g the number of distinct gray levels in the quantized image. $\sum_i$ and $\sum_j$, $\sum_{i=1}^{N_g}$ and $\sum_{i=1}^{N_g}$, respectively, then define

$$\begin{aligned} p_y(j) &= \sum_{i=1}^{N_g} p(i,j) \\ p_{x+y}(k) &= \sum_{\substack{i=1 \\ i+j=k}}^{N_g} \sum_{j=1}^{N_g} p(i,j), \qquad k = 2,3,\cdots 2N_g \\ p_{x-y}(k) &= \sum_{\substack{i=1 \\ |i-j|=k}}^{N_g} \sum_{j=1}^{N_g} p(i,j), \qquad k = 0,\cdots N_g - 1 \end{aligned} \tag{1}$$

Based on the GLCM the following texture features are introduced:

Angular Second Moment (Energy)

$$f_1 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p(i,j)^2 \tag{2}$$

Contrast

$$f_2 = \sum_{n=0}^{N_g-1} n^2 \left\{ \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p(i,j), |i-j| = n \right\} \tag{3}$$

Correlation

$$f_3 = \frac{\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (ij) p(i,j) - \mu_x \mu_y}{\sigma_x \sigma_y} \tag{4}$$

where μ_x, μ_y, σ_x, and σ_y are the means and standard deviations of the marginal distributions associated with $p(i,j)$.

Sum of Squares (Variance)

$$f_4 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i-\mu)^2 p(i,j) \tag{5}$$

Inverse Difference Moment (Homogeneity)

$$f_5 = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{1}{1+(i-j)^2} p(i,j) \tag{6}$$

Sum Average

$$f_6 = \sum_{i=2}^{2N_g} i p_{x+y}(i) \tag{7}$$

where $P_{x+y}(k)$ is the probability of co-occurrence matrix coordinates summing to k. Sum Variance

$$f_7 = \sum_{i=2}^{2N_g} (i - f_8)^2 \, p_{x+y}(i) \tag{8}$$

Sum Entropy

$$f_8 = -\sum_{k=2}^{2N_g} P_{x+y}(k) \log(P_{x+y}(k)) \tag{9}$$

Entropy

$$f_9 = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p(i,j) \log(p(i,j)) \tag{10}$$

Difference Variance

$$f_{10} = \text{variance of } p_{x-y} \tag{11}$$

Difference Entropy

$$f_{11} = -\sum_{i=0}^{N_g-1} P_{x-y}(i) \log(p_{x-y}(i)) \tag{12}$$

Information Measures of Correlation

$$f_{12} = \frac{HXY - HXY1}{\max\{HX, HY\}} \tag{13}$$

$$f_{13} = \sqrt{1 - \exp[-2(HXY2 - HXY)]} \tag{14}$$

Where HXY is the entropy of $p(i,j)$, HX and HY are entropies of the marginal distributions, and HXY1 and HXY2 are

$$HXY = -\sum_i \sum_j p(i,j)\log(p(i,j))$$
$$HXY1 = -\sum_k p(i,j)\log(p_x(i)p_y(j))$$
$$HXY2 = -\sum_k p_x(i)p_y(j)\log(p_x(i)p_y(j))$$

Maximal Correlation Coefficient

$$f_{13} = \sqrt{\text{second largest eigenvalue of Q}} \tag{15}$$

Where Q is

$$Q(i,j) = \sum_k \frac{p(i,k)\ p(j,k)}{p_x(i)\ p_y(k)} \tag{16}$$

7. Fuzzy Clustering Exploration

This section presents the full results of the Fuzzy Cmeans exploration for the Orange, Apple and Banana cases considering 2 to 10 clustering solutions with the Partition Coefficient (PC), Partition Entropy (PE) and Xie-Beni index as cluster quality measures. There is a separate table for each case (Tables 3, 4, and 5 for Orange, Apple and Banana, respectively), with the best values of the cluster quality indices in bold. Note that even though according to these measures the best configurations are obtained with two clusters, it does not imply that the classes associated with the food states are binary. The time interval between the hinges of the membership function determines a transition zone and its length indicates the speed at which the state transition for each case takes place.

8. Multidimensional Scaling

Multidimensional Scaling (MDS) is a statistical technique for visualizing pairwise dissimilarities between objects in a low-dimensional Euclidean space [34]. Given n objects with pairwise dissimilarities δ_{ij}, MDS seeks coordinates $\mathbf{y}_1, \dots, \mathbf{y}_n \in \mathbb{R}^k$ that minimize the *stress* function:

$$S(\mathbf{Y}) = \sqrt{\sum_{i<j} w_{ij}(d_{ij}(\mathbf{Y}) - \delta_{ij})^2} \tag{17}$$

where $d_{ij}(\mathbf{Y}) = \|\mathbf{y}_i - \mathbf{y}_j\|$ represents the Euclidean distance between points i and j, and w_{ij} are optional weights [16].

8.1. Classical Metric MDS

For Euclidean distances, the solution derives from the Gram matrix $\mathbf{G} = \mathbf{Y}\mathbf{Y}^\top$. Let $\mathbf{D}^{(2)} = (d_{ij}^2)$ be the squared distance matrix. The centered inner product matrix is computed as:

$$\mathbf{G} = -\frac{1}{2}\mathbf{J}\mathbf{D}^{(2)}\mathbf{J} \tag{18}$$

ORANGE			
	Color (Kolmogorov-Smirnov D)		
cluster	PC	PE	Xie-Beni index
2	**0.9425**	**0.1021**	**0.0246**
3	0.9414	0.1287	0.0287
4	0.9253	0.1729	0.0277
5	0.9080	0.2185	0.0520
6	0.8886	0.2674	0.0839
7	0.8003	0.4044	0.1816
8	0.7781	0.4532	0.3028
9	0.7892	0.4443	0.2046
10	0.7538	0.5144	0.2664
	Texture (Haralick descriptors)		
cluster	PC	PE	Xie-Beni index
2	**0.8927**	**0.1898**	**0.0610**
3	0.8245	0.3237	0.0959
4	0.8203	0.3612	0.2794
5	0.8417	0.3449	0.1364
6	0.8315	0.3807	0.1024
7	0.8175	0.4218	0.0794
8	0.7622	0.5289	0.5266
9	0.7911	0.4858	0.1951
10	0.7500	0.5660	0.3080
	Shape (simple descriptors)		
cluster	PC	PE	Xie-Beni index
2	**0.9151**	**0.1554**	**0.0326**
3	0.8407	0.3000	0.0841
4	0.7974	0.3921	0.0683
5	0.8006	0.4094	0.1092
6	0.7784	0.4667	0.0844
7	0.7216	0.5718	0.2873
8	0.6927	0.6393	0.3270
9	0.6603	0.7091	0.2980
10	0.6414	0.7562	0.3290
	Shape (Zernike)		
cluster	PC	PE	Xie-Beni index
2	**0.9199**	**0.1369**	**0.0266**
3	0.9029	0.1868	0.0374
4	0.8730	0.2546	0.0471
5	0.8437	0.3139	0.1036
6	0.8404	0.3319	0.0693
7	0.8269	0.3668	0.0544
8	0.8123	0.3966	0.0635
9	0.7842	0.4477	0.0937
10	0.7846	0.4555	0.1794

Table 3. Orange case. Fuzzy cMeans cluster quality measures using different visual properties for 2 to 10 cluster solutions (Best values in bold). PC: Partition coefficient, PE: Partition Entropy

APPLE			
	Color (Kolmogorov-Smirnov D)		
cluster	PC	PE	Xie-Beni index
2	**0.8537**	**0.2551**	**0.0977**
3	0.7409	0.4726	0.1475
4	0.7112	0.5620	0.2143
5	0.6935	0.6270	0.2241
6	0.6812	0.6767	0.1762
7	0.6633	0.7523	0.2093
8	0.6640	0.7556	0.1530
9	0.6700	0.7639	0.1240
10	0.6471	0.8247	0.4139
	Texture (Haralick descriptors)		
cluster	PC	PE	Xie-Beni index
2	**0.8767**	**0.2209**	**0.0796**
3	0.7816	0.3998	0.2220
4	0.7767	0.4554	0.1343
5	0.7446	0.5483	0.1226
6	0.6151	0.7677	1.3436
7	0.5955	0.8420	1.0561
8	0.5630	0.9299	0.7618
9	0.5430	0.9723	0.7359
10	0.5595	0.9824	0.5735
	Shape (simple descriptors)		
cluster	PC	PE	Xie-Beni index
2	**0.9408**	**0.1088**	**0.0212**
3	0.8934	0.2163	0.0420
4	0.7733	0.4182	0.2977
5	0.7585	0.4652	0.2246
6	0.7014	0.5946	0.2202
7	0.6463	0.7058	0.6539
8	0.6197	0.7803	0.5616
9	0.6081	0.8054	0.4501
10	0.5812	0.8918	0.3596
	Shape (Zernike)		
cluster	PC	PE	Xie-Beni index
2	**0.9069**	**0.1698**	**0.0367**
3	0.8467	0.2863	0.0990
4	0.8390	0.3126	0.0641
5	0.7780	0.4328	0.1734
6	0.7861	0.4315	0.1061
7	0.7785	0.4636	0.1321
8	0.7741	0.4774	0.1006
9	0.7726	0.4873	0.0832
10	0.7602	0.5167	0.1650

Table 4. Apple case. Fuzzy cMeans cluster quality measures using different visual properties for 2 to 10 cluster solutions (Best values in bold). PC: Partition coefficient, PE: Partition Entropy

BANANA			
	Color (Kolmogorov-Smirnov D)		
cluster	PC	PE	Xie-Beni index
2	**0.8992**	**0.1734**	**0.0537**
3	0.8131	0.3336	0.1438
4	0.8141	0.3649	0.0623
5	0.8310	0.3555	0.0748
6	0.8221	0.3864	0.0554
7	0.8107	0.4243	0.0597
8	0.7908	0.4731	0.1431
9	0.7656	0.5334	0.2190
10	0.7656	0.5363	0.2426
	Texture (Haralick descriptors)		
cluster	PC	PE	Xie-Beni index
2	**0.8794**	**0.2183**	**0.0566**
3	0.8139	0.3534	0.1891
4	0.8060	0.3967	0.0901
5	0.7906	0.4466	0.0796
6	0.7791	0.4912	0.0755
7	0.7104	0.6210	0.6171
8	0.7128	0.6317	0.5407
9	0.6936	0.6772	0.4736
10	0.7308	0.6401	0.1821
	Shape (simple descriptors)		
cluster	PC	PE	Xie-Beni index
2	**0.8671**	**0.2300**	**0.0622**
3	0.7907	0.3839	0.1000
4	0.7678	0.4478	0.0889
5	0.7508	0.5009	0.0682
6	0.7298	0.5550	0.1003
7	0.7313	0.5683	0.0746
8	0.7156	0.6103	0.1154
9	0.7178	0.6125	0.0854
10	0.6972	0.6613	0.1086
	Shape (Zernike)		
cluster	PC	PE	Xie-Beni index
2	**0.8656**	**0.2290**	**0.0648**
3	0.7955	0.3694	0.0700
4	0.7795	0.4261	0.0948
5	0.7168	0.5497	0.1234
6	0.7734	0.4701	0.0749
7	0.7446	0.5303	0.1676
8	0.7401	0.5458	0.1284
9	0.7409	0.5564	0.1061
10	0.7254	0.5936	0.1520

Table 5. Banana case. Fuzzy cMeans cluster quality measures using different visual properties for 2 to 10 cluster solutions (Best values in bold). PC: Partition coefficient, PE: Partition Entropy

where $\mathbf{J} = \mathbf{I}_n - \frac{1}{n}\mathbf{1}\mathbf{1}^\top$ is the centering matrix [6]. The spectral decomposition $\mathbf{G} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^\top$ yields the embedding coordinates:

$$\mathbf{Y} = \mathbf{U}_k \mathbf{\Lambda}_k^{1/2} \tag{19}$$

where $\mathbf{\Lambda}_k$ contains the k largest eigenvalues and $\mathbf{U}_k$ their corresponding eigenvectors.

8.2. Optimality Conditions

The MDS embedding preserves original distances through the relation:

$$d_{ij}^2 = \|\mathbf{y}_i\|^2 + \|\mathbf{y}_j\|^2 - 2\mathbf{y}_i^\top \mathbf{y}_j \tag{20}$$

with the centroid constraint $\sum_{i=1}^n \mathbf{y}_i = \mathbf{0}$ to ensure identifiability [7].

8.3. Non-Metric MDS

When only ordinal dissimilarities are available, non-metric MDS minimizes the stress (S_{nm}) through a monotonic transformation f:

$$S_{\text{nm}} = \sqrt{\frac{\sum_{i<j}(d_{ij} - f(\delta_{ij}))^2}{\sum_{i<j} d_{ij}^2}} \tag{21}$$

and uses isotonic regression to estimate f [28].

8.4. Algorithmic Complexity

The classical MDS solution requires $O(n^3)$ operations for eigendecomposition, making it computationally intensive for large datasets [8].