DBMSolver: A Training-free Diffusion Bridge Sampler for High-Quality Image-to-Image Translation

Supplementary Material

1. Diffusion Models

The reverse diffusion process is given by the PF ODE [1, 11]:

$$\mathrm{d}\boldsymbol{x}_t = \left[f(\boldsymbol{x}_t, t) - \frac{1}{2} g(t)^2 \nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}) \right] \mathrm{d}t, \tag{1}$$

where the marginal distribution of $\boldsymbol{x}_t$ at t is $p_t(\mathbf{x})$, and $\nabla_{\boldsymbol{x}_t} \log p_t(\mathbf{x})$ is its *score function*, which is learned by a neural network [4]. Furthermore, the drift and diffusion coefficients are:

$$f(\boldsymbol{x}_t, t) = \boldsymbol{x}_t \frac{\mathrm{d}}{\mathrm{d}t} \log \alpha_t, \text{ and } g(t)^2 = -2\sigma_t^2 \frac{\mathrm{d}}{\mathrm{d}t} \log \left(\frac{\alpha_t}{\sigma_t} \right),$$

where $\alpha_t := \alpha(t)$ and $\sigma_t := \sigma(t)$, for time $t \in [0, T]$ (where $T > 0$).
Different formulations of α_t and σ_t give rise to different formulations for the diffusion process. Prior works hand-design these to obtain the *variance-preserving* (VP) [11, 15], *variance-exploding* (VE) [6], and *TrigFlow* [9] formulations. We contrast the design choices of such diffusion formulations in Table 1.

Table 1. Design choices for widely-used diffusion formulations. DDBM [15] mainly utilizes the VP and VE bridges, while TrigFlow is utilized by [9].

Formulation	α_t	σ_t	$f(\boldsymbol{x}_t, t)$	$g(t)^2$	$\text{SNR}_t = \alpha_t^2/\sigma_t^2$	Domain of t
VP [11, 15]	$e^{-(0.5t^2+0.05t)}$	$\sqrt{1-e^{-(t^2+0.1t)}}$	$-(t+0.05)\boldsymbol{x}_t$	$2t+0.1$	$1/(e^{(t^2+0.1t)}-1)$	$[0.0001, 1]$
VE [6]	1	t	$\mathbf{0}$	$2t$	$1/t^2$	$[0.002, 80]$
TrigFlow [9]	$\cos(t)$	$\sin(t)$	$-\tan(t)\,\boldsymbol{x}_t$	$2\tan(t)$	$\cot^2(t)$	$[0, \pi/2]$

2. Proofs & Derivations

2.1. Proof of Proposition 1 of the main paper

Given a well-trained DBM $\boldsymbol{D}_\theta(\cdot)$ that approximates data sample $\boldsymbol{x}_0$, we can simplify Equation 2 of the main paper as:

$$
\begin{aligned}
\mathrm{d}\boldsymbol{x}_t &= \big(f(\boldsymbol{x}_t, t) - g(t)^2 [\nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}_t \mid \boldsymbol{x}_T) - \nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}_T \mid \boldsymbol{x}_t)] \big)\,\mathrm{d}t + g(t)\,\mathrm{d}\mathbf{w}_t \\
&= \left(\boldsymbol{x}_t \frac{\mathrm{d}\log\alpha_t}{\mathrm{d}t} + 2\sigma_t^2 \frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} [\nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}_t \mid \boldsymbol{x}_T) - \nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}_T \mid \boldsymbol{x}_t)] \right) \mathrm{d}t \\
&\quad + \sigma_t \sqrt{-2\frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t}}\,\mathrm{d}\mathbf{w}_t \\
&= \quad \boldsymbol{x}_t \frac{\mathrm{d}\log\alpha_t}{\mathrm{d}t}\mathrm{d}t - 2\sigma_t^2 \frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \left[\frac{\frac{\alpha_t}{\alpha_T}\boldsymbol{x}_T - \boldsymbol{x}_t}{\sigma_t^2\left(\frac{\alpha_t^2\sigma_T^2}{\sigma_t^2\alpha_T^2} - 1\right)} \right] \mathrm{d}t \\
&\quad + 2\sigma_t^2 \frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \left[\frac{\frac{\alpha_T^2\sigma_t^2}{\sigma_T^2\alpha_t^2}\frac{\alpha_t}{\alpha_T}\boldsymbol{x}_T + \alpha_t \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_t)\left(1 - \frac{\alpha_T^2\sigma_t^2}{\sigma_T^2\alpha_t^2}\frac{\alpha_t}{\alpha_T}\right) - \boldsymbol{x}_t}{\sigma_t^2\left(1 - \frac{\alpha_T^2\sigma_t^2}{\sigma_T^2\alpha_t^2}\frac{\alpha_t}{\alpha_T}\right)} \right] \mathrm{d}t \\
&\quad + \sigma_t \sqrt{-2\frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t}}\,\mathrm{d}\mathbf{w}_t \\
&= \left(\boldsymbol{x}_t \frac{\mathrm{d}\log\alpha_t}{\mathrm{d}t} + 2\sigma_t^2 \frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \left[\frac{\alpha_t \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_t) - \boldsymbol{x}_t}{\sigma_t^2} \right] \right) \mathrm{d}t + \sigma_t \sqrt{-2\frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t}}\,\mathrm{d}\mathbf{w}_t \\
&= \underbrace{\left(\frac{\mathrm{d}\log\alpha_t}{\mathrm{d}t} - 2\frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \right)}_{L(t)\ \text{(Linear Term)}} \boldsymbol{x}_t\,\mathrm{d}t \\
&\quad + \underbrace{\left(2\,\alpha_t \frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_t) + \sigma_t \sqrt{-2\frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t}}\,\frac{\mathrm{d}\mathbf{w}_t}{\mathrm{d}t} \right)}_{N(\boldsymbol{x}_t, t)\ \text{(Non-linear Term)}} \mathrm{d}t,
\end{aligned} \tag{2}
$$

where $L(t)$ is the *linear* term, and $N(\boldsymbol{x}_t, t)$ is the *non-linear* term. This shows the semi-linearity of Proposition 2 of the main paper. Thanks to the semi-linearity, we can make use of the Exponential Integrators method [3] to solve Equation (2), as explained next.
Given initial value $\boldsymbol{x}_a$, where $b = a + \Delta\tau$ and $0 < b < a < T$, we obtain the solution to $\boldsymbol{x}_b$ as:

$$\boldsymbol{x}_b = e^{\int_a^b L(r)\,\mathrm{d}r}\,\boldsymbol{x}_a + \int_a^b e^{\int_{a+\tau}^b L(h)\,\mathrm{d}h} \cdot N(\boldsymbol{x}_\tau, \tau)\,\mathrm{d}\tau. \tag{3}$$

To simplify the integral of the linear term $L(r)$ from a to b, we first expand $L(r)$ using the properties of logarithms:

$$\begin{aligned} L(r) &= \frac{\mathrm{d}\log\alpha_r}{\mathrm{d}r} - 2\frac{\mathrm{d}\log\left(\frac{\alpha_r}{\sigma_r}\right)}{\mathrm{d}r} \\ &= \frac{\mathrm{d}}{\mathrm{d}r}\left[\log\alpha_r - 2(\log\alpha_r - \log\sigma_r)\right] \\ &= \frac{\mathrm{d}}{\mathrm{d}r}\left[2\log\sigma_r - \log\alpha_r\right] \\ &= \frac{\mathrm{d}}{\mathrm{d}r}\left[\log\left(\frac{\sigma_r^2}{\alpha_r}\right)\right]. \end{aligned} \tag{4}$$

Because $L(r)$ simplifies to a single exact derivative, we can easily evaluate its integral with respect to r from a to b:

$$\begin{aligned} \int_a^b L(r)\,\mathrm{d}r &= \int_a^b \frac{\mathrm{d}}{\mathrm{d}r}\left[\log\left(\frac{\sigma_r^2}{\alpha_r}\right)\right]\mathrm{d}r \\ &= \left[\log\frac{\sigma_r^2}{\alpha_r}\right]_a^b \\ &= \log\left(\frac{\sigma_b^2}{\alpha_b}\right) - \log\left(\frac{\sigma_a^2}{\alpha_a}\right) \\ &= \log\left(\frac{\alpha_a}{\alpha_b}\cdot\frac{\sigma_b^2}{\sigma_a^2}\right). \end{aligned} \tag{5}$$

Next, let's define $\lambda_t := \log\frac{\alpha_t}{\sigma_t}$, with the inverse function $t_\lambda(\cdot)$, that satisfies $t_\lambda(\lambda_t) = t$. Through the *change-of-variables* method for λ, we can denote $\alpha_\lambda := \alpha_{t_\lambda(\lambda)}$, $\boldsymbol{x}_\lambda := \boldsymbol{x}_{t_\lambda(\lambda)}$, $\mathbf{D}_\theta(\boldsymbol{x}_\lambda) := \mathbf{D}_\theta(\boldsymbol{x}_{t_\lambda(\lambda)})$, $\mathbf{w}_\lambda := \mathbf{w}_{t_\lambda(\lambda)}$, $\mathrm{d}\mathbf{w}_\lambda := \sqrt{-\frac{\mathrm{d}\lambda}{\mathrm{d}t}}\,\mathrm{d}\mathbf{w}_{t_\lambda(\lambda)}$, and $N(\boldsymbol{x}_\lambda, \lambda) := N(\boldsymbol{x}_{t_\lambda(\lambda)}, t_\lambda(\lambda))$.

Thus, we rewrite Equation (3) as:

$$\begin{aligned}
\boldsymbol{x}_b &= \frac{\alpha_a}{\alpha_b}\frac{\sigma_b^2}{\sigma_a^2}\boldsymbol{x}_a + \int_{\lambda_a}^{\lambda_b} \frac{\alpha_\lambda}{\alpha_b}\frac{\sigma_b^2}{\sigma_\lambda^2} \cdot N(\boldsymbol{x}_\lambda, \lambda)\,\mathrm{d}\lambda \\
&= \frac{\alpha_a}{\alpha_b}\frac{\sigma_b^2}{\sigma_a^2}\boldsymbol{x}_a + \int_{\lambda_a}^{\lambda_b} \frac{\alpha_\lambda}{\alpha_b}\frac{\sigma_b^2}{\sigma_\lambda^2}\left(2\,\alpha_\lambda \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda) + \sqrt{2}\,\sigma_\lambda \frac{\mathrm{d}\mathbf{w}_\lambda}{\mathrm{d}\lambda}\right)\mathrm{d}\lambda \\
&= \frac{\alpha_a}{\alpha_b}\frac{\sigma_b^2}{\sigma_a^2}\boldsymbol{x}_a + 2\,\frac{\sigma_b^2}{\alpha_b}\int_{\lambda_a}^{\lambda_b} \frac{\alpha_\lambda^2}{\sigma_\lambda^2}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda)\,\mathrm{d}\lambda + \sqrt{2}\,\frac{\sigma_b^2}{\alpha_b}\int_{\lambda_a}^{\lambda_b} \frac{\alpha_\lambda}{\sigma_\lambda}\mathrm{d}\mathbf{w}_\lambda \\
&= \frac{\alpha_a}{\alpha_b}\frac{\sigma_b^2}{\sigma_a^2}\boldsymbol{x}_a + 2\,\alpha_b\, e^{-2\lambda_b}\underbrace{\int_{\lambda_a}^{\lambda_b} e^{2\lambda}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda)\,\mathrm{d}\lambda}_{\text{Use Taylor Expansion}} + \sqrt{2}\,\alpha_b\, e^{-2\lambda_b}\underbrace{\int_{\lambda_a}^{\lambda_b} e^{\lambda}\,\mathrm{d}\mathbf{w}_\lambda}_{\text{Itô Integral}}\,. \qquad (6)
\end{aligned}$$

The integral $\int_{\lambda_a}^{\lambda_b} e^{2\lambda}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda)\,\mathrm{d}\lambda$ can be computed by performing Taylor Expansion:

$$\int_{\lambda_a}^{\lambda_b} e^{2\lambda}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda)\,\mathrm{d}\lambda \approx \sum_{n=0}^{k-1}\boldsymbol{D}_{\boldsymbol{\theta}}^{(n)}(\boldsymbol{x}_{\lambda_a})\int_{\lambda_a}^{\lambda_b} e^{2\lambda}\frac{(\lambda-\lambda_a)^n}{n!}\,\mathrm{d}\lambda + \mathcal{O}((\lambda_b-\lambda_a)^{k+1}), \qquad (7)$$

where $k \geq 1$, and $\boldsymbol{D}_{\boldsymbol{\theta}}^{(n)}(\boldsymbol{x}_{\lambda_a}) := \frac{\mathrm{d}^n \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a})}{\mathrm{d}\lambda^n}$ is the n^{th}-order derivative of $\boldsymbol{D}_{\boldsymbol{\theta}}(\cdot)$ w.r.t. λ.
Furthermore, we can compute the Itô integral [10] as:

$$\int_{\lambda_a}^{\lambda_b} e^{\lambda}\,\mathrm{d}\mathbf{w}_\lambda = \left(\sqrt{\int_{\lambda_a}^{\lambda_b} e^{2\lambda}\,\mathrm{d}\lambda}\right)\mathbf{z}_b = \frac{e^{\lambda_b}}{\sqrt{2}}\sqrt{1-e^{2(\lambda_a-\lambda_b)}}\,\mathbf{z}_b,$$

where $\mathbf{z}_b \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I})$.

2.1.1. 1st-order Bridge SDE Solution

Substituting $k = 1$ for Equation (7), we can ultimately simplify Equation (6) as:

$$\begin{aligned}
\boldsymbol{x}_b &= \frac{\alpha_b}{\alpha_a}e^{2(\lambda_a-\lambda_b)}\boldsymbol{x}_a + \alpha_b\left(1-e^{2(\lambda_a-\lambda_b)}\right)\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a}) + \sigma_b\sqrt{1-e^{2(\lambda_a-\lambda_b)}}\,\mathbf{z}_b \\
&= \frac{\alpha_b}{\alpha_a}\frac{\text{SNR}_a}{\text{SNR}_b}\boldsymbol{x}_a + \alpha_b\left(1-\frac{\text{SNR}_a}{\text{SNR}_b}\right)\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a}) + \sigma_b\sqrt{1-\frac{\text{SNR}_a}{\text{SNR}_b}}\,\mathbf{z}_b, \qquad (8)
\end{aligned}$$

where $\text{SNR}_t := \alpha_t^2/\sigma_t^2 = e^{2\lambda_t}$, giving us the 1^{st}-order solution to the Bridge SDE.
Thus, given the initial value $\boldsymbol{x}_a$, where $b = a + \Delta t$ and $0 \leq b < a \leq T$, the solution to $\boldsymbol{x}_b$ is:

$$\boldsymbol{x}_b = \frac{\alpha_b}{\alpha_a}\frac{\text{SNR}_a}{\text{SNR}_b}\boldsymbol{x}_a + \alpha_b\left(1-\frac{\text{SNR}_a}{\text{SNR}_b}\right)\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_a) + \sigma_b\sqrt{1-\frac{\text{SNR}_a}{\text{SNR}_b}}\,\mathbf{z}_b. \qquad (9)$$

■

2.1.2. 2^{nd}-order Bridge SDE Solution

Similar to the process above, by substituting $k = 2$ for Equation (7), we simplify Equation (6) as:

$$\boldsymbol{x}_b = \frac{\alpha_b}{\alpha_a} e^{2(\lambda_a - \lambda_b)} \boldsymbol{x}_a + \alpha_b \left(1 - e^{2(\lambda_a - \lambda_b)}\right) \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a}) \tag{10}$$

$$+ \alpha_b \left(\lambda_b - \lambda_a + e^{2(\lambda_a - \lambda_b)} - 1\right) \boldsymbol{D}_{\boldsymbol{\theta}}^{(1)}(\boldsymbol{x}_{\lambda_a}) + \sigma_b \sqrt{1 - e^{2(\lambda_a - \lambda_b)}}\, \mathbf{z}_b$$

$$= \frac{\text{SNR}_a}{\text{SNR}_b} \frac{\alpha_b}{\alpha_a} \boldsymbol{x}_a + \alpha_b \left(1 - \frac{\text{SNR}_a}{\text{SNR}_b}\right) \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a}) \tag{11}$$

$$+ \alpha_b \left(\lambda_b - \lambda_a + \frac{\text{SNR}_a}{\text{SNR}_b} - 1\right) \boldsymbol{D}_{\boldsymbol{\theta}}^{(1)}(\boldsymbol{x}_{\lambda_a}) + \sigma_b \sqrt{1 - \frac{\text{SNR}_a}{\text{SNR}_b}}\, \mathbf{z}_b, \tag{12}$$

where $\boldsymbol{D}_{\boldsymbol{\theta}}^{(n)}(\boldsymbol{x}_{\lambda_a}) := \frac{\mathrm{d}^n \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a})}{\mathrm{d}\lambda^n}$ is the n^{th}-order derivative of $\boldsymbol{D}_{\boldsymbol{\theta}}(\cdot)$ w.r.t. λ, giving us the 2^{nd}-order solution to the Bridge SDE.

■

2.1.3. Ablation Study on the Bridge SDE Solutions

We provide an ablation study on the effect of $k = 1$ (Eq. (9)) and $k = 2$ (Eq. (12)). In Section 3 of the main paper, we justify our choice of $k = 1$ for the **initial Bridge SDE step** in DBMSolver. As $k = 2$ step is higher-order, it requires an extra NFE. However, because this initial step is taken over a very small interval $\mathrm{d}t = \epsilon$, $k = 2$'s higher-order solution renders the gains trivial, providing negligible benefits.
To validate this claim, we conduct an ablation study on the DIODE (256×256) dataset using DBMSolver. For a fixed number of sampling steps (with the same 2^{nd}-order Bridge ODE step), we compare $k = 1$ and $k = 2$ and report the results in Table 2. For 4 sampling steps, both settings achieve an FID of 3.38. However, $k = 1$ requires only 6 NFEs, whereas $k = 2$ requires 7 NFEs due to its higher-order computation. With 11 sampling steps, the FIDs for $k = 1$ and $k = 2$ are 2.06 and 2.07, respectively. Although the IS metric for $k = 2$ is slightly better (6.04 IS) compared to that of $k = 1$ (6.00 IS), the improvement is marginal compared to the computational cost incurred for the extra NFE .
These results confirm that using $k = 2$ offers no meaningful improvement in generation quality over $k = 1$, supporting our choice to use $k = 1$ for the initial Bridge SDE step.

Table 2. Ablation Study on value of k for the Bridge SDE solution on DIODE (256×256). We see that compared to using $k = 2$, using $k = 1$ for the solution for DBMSolver is sufficiently good and requires lower computational resources.

Value of k	Steps	NFEs	FID ↓	IS ↑	LPIPS ↓	MSE ↓
$k = 2$	4	7	3.38	**6.04**	0.197	**0.015**
$k = 2$	11	21	2.07	6.02	0.198	0.018
$\boldsymbol{k = 1}$ **(Proposed)**	4	6	3.38	6.00	**0.196**	**0.015**
$\boldsymbol{k = 1}$ **(Proposed)**	11	20	**2.06**	6.00	0.198	0.018

2.2. Proof of Proposition 2 of the main paper

Given a well-trained DBM $\boldsymbol{D}_\theta(\cdot)$ that approximates data sample $\boldsymbol{x}_0$, we can re-write the Bridge PF ODE (Equation 5 of the main paper) as:

$$
\begin{aligned}
\frac{\mathrm{d}\boldsymbol{x}_t}{\mathrm{d}t} &= f(\boldsymbol{x}_t, t) - g(t)^2 \left(\frac{1}{2} \nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}_t \mid \boldsymbol{x}_T) - \nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}_T \mid \boldsymbol{x}_t) \right) \\
&= \boldsymbol{x}_t \frac{\mathrm{d} \log \alpha_t}{\mathrm{d}t} + 2\sigma_t^2 \frac{\mathrm{d} \log \left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \left(\frac{1}{2} \nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}_t \mid \boldsymbol{x}_T) - \nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}_T \mid \boldsymbol{x}_t) \right) \\
&= \boldsymbol{x}_t \frac{\mathrm{d} \log \alpha_t}{\mathrm{d}t} + \sigma_t^2 \frac{\mathrm{d} \log \left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \cdot \frac{\frac{\alpha_T^2 \sigma_t^2}{\sigma_T^2 \alpha_t^2} \frac{\alpha_t}{\alpha_T} \boldsymbol{x}_T + \alpha_t \left(1 - \frac{\alpha_T^2 \sigma_t^2}{\sigma_T^2 \alpha_t^2}\right) \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_t) - \boldsymbol{x}_t}{\sigma_t^2 \left(1 - \frac{\alpha_T^2 \sigma_t^2}{\sigma_T^2 \alpha_t^2}\right)} \\
&\quad - 2\sigma_t^2 \frac{\mathrm{d} \log \left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \cdot \frac{\frac{\alpha_t}{\alpha_T} \boldsymbol{x}_T - \boldsymbol{x}_t}{\sigma_t^2 \left(\frac{\alpha_t^2 \sigma_T^2}{\sigma_t^2 \alpha_T^2} - 1 \right)} \\
&= \boldsymbol{x}_t \frac{\mathrm{d} \log \alpha_t}{\mathrm{d}t} + \frac{\mathrm{d} \log \left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \left[\alpha_t \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_t) - \boldsymbol{x}_t - \frac{\frac{\alpha_t}{\alpha_T} \boldsymbol{x}_T - \boldsymbol{x}_t}{\frac{\alpha_t^2 \sigma_T^2}{\sigma_t^2 \alpha_T^2} - 1} \right] \\
&= \boldsymbol{x}_t \left(\frac{\mathrm{d} \log \alpha_t}{\mathrm{d}t} - \frac{\mathrm{d} \log \left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} + \frac{1}{\frac{\alpha_t^2 \sigma_T^2}{\sigma_t^2 \alpha_T^2} - 1} \frac{\mathrm{d} \log \left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \right) \\
&\quad + \alpha_t \frac{\mathrm{d} \log \left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \left[\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_t) - \frac{\boldsymbol{x}_T / \alpha_T}{\frac{\alpha_t^2 \sigma_T^2}{\sigma_t^2 \alpha_T^2} - 1} \right].
\end{aligned}
$$

We further simplify the equation above as:

$$\frac{\mathrm{d}\boldsymbol{x}_t}{\mathrm{d}t} = \underbrace{\boldsymbol{x}_t \left(\frac{\mathrm{d}\log\sigma_t}{\mathrm{d}t} + \frac{1}{\frac{\alpha_t^2\sigma_T^2}{\sigma_t^2\alpha_T^2} - 1} \frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t} \right)}_{L(t)\text{ (Linear Term)}} + \underbrace{\alpha_t \left[\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_t) - \frac{\boldsymbol{x}_T/\alpha_T}{\frac{\alpha_t^2\sigma_T^2}{\sigma_t^2\alpha_T^2} - 1} \right] \frac{\mathrm{d}\log\left(\frac{\alpha_t}{\sigma_t}\right)}{\mathrm{d}t}}_{N(\boldsymbol{x}_t,t)\text{ (Non-linear Term)}} . \tag{13}$$

where $L(t)$ is the *linear* term, and $N(\boldsymbol{x}_t, t)$ is the *non-linear* term. Thus, we can clearly observe the semi-linearity of Equation 5 of the main paper. Similar to the derivation above, we can once again make use of the EI method to
solve Equation (13):
Given an initial value $\boldsymbol{x}_a$ where $b = a + \Delta\tau$ and $0 < b < a < T$, we obtain the solution to $\boldsymbol{x}_b$ as:

$$\boldsymbol{x}_b = e^{\int_a^b L(r)\,\mathrm{d}r}\,\boldsymbol{x}_a + \int_a^b e^{\int_{a+\tau}^b L(h)\,\mathrm{d}h} \cdot N(\boldsymbol{x}_\tau, \tau)\,\mathrm{d}\tau. \tag{14}$$

To simplify the equation, we first integrate the linear term $L(r)$ with respect to r from a to b:

$$\int_a^b L(r)\,\mathrm{d}r = \left[\log\left(\frac{\alpha_r \sqrt{e^{2(\lambda_r - \lambda_T)} - 1}}{e^{2\lambda_r}} \right) \right]_a^b = \log\left(\frac{\alpha_b}{\alpha_a} e^{2(\lambda_a - \lambda_b)} \sqrt{\frac{e^{2(\lambda_b - \lambda_T)} - 1}{e^{2(\lambda_a - \lambda_T)} - 1}} \right), \tag{15}$$

where $\lambda_t := \log\frac{\alpha_t}{\sigma_t}$ has the inverse function $t_\lambda(\cdot)$ which satisfies $t_\lambda(\lambda_t) = t$.
Next, we can use the *change-of-variables* method for λ. We denote $\alpha_\lambda := \alpha_{t_\lambda(\lambda)}$, $\boldsymbol{x}_\lambda := \boldsymbol{x}_{t_\lambda(\lambda)}$, and $N(\boldsymbol{x}_\lambda, \lambda) := N(\boldsymbol{x}_{t_\lambda(\lambda)}, t_\lambda(\lambda))$.
Substituting Equation (15) along with the value of $N(\boldsymbol{x}_\lambda, \lambda)$ back into Equation (14), we get:

$$\begin{aligned}\boldsymbol{x}_b = {} & \frac{\alpha_b}{\alpha_a} e^{2(\lambda_a - \lambda_b)} \sqrt{\frac{e^{2(\lambda_b - \lambda_T)} - 1}{e^{2(\lambda_a - \lambda_T)} - 1}}\,\boldsymbol{x}_a \\ & + \alpha_b\, e^{-2\lambda_b} \underbrace{\int_{\lambda_a}^{\lambda_b} \frac{e^{2\lambda}}{\alpha_\lambda} \sqrt{\frac{e^{2(\lambda_b - \lambda_T)} - 1}{e^{2(\lambda - \lambda_T)} - 1}} \cdot \alpha_\lambda \left[\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda) - \frac{\boldsymbol{x}_T/\alpha_T}{e^{2(\lambda - \lambda_T)} - 1} \right] \mathrm{d}\lambda}_{\text{Separate and simplify integral further}}\end{aligned} \tag{16}$$

When we simplify the integral in Equation (16), we get:

$$\sqrt{e^{2(\lambda_b-\lambda_T)}-1}\int_{\lambda_a}^{\lambda_b}\left[\frac{e^{2\lambda}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda)}{\sqrt{e^{2(\lambda-\lambda_T)}-1}}-\frac{\boldsymbol{x}_T\,e^{2\lambda}}{\alpha_T\,\left(e^{2(\lambda-\lambda_T)}-1\right)^{3/2}}\right]\mathrm{d}\lambda. \tag{17}$$

Next, we separately integrate the individual terms of Equation (17).
For the integral associated with $\boldsymbol{x}_T$, we use *integration-by-parts* to simplify it:

$$\int_{\lambda_a}^{\lambda_b}\frac{\boldsymbol{x}_T\,e^{2\lambda}}{\alpha_T\,\left(e^{2(\lambda-\lambda_T)}-1\right)^{3/2}}\mathrm{d}\lambda=-\frac{\boldsymbol{x}_T}{\alpha_T}\frac{e^{2\lambda_T}}{\sqrt{e^{2(\lambda_b-\lambda_T)}-1}}\left(1-\sqrt{\frac{e^{2(\lambda_b-\lambda_T)}-1}{e^{2(\lambda_a-\lambda_T)}-1}}\right) \tag{18}$$

Using the result in Equation (18), we can simplify Equation (16) as:

$$\begin{aligned}\boldsymbol{x}_b=&\frac{\alpha_b}{\alpha_a}e^{2(\lambda_a-\lambda_b)}\sqrt{\frac{\rho(\lambda_b,\lambda_T)}{\rho(\lambda_a,\lambda_T)}}\boldsymbol{x}_a+\frac{\alpha_b}{\alpha_T}e^{2(\lambda_T-\lambda_b)}\left(1-\sqrt{\frac{\rho(\lambda_b,\lambda_T)}{\rho(\lambda_a,\lambda_T)}}\right)\boldsymbol{x}_T\\&+\alpha_b\,e^{-2\lambda_b}\sqrt{\rho(\lambda_b,\lambda_T)}\underbrace{\int_{\lambda_a}^{\lambda_b}\frac{e^{2\lambda}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda)}{\sqrt{\rho(\lambda,\lambda_T)}}\mathrm{d}\lambda}_{\text{Use Taylor Expansion}},\end{aligned} \tag{19}$$

where $\rho(m,n):=e^{2(m-n)}-1$.
Finally, we perform Taylor Expansion to obtain the solution to the integral in Equation (19):

$$\int_{\lambda_a}^{\lambda_b}\frac{e^{2\lambda}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda)}{\sqrt{\rho(\lambda,\lambda_T)}}\mathrm{d}\lambda\approx\sum_{n=0}^{k-1}\underbrace{\boldsymbol{D}_{\boldsymbol{\theta}}^{(n)}(\boldsymbol{x}_{\lambda_a})}_{\text{Estimated}}\underbrace{\int_{\lambda_a}^{\lambda_b}\frac{e^{2\lambda}}{\sqrt{\rho(\lambda,\lambda_T)}}\frac{(\lambda-\lambda_a)^n}{n!}\mathrm{d}\lambda}_{\text{Analytically Computed (Section 2.3)}}+\underbrace{\mathcal{O}((\lambda_b-\lambda_a)^{k+1})}_{\text{Omitted}},$$

where $k\geq 1$, and $\boldsymbol{D}_{\boldsymbol{\theta}}^{(n)}(\boldsymbol{x}_{\lambda_a}):=\frac{\mathrm{d}^n\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a})}{\mathrm{d}\lambda^n}$ is the n^{th}-order derivative of $\boldsymbol{D}_{\boldsymbol{\theta}}(\cdot)$ w.r.t. λ.
This is the same as Equation 10 of the main paper.
Thus, we can derive an exact solution for $\boldsymbol{x}_b$. For completeness, we derive the 1^{st}- and 2^{nd}-order solutions below.

■

2.3. Deriving Solutions for Proposition 2 of the main paper

2.3.1. 1st-order Bridge ODE Solution

We use Taylor Expansion to find the solution. By using $k = 1$, the 1^{st}-order solution is as follows:

$$\begin{aligned}\int_{\lambda_a}^{\lambda_b} \frac{e^{2\lambda}\boldsymbol{D_\theta}(\boldsymbol{x}_\lambda)}{\sqrt{\rho(\lambda,\lambda_T)}}\,\mathrm{d}\lambda &\approx \boldsymbol{D}_{\boldsymbol{\theta}}^{(0)}(\boldsymbol{x}_{\lambda_a})\int_{\lambda_a}^{\lambda_b}\frac{e^{2\lambda}}{\sqrt{\rho(\lambda,\lambda_T)}}\frac{(\lambda-\lambda_a)^0}{0!}\mathrm{d}\lambda \\ &= \boldsymbol{D_\theta}(\boldsymbol{x}_{\lambda_a})\int_{\lambda_a}^{\lambda_b}\frac{e^{2\lambda}}{\sqrt{\rho(\lambda,\lambda_T)}}\,\mathrm{d}\lambda \\ &= \boldsymbol{D_\theta}(\boldsymbol{x}_{\lambda_a})\,e^{2\lambda_T}\sqrt{\rho(\lambda_b,\lambda_T)}\left(1-\sqrt{\frac{\rho(\lambda_a,\lambda_T)}{\rho(\lambda_b,\lambda_T)}}\right). \end{aligned} \tag{20}$$

Substituting Equation (20) back into Equation (19), we get the following 1^{st}-order formulation of $\boldsymbol{x}_b$:

$$\begin{aligned}\boldsymbol{x}_b = \frac{\alpha_b}{\alpha_a}e^{2(\lambda_a-\lambda_b)}\sqrt{\frac{\rho(\lambda_b,\lambda_T)}{\rho(\lambda_a,\lambda_T)}}\,\boldsymbol{x}_a + \frac{\alpha_b}{\alpha_T}e^{2(\lambda_T-\lambda_b)}\left(1-\sqrt{\frac{\rho(\lambda_b,\lambda_T)}{\rho(\lambda_a,\lambda_T)}}\right)\boldsymbol{x}_T \\ + \alpha_b\,e^{2(\lambda_T-\lambda_b)}\rho(\lambda_b,\lambda_T)\left(1-\sqrt{\frac{\rho(\lambda_a,\lambda_T)}{\rho(\lambda_b,\lambda_T)}}\right)\boldsymbol{D_\theta}(\boldsymbol{x}_{\lambda_a}).\end{aligned} \tag{21}$$

$$\boldsymbol{x}_b = m(a,b)\boldsymbol{x}_a + n(a,b)\boldsymbol{x}_T + o(a,b)\boldsymbol{D_\theta}(\boldsymbol{x}_a) \tag{22}$$

■

Relation to DBIM Sampler [14]. Our solution is a generalized form of the DBIM-1 sampler's deterministic step. Simplifying the DBIM-1 deterministic step's formulation for $\boldsymbol{x}_b$

(with $\rho = 0$), we get:

$$
\begin{aligned}
\boldsymbol{x}_b &= \frac{\alpha_b}{\alpha_T}\frac{\text{SNR}_T}{\text{SNR}_b}\boldsymbol{x}_T + \alpha_b\left(1 - \frac{\text{SNR}_T}{\text{SNR}_b}\right)\boldsymbol{D}_\theta(\boldsymbol{x}_a) \\
&\quad + \sigma_b\sqrt{1 - \frac{\text{SNR}_T}{\text{SNR}_b}}\left[\frac{\boldsymbol{x}_a - \frac{\alpha_a}{\alpha_T}\frac{\text{SNR}_T}{\text{SNR}_a}\boldsymbol{x}_T - \alpha_a\left(1 - \frac{\text{SNR}_T}{\text{SNR}_a}\right)\boldsymbol{D}_\theta(\boldsymbol{x}_a)}{\sigma_a\sqrt{1 - \frac{\text{SNR}_T}{\text{SNR}_a}}}\right] \\
&= \frac{\alpha_b}{\alpha_a}\frac{\text{SNR}_a}{\text{SNR}_b}\sqrt{\frac{\frac{\text{SNR}_b}{\text{SNR}_T} - 1}{\frac{\text{SNR}_a}{\text{SNR}_T} - 1}}\boldsymbol{x}_a + \frac{\alpha_b}{\alpha_T}\frac{\text{SNR}_T}{\text{SNR}_b}\left(1 - \frac{\sigma_b\sqrt{1 - \frac{\text{SNR}_T}{\text{SNR}_b}}}{\sigma_a\sqrt{1 - \frac{\text{SNR}_T}{\text{SNR}_a}}}\frac{\alpha_a}{\alpha_T}\frac{\text{SNR}_T}{\text{SNR}_a}\frac{\alpha_T}{\alpha_b}\frac{\text{SNR}_b}{\text{SNR}_T}\right)\boldsymbol{x}_T \\
&\quad + \alpha_b\frac{\text{SNR}_T}{\text{SNR}_b}\left(\frac{\text{SNR}_b}{\text{SNR}_T} - 1\right)\left(1 - \sqrt{\frac{\frac{\text{SNR}_a}{\text{SNR}_T} - 1}{\frac{\text{SNR}_b}{\text{SNR}_T} - 1}}\right)\boldsymbol{D}_\theta(\boldsymbol{x}_a).
\end{aligned} \tag{23}
$$

By substituting the equation above with $e^{2\lambda_t} := \text{SNR}_t$, we in fact see that Equation (23) simplifies to our 1^{st}-order formulation of $\boldsymbol{x}_b$ in Equation (22). Thus, we see that DBIM is actually a 1^{st}-order formulation of our solution to the Bridge PF ODE (*i.e.*, $k = 1$). DBMSolver's advantage is that it instead utilizes a more precise, 2^{nd}-order solution that has lower error bounds compared to DBIM.

2.3.2. 2^{nd}-order Bridge ODE Solution

Similar to the derivation in Section 2.3.1, we use Taylor Expansion to find the solution when $k = 2$:

$$
\int_{\lambda_a}^{\lambda_b} \frac{e^{2\lambda}\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_\lambda)}{\sqrt{\rho(\lambda, \lambda_T)}} \approx \underbrace{\boldsymbol{D}_{\boldsymbol{\theta}}^{(0)}(\boldsymbol{x}_{\lambda_a})\int_{\lambda_a}^{\lambda_b}\frac{e^{2\lambda}}{\sqrt{\rho(\lambda, \lambda_T)}}\,\mathrm{d}\lambda}_{\text{Solution derived in Equation (20)}} + \boldsymbol{D}_{\boldsymbol{\theta}}^{(1)}(\boldsymbol{x}_{\lambda_a})\underbrace{\int_{\lambda_a}^{\lambda_b}\frac{e^{2\lambda}(\lambda - \lambda_a)}{\sqrt{\rho(\lambda, \lambda_T)}}\,\mathrm{d}\lambda}_{\text{Solution derived below}}. \tag{24}
$$

The second term's integral can be solved as:

$$
\begin{aligned}
\int_{\lambda_a}^{\lambda_b} \frac{e^{2\lambda}(\lambda - \lambda_a)}{\sqrt{\rho(\lambda, \lambda_T)}}\,\mathrm{d}\lambda = e^{2\lambda_T}\left[\tan^{-1}\left(\sqrt{\rho(\lambda_b, \lambda_T)}\right) - \tan^{-1}\left(\sqrt{\rho(\lambda_a, \lambda_T)}\right)\right] \\
+ e^{2\lambda_T}\left[(\lambda_b - \lambda_a - 1)\sqrt{\rho(\lambda_b, \lambda_T)} + \sqrt{\rho(\lambda_a, \lambda_T)}\right].
\end{aligned} \tag{25}
$$

By substituting Equations 20 and 25 into Equation (24), we reach the 2^{nd}-order Taylor

Expansion:

$$e^{2\lambda_T}\sqrt{\rho(\lambda_b,\lambda_T)}\left(1-\sqrt{\frac{\rho(\lambda_a,\lambda_T)}{\rho(\lambda_b,\lambda_T)}}\right)\left[\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a})-\boldsymbol{D}_{\boldsymbol{\theta}}^{(1)}(\boldsymbol{x}_{\lambda_a})\right]$$
$$+\,e^{2\lambda_T}\sqrt{\rho(\lambda_b,\lambda_T)}\left[\lambda_b-\lambda_a+\frac{\tan^{-1}\left(\sqrt{\rho(\lambda_b,\lambda_T)}\right)-\tan^{-1}\left(\sqrt{\rho(\lambda_a,\lambda_T)}\right)}{\sqrt{\rho(\lambda_b,\lambda_T)}}\right]\boldsymbol{D}_{\boldsymbol{\theta}}^{(1)}(\boldsymbol{x}_{\lambda_a}). \tag{26}$$

Finally, substituting Equation (26) back into Equation (19), we get the following formulation of $\boldsymbol{x}_b$:

$$\begin{aligned}\boldsymbol{x}_b=\ &\frac{\alpha_b}{\alpha_a}e^{2(\lambda_a-\lambda_b)}\sqrt{\frac{\rho(\lambda_b,\lambda_T)}{\rho(\lambda_a,\lambda_T)}}\,\boldsymbol{x}_a+\frac{\alpha_b}{\alpha_T}e^{2(\lambda_T-\lambda_b)}\left(1-\sqrt{\frac{\rho(\lambda_b,\lambda_T)}{\rho(\lambda_a,\lambda_T)}}\right)\boldsymbol{x}_T\\
&-\alpha_b\,\rho(\lambda_T,\lambda_b)\left[1-\sqrt{\frac{\rho(\lambda_a,\lambda_T)}{\rho(\lambda_b,\lambda_T)}}\right]\left(\boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a})-\boldsymbol{D}_{\boldsymbol{\theta}}^{(1)}(\boldsymbol{x}_{\lambda_a})\right)\\
&-\alpha_b\,\rho(\lambda_T,\lambda_b)\left[\lambda_b-\lambda_a+\frac{\tan^{-1}\left(\sqrt{\rho(\lambda_b,\lambda_T)}\right)-\tan^{-1}\left(\sqrt{\rho(\lambda_a,\lambda_T)}\right)}{\sqrt{\rho(\lambda_b,\lambda_T)}}\right]\boldsymbol{D}_{\boldsymbol{\theta}}^{(1)}(\boldsymbol{x}_{\lambda_a}),\end{aligned} \tag{27}$$

where $\boldsymbol{D}_{\boldsymbol{\theta}}^{(n)}(\boldsymbol{x}_{\lambda_a}) := \frac{\mathrm{d}^n \boldsymbol{D}_{\boldsymbol{\theta}}(\boldsymbol{x}_{\lambda_a})}{\mathrm{d}\lambda^n}$ is the n^{th}-order derivative of $\boldsymbol{D}_{\boldsymbol{\theta}}(\cdot)$ w.r.t. λ. Note that $\tilde{\boldsymbol{x}}_{\lambda_m}$, the perturbed image at λ_m, is obtained via Equation (22).

■

3. Experiment Details

3.1. Training Details

We provide thorough details for the diffusion bridge models and their training procedures in Table 3.

3.2. Sampling Details

To ensure consistency across different Image-to-Image Translation tasks, the sampling hyperparameters used in our experiments are summarized in Table 4. The following table outlines the discretization schedules and step counts applied to each dataset.

Table 3. Training details for the various Image-to-Image Translation tasks.

Dataset	Edges2Handbags [5]	DIODE [13]	Inpainting on Conditional ImageNet [2]	CelebAMask-HQ [7]	Face2Comics [12]
Hyperparameters and Training Details					
Bridge Formulation	VP	VP	I^2SB [8]	VP	VP
Noise Conditioning, c_{noise}	$250 \ln t$	$250 \ln t$	$1000\, t$	$250 \ln t$	$250 \ln t$
Learning Rate	1e-4	1e-4	1e-4	1e-4	1e-4
EMA Rate	0.9999	0.9999	0.9999	0.9993	0.9993
Noise Discretization Schedule	Karras	Karras	Karras	Karras	Karras
Noise Discretization Steps	40	40	40	40	40
Batch Size	256	64	256	64	64
Training Iterations	400k	400k	400k	120k	120k
Number and Type of GPUs	4 A100	4 A100	8 A800	8 A6000	8 A6000
Model Details					
Model Channels	192	256	256	256	256
Dropout	10%	10%	10%	10%	10%
Time Embedding	Cosine	Cosine	Cosine	Cosine	Cosine
Channel Multiplier	(1, 2, 3, 4)	(1, 1, 2, 2, 4, 4)	(1, 1, 2, 2, 4, 4)	(1, 1, 2, 2, 4, 4)	(1, 1, 2, 2, 4, 4)
Number of Residual Layers	3	2	2	2	2
Attention Resolutions	(8, 16, 32)	(8, 16, 32)	(8, 16, 32)	(8, 16, 32)	(8, 16, 32)
Model Capacity (Mparams)	284	534	534	534	534

Table 4. Sampling details for the various Image-to-Image Translation tasks.

Dataset	Edges2Handbags [5]	DIODE [13]	Inpainting on Conditional ImageNet [2]	CelebAMask-HQ [7]	Face2Comics [12]
Hyperparameters for Sampling					
Discretization Schedule	Karras	Karras	Uniform	Uniform	Karras
Discretization Steps	40	40	40	40	40

4. More Qualitative Results

Beyond the quantitative metrics presented earlier, we include additional qualitative results in Figures 1–3 to further highlight the perceptual advantages of DBMSolver. We generate images with different random seeds to observe the image generation quality across multiple instances, as portrayed in Figure 1. We observe that different random seeds lead to the generation of different images. This is because the random seed affects the initial Bridge SDE step of DBMSolver, which utilizes random gaussian noise.

In Figures 2 and 3, we continue to observe that our DBMSolver produces significantly better qualitative results compared to other baselines. DBMSolver achieves 34.76 FID in 6 NFEs, while DBIM-1 achieves 44.92 FID on CelebAMask-HQ (256×256). On the Class-conditional Inpainting task on ImageNet (256×256), we achieve 4.98 FID in 6 NFEs while DBIM-1 achieves 5.36 FID, demonstrating our DBMSolver's improvements.

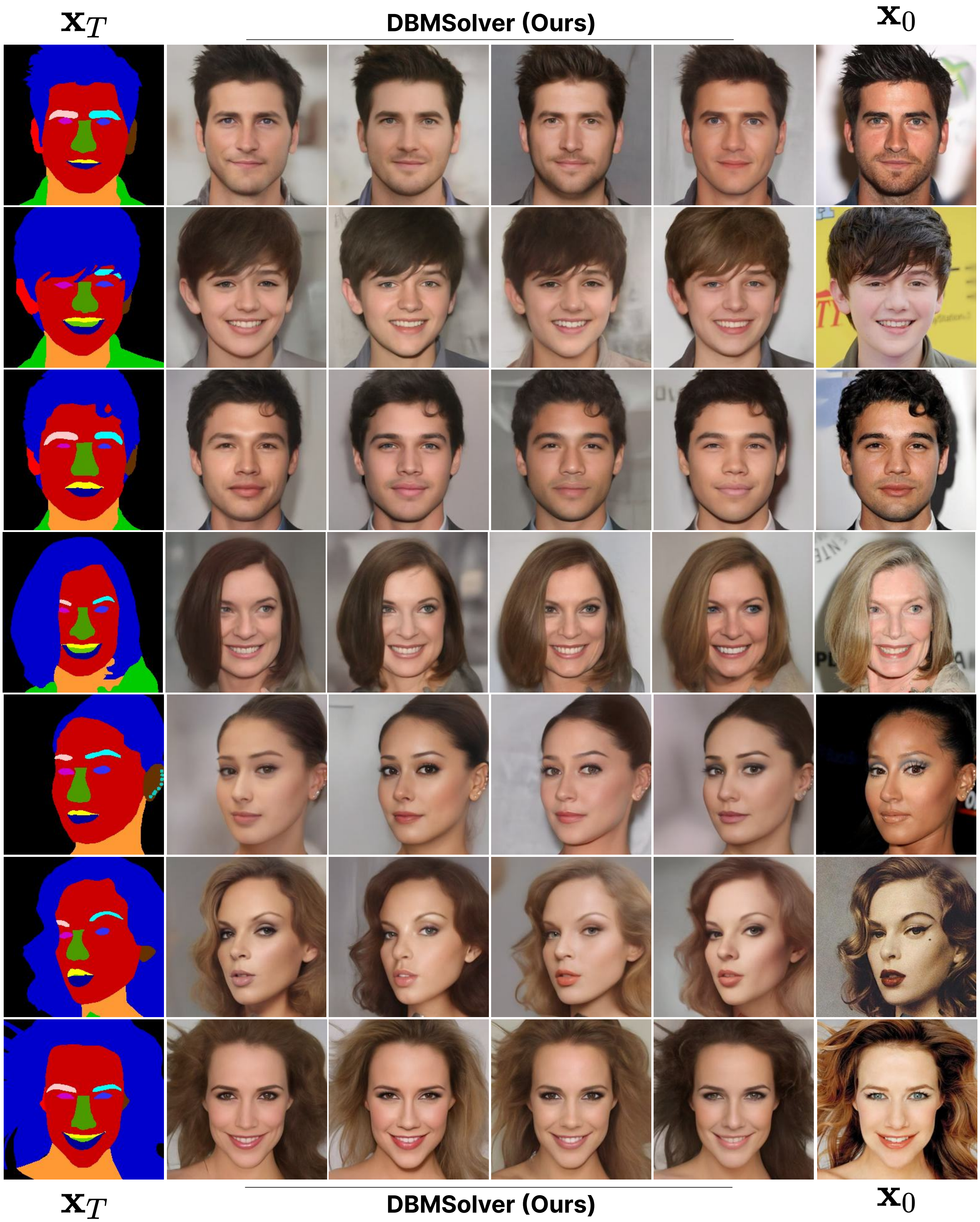

Figure 1. Additional CelebAMask-HQ samples for DBMSolver with 6 NFEs, with different initial SDE steps.

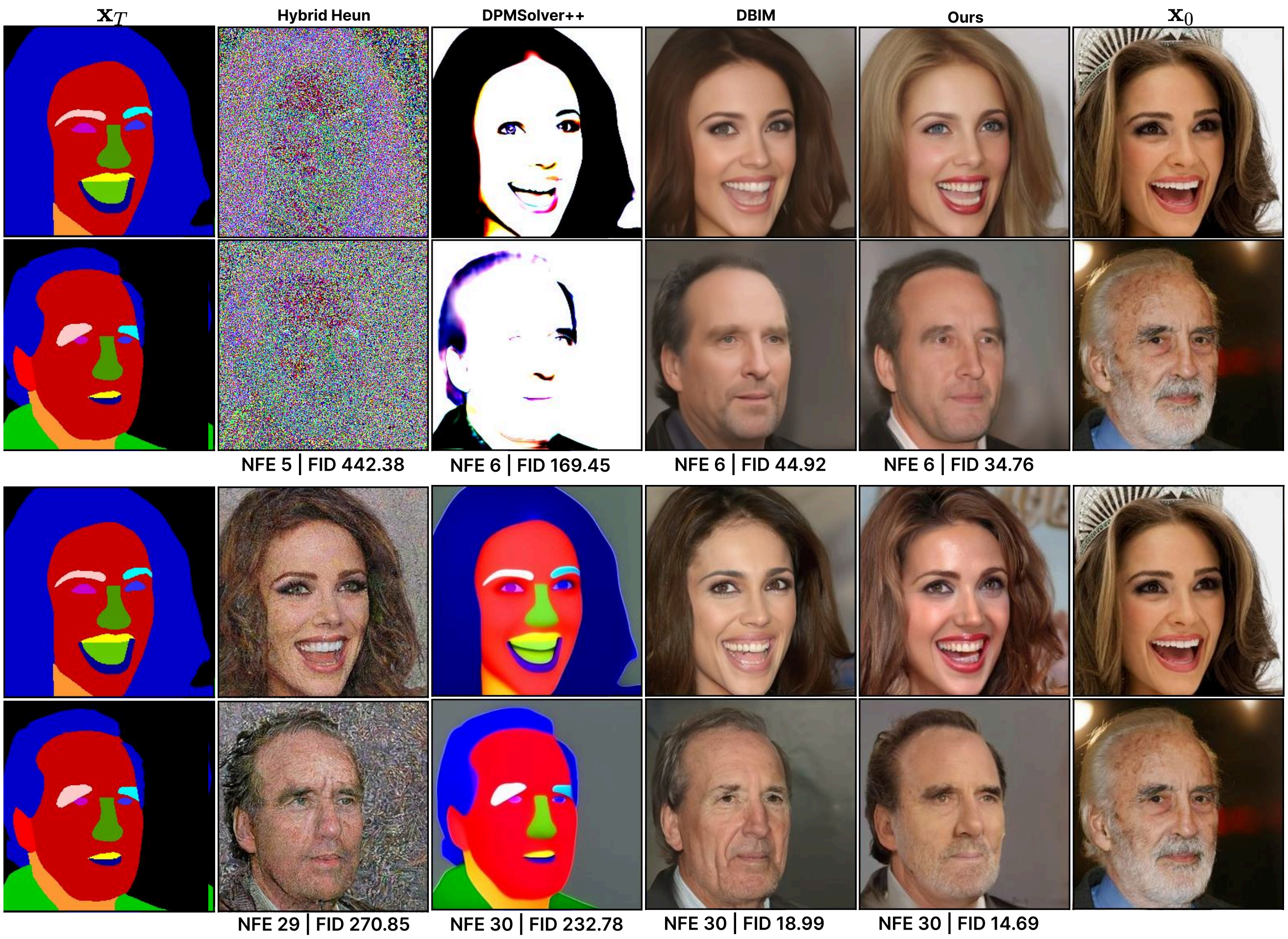

Figure 2. Additional qualitative comparison for Label-to-Face Generation on CelebAMask-HQ.

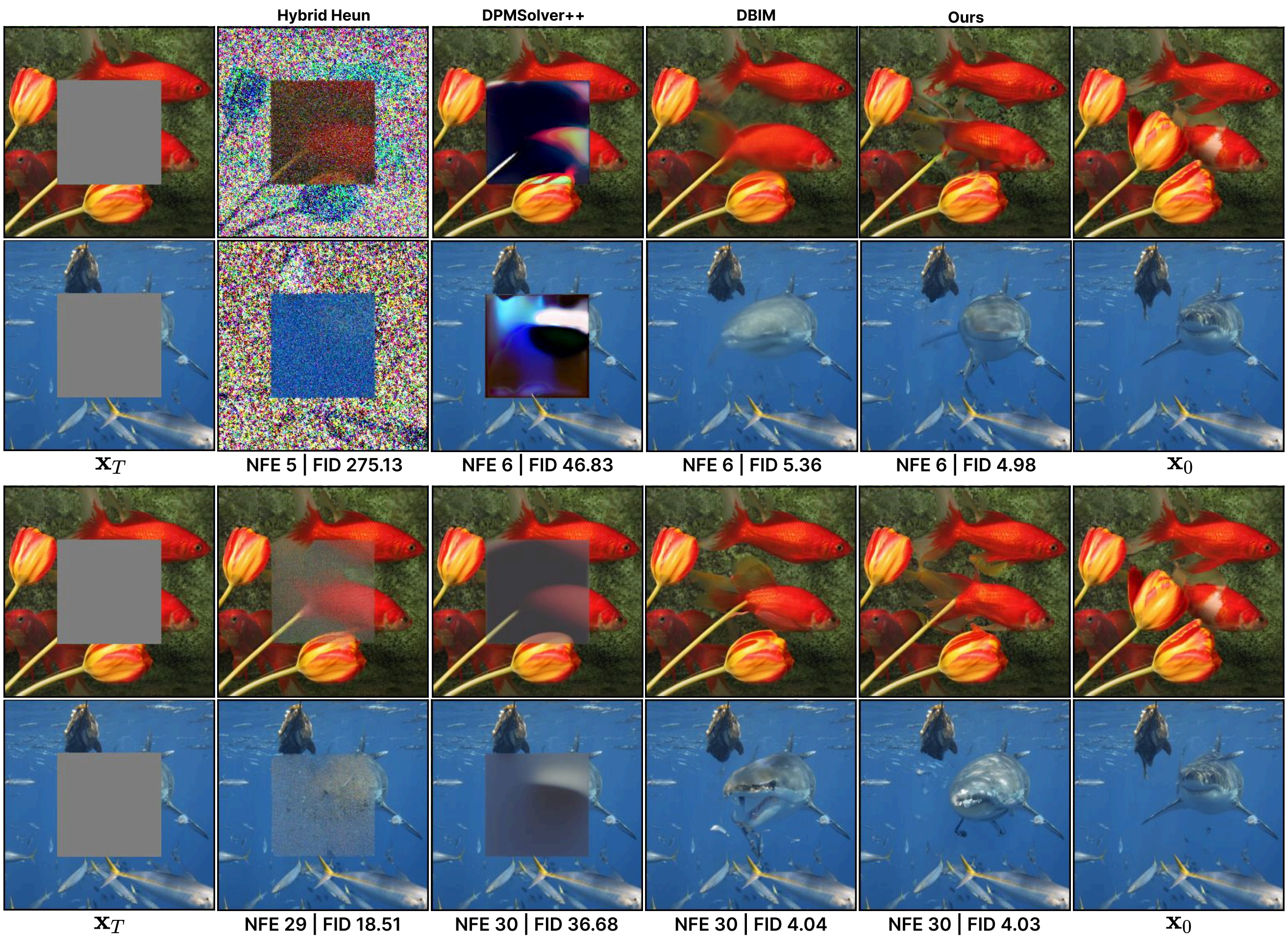

Figure 3. Additional qualitative comparison for Class-Conditional Inpainting on ImageNet.

References

[1] Brian D.O. Anderson. Reverse-time diffusion equation models. *Stochastic Processes and their Applications*, 12(3), 1982. 1

[2] Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Li Fei-Fei. Imagenet: A large-scale hierarchical image database. In *2009 IEEE Conference on Computer Vision and Pattern Recognition*, 2009. 12

[3] Marlis Hochbruck and Alexander Ostermann. Exponential integrators. *Acta Numerica*, 19, 2010. 3

[4] Aapo Hyvärinen. Estimation of non-normalized statistical models by score matching. *Journal of Machine Learning Research*, 6 (24), 2005. 1

[5] Phillip Isola, Jun-Yan Zhu, Tinghui Zhou, and Alexei A Efros. Image-to-image translation with conditional adversarial networks. *CVPR*, 2017. 12

[6] Tero Karras, Miika Aittala, Timo Aila, and Samuli Laine. Elucidating the design space of diffusion-based generative models. *Advances in neural information processing systems*, 35, 2022. 1

[7] Cheng-Han Lee, Ziwei Liu, Lingyun Wu, and Ping Luo. Maskgan: Towards diverse and interactive facial image manipulation. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2020. 12

[8] Guan-Horng Liu, Arash Vahdat, De-An Huang, Evangelos A Theodorou, Weili Nie, and Anima Anandkumar. I2sb: Image-to-image schr\" odinger bridge. *arXiv preprint arXiv:2302.05872*, 2023. 12

[9] Cheng Lu and Yang Song. Simplifying, stabilizing and scaling continuous-time consistency models. *arXiv preprint arXiv:2410.11081*, 2024. 1

[10] L Chris G Rogers and David Williams. *Diffusions, Markov processes, and martingales: Itô calculus*. Cambridge university press, 2000. 4

[11] Yang Song, Jascha Sohl-Dickstein, Diederik P Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. Score-based generative modeling through stochastic differential equations. *arXiv preprint arXiv:2011.13456*, 2020. 1

[12] Sxela. Face2comics. `https://github.com/Sxela/face2comics`, 2021. 12

[13] Igor Vasiljevic, Nick Kolkin, Shanyi Zhang, Ruotian Luo, Haochen Wang, Falcon Z. Dai, Andrea F. Daniele, Mohammadreza Mostajabi, Steven Basart, Matthew R. Walter, and Gregory Shakhnarovich. DIODE: A Dense Indoor and Outdoor DEpth Dataset. *CoRR*, 2019. 12

[14] Kaiwen Zheng, Guande He, Jianfei Chen, Fan Bao, and Jun Zhu. Diffusion bridge implicit models. *arXiv preprint arXiv:2405.15885*, 2024. 9

[15] Linqi Zhou, Aaron Lou, Samar Khanna, and Stefano Ermon. Denoising diffusion bridge models. *arXiv preprint arXiv:2309.16948*, 2023. 1