

Revisiting 3D Reconstruction Kernels as Low-Pass Filters

Supplementary Material

A. Jinc Kernel

In this section, we perform detailed derivation of Jinc kernel proposed in Section 4.1.

A.1. Frequency Domain Definition

The ideal low-pass filter (ILPF) in three dimensions is a theoretical construct used in signal processing for multi-dimensional signals, such as volumetric data or 3D images. Its frequency domain transfer function is a spherical indicator function, allowing frequency components within a cutoff radius to pass unattenuated while blocking higher frequencies. The spatial domain response, or impulse response, is obtained via the inverse Fourier transform. We use the following conventions:

$$H(\mathbf{f}) = \iiint h(\mathbf{r}) e^{-j2\pi\mathbf{f}\cdot\mathbf{r}} dx dy dz, \quad (1)$$

$$h(\mathbf{r}) = \iiint H(\mathbf{f}) e^{j2\pi\mathbf{f}\cdot\mathbf{r}} df_x df_y df_z, \quad (2)$$

where $j = \sqrt{-1}$.

The transfer function of the 3D ideal low-pass filter with cutoff frequency f_c is:

$$H(\mathbf{f}) = \begin{cases} 1, & \|\mathbf{f}\| \leq f_c, \\ 0, & \|\mathbf{f}\| > f_c, \end{cases} \quad (3)$$

where $\|\mathbf{f}\| = \sqrt{f_x^2 + f_y^2 + f_z^2}$. This defines a spherical passband in the frequency domain.

A.2. Derivation of the Spatial Response

Due to the radial symmetry of $H(\mathbf{f})$, the spatial response $h(\mathbf{r})$ depends only on $r = \|\mathbf{r}\| = \sqrt{x^2 + y^2 + z^2}$. We compute $h(r)$ using spherical coordinates in the frequency domain. The inverse Fourier transform becomes:

$$\begin{aligned} h(r) &= \int_0^{f_c} \int_0^\pi \int_0^{2\pi} e^{j2\pi f r \cos\theta} f^2 \sin\theta d\phi d\theta df, \\ &= 2\pi \int_0^{f_c} f^2 \int_0^\pi e^{j2\pi f r \cos\theta} \sin\theta d\theta df, \\ &= 2\pi \int_0^{f_c} f^2 \cdot \frac{2 \sin(2\pi f r)}{2\pi f r} df, \\ &= \frac{4\pi}{2\pi r} \int_0^{f_c} f^2 \cdot \frac{\sin(2\pi f r)}{f} df, \\ &= \frac{2}{r} \int_0^{f_c} f \sin(2\pi f r) df, \\ &= \frac{1}{2\pi^2 r^3} [\sin(2\pi f_c r) - 2\pi f_c r \cos(2\pi f_c r)]. \end{aligned} \quad (4)$$

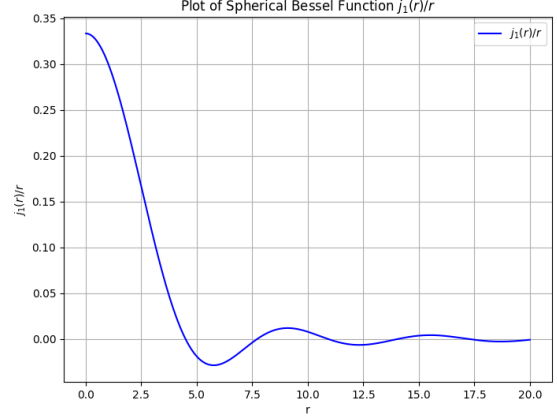


Figure 1. The sketch map of half of the ILPF.

The spherical Bessel function of the first kind $j_1(\alpha)$ is given as:

$$j_1(\alpha) = \frac{\sin \alpha - \alpha \cos \alpha}{\alpha^2}. \quad (5)$$

So, we have:

$$h(r) = \frac{1}{2\pi^2 r^3} \cdot (2\pi f_c r)^2 j_1(2\pi f_c r) = \frac{2f_c^2}{r} j_1(2\pi f_c r).$$

The curve of $j_1(r)/r$ is visualized in Figure 1

A.3. Anisotropy ILPF

We have discussed about the isotropy ILPF above. Now we introduce the definition of the anisotropy ILPF:

$$h(\mathbf{x}) = \frac{j_1(\sqrt{(\mathbf{x} - \boldsymbol{\mu})^\top \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu})})}{\sqrt{(\mathbf{x} - \boldsymbol{\mu})^\top \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu})}}, \quad (6)$$

where $\boldsymbol{\mu}$ is the center position of ILPF and $\Sigma = R S S^\top R^\top$ is the covariance matrix. Here we ignore the outoff frequency, which can be absorbed into the scale matrix S .

B. Jinc Splatting

In this section, we perform detailed derivation for Jinc Splatting in Section 4.2.

B.1. Integration Along a Line

This section derives the integral of the spatial response of a three-dimensional ideal low-pass filter along an arbitrary straight line $\mathbf{x}(t) = \mathbf{a} + t\mathbf{b}$, where $\mathbf{a} = (a_x, a_y, a_z)$, $\mathbf{b} = (b_x, b_y, b_z)$, and $t \in (-\infty, \infty)$.

We need to compute:

$$\begin{aligned}
I &= \int_{-\infty}^{\infty} h(\mathbf{x}(t)) dt, \\
&= \int_{-\infty}^{\infty} \frac{j_1(\sqrt{(\mathbf{x}(t) - \boldsymbol{\mu})^\top \Sigma^{-1} (\mathbf{x}(t) - \boldsymbol{\mu})})}{\sqrt{(\mathbf{x}(t) - \boldsymbol{\mu})^\top \Sigma^{-1} (\mathbf{x}(t) - \boldsymbol{\mu})}} dt, \\
&= \int_{-\infty}^{\infty} \frac{j_1\left(\sqrt{((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b})^\top \Sigma^{-1} ((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b})}\right)}{\sqrt{((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b})^\top \Sigma^{-1} ((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b})}} dt.
\end{aligned}$$

Since $\Sigma = R S S^\top R^\top$, we define $\mathbf{m} = S^{-1} R^{-1} (\mathbf{a} - \boldsymbol{\mu})$ and $\mathbf{n} = S^{-1} R^{-1} \mathbf{b}$. Thus, we have:

$$\begin{aligned}
&((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b})^\top \Sigma^{-1} ((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b}) \\
&= (\mathbf{m} + t\mathbf{n})^\top (\mathbf{m} + t\mathbf{n}), \\
&= \|\mathbf{m}\|^2 + 2t(\mathbf{m} \cdot \mathbf{n}) + t^2 \|\mathbf{n}\|^2.
\end{aligned} \tag{7}$$

Substitute $s = t + \mathbf{m} \cdot \mathbf{n} / \|\mathbf{n}\|^2$, the integration transfers to:

$$\begin{aligned}
I &= \int_{-\infty}^{\infty} \frac{j_1\left(\sqrt{((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b})^\top \Sigma^{-1} ((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b})}\right)}{\sqrt{((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b})^\top \Sigma^{-1} ((\mathbf{a} - \boldsymbol{\mu}) + t\mathbf{b})}} dt, \\
&= \int_{-\infty}^{\infty} \frac{j_1\left(\sqrt{\|\mathbf{m}\|^2 + 2t(\mathbf{m} \cdot \mathbf{n}) + t^2 \|\mathbf{n}\|^2}\right)}{\sqrt{\|\mathbf{m}\|^2 + 2t(\mathbf{m} \cdot \mathbf{n}) + t^2 \|\mathbf{n}\|^2}} dt, \\
&= \int_{-\infty}^{\infty} \frac{j_1\left(\frac{\|\mathbf{n}\| \sqrt{s^2 + \frac{\|\mathbf{m} \times \mathbf{n}\|^2}{\|\mathbf{n}\|^4}}}{\|\mathbf{n}\| \sqrt{s^2 + \frac{\|\mathbf{m} \times \mathbf{n}\|^2}{\|\mathbf{n}\|^4}}}\right)}{\|\mathbf{n}\| \sqrt{s^2 + \frac{\|\mathbf{m} \times \mathbf{n}\|^2}{\|\mathbf{n}\|^4}}} ds, \\
&= \int_{-\infty}^{\infty} \frac{j_1\left(\frac{\frac{\|\mathbf{m} \times \mathbf{n}\|}{\|\mathbf{n}\|} \sqrt{v^2 + 1}}{\|\mathbf{n}\| \sqrt{v^2 + 1}}\right)}{\|\mathbf{n}\| \sqrt{v^2 + 1}} dv, \\
&= \int_{-\infty}^{\infty} \frac{j_1(\alpha \sqrt{v^2 + 1})}{\|\mathbf{n}\| \sqrt{v^2 + 1}} dv, \\
&= \frac{\pi J_1(\alpha)}{\|\mathbf{n}\| \alpha}
\end{aligned} \tag{8}$$

B.2. Gradient Backward Propagation

This section derives the backward propagation for Jinc Splatting in Section 4.2.4.

B.2.1. The Partial Derivative of I with respect to α

First, we have the differential of $j_1(x)$:

$$j_1'(x) = j_0(x) - \frac{2}{x} j_1(x), \tag{9}$$

$$j_0(x) = \frac{\sin(x)}{x}, \quad j_1(x) = \frac{\sin(x) - x \cos(x)}{x^2}. \tag{10}$$

Thus:

$$\begin{aligned}
\frac{\partial I}{\partial \alpha} &= \int_{-\infty}^{\infty} \frac{1}{\|\mathbf{n}\| \sqrt{v^2 + 1}} \frac{\partial j_1(\alpha \sqrt{v^2 + 1})}{\partial \alpha} dv \\
&= \int_{-\infty}^{\infty} \frac{\partial j_1(\alpha \sqrt{v^2 + 1})}{\|\mathbf{n}\| \partial(\alpha \sqrt{v^2 + 1})} dv \\
&= \frac{1}{\|\mathbf{n}\|} \int_{-\infty}^{\infty} j_0(\alpha \sqrt{v^2 + 1}) - \frac{2j_1(\alpha \sqrt{v^2 + 1})}{\alpha \sqrt{v^2 + 1}} dv \\
&= \frac{1}{\|\mathbf{n}\|} \left[\frac{\pi J_0(\alpha)}{\alpha} - \frac{2\pi J_1(\alpha)}{\alpha^2} \right].
\end{aligned} \tag{11}$$

B.2.2. The Partial Derivative of α with respect to $\boldsymbol{\mu}$

The partial derivative of α with respect to $\boldsymbol{\mu}$ can be formulated as:

$$\begin{aligned}
\frac{\partial \alpha}{\partial \boldsymbol{\mu}} &= \frac{1}{2\alpha} \frac{\partial}{\partial \boldsymbol{\mu}} \left(\|\mathbf{m}\|^2 - \frac{(\mathbf{m}^\top \mathbf{n})^2}{\|\mathbf{n}\|^2} \right) \\
&= \frac{1}{2\alpha} \left(\frac{\partial(\mathbf{m}^\top \mathbf{m})}{\partial \boldsymbol{\mu}} - \frac{\partial}{\partial \boldsymbol{\mu}} \left(\frac{(\mathbf{m}^\top \mathbf{n})^2}{\|\mathbf{n}\|^2} \right) \right) \\
&= \frac{1}{2\alpha} \left(2 \left(\frac{\partial \mathbf{m}}{\partial \boldsymbol{\mu}} \right)^\top \mathbf{m} - \frac{2(\mathbf{m}^\top \mathbf{n})}{\|\mathbf{n}\|^2} \left(\frac{\partial \mathbf{m}}{\partial \boldsymbol{\mu}} \right)^\top \mathbf{n} \right) \\
&= \frac{1}{2\alpha} \left(-2(\mathbf{S}^{-1} \mathbf{R}^{-1})^\top \mathbf{m} + \frac{2(\mathbf{m}^\top \mathbf{n})}{\|\mathbf{n}\|^2} (\mathbf{S}^{-1} \mathbf{R}^{-1})^\top \mathbf{n} \right) \\
&= \frac{1}{\alpha} \left(-(\mathbf{S}^{-1} \mathbf{R}^{-1})^\top \mathbf{m} + \frac{\mathbf{m}^\top \mathbf{n}}{\|\mathbf{n}\|^2} (\mathbf{S}^{-1} \mathbf{R}^{-1})^\top \mathbf{n} \right) \\
&= -\frac{1}{\alpha} (\mathbf{S}^{-1} \mathbf{R}^{-1})^\top \left(\mathbf{m} - \frac{\mathbf{m}^\top \mathbf{n}}{\|\mathbf{n}\|^2} \mathbf{n} \right) \\
&= -\frac{1}{\alpha} (\mathbf{R} \mathbf{S})^{-\top} \left(\mathbf{m} - \frac{\mathbf{m}^\top \mathbf{n}}{\|\mathbf{n}\|^2} \mathbf{n} \right).
\end{aligned} \tag{12}$$

B.2.3. The Partial Derivative of α with respect to Σ

The partial derivative of α with respect to Σ can be formulated as:

$$\frac{\partial \alpha}{\partial \Sigma} = \frac{1}{2\alpha} \left(\frac{\partial(\mathbf{m}^\top \mathbf{m})}{\partial \Sigma} - \frac{\partial}{\partial \Sigma} \left(\frac{(\mathbf{m}^\top \mathbf{n})^2}{\|\mathbf{n}\|^2} \right) \right). \tag{13}$$

We introduce the standard form of matrix differential in the numerator layout convention:

$$df = \text{tr} \left(\left(\frac{\partial f}{\partial \mathbf{X}} \right)^\top \mathbf{X} \right) \tag{14}$$

Similar to the partial derivative of α with respect to $\boldsymbol{\mu}$, given the standard form of matrix differential, we can get that:

$$\begin{aligned}
\frac{\partial \alpha}{\partial \Sigma} &= \frac{\Sigma^{-1}}{2\alpha (\mathbf{n}^\top \Sigma^{-1} \mathbf{n})^2} \left(-(\mathbf{n}^\top \Sigma^{-1} \mathbf{n})^2 \mathbf{m} \mathbf{m}^\top \right. \\
&\quad \left. + 2(\mathbf{m}^\top \Sigma^{-1} \mathbf{n})(\mathbf{n}^\top \Sigma^{-1} \mathbf{n}) \mathbf{n} \mathbf{m}^\top - (\mathbf{m}^\top \Sigma^{-1} \mathbf{n})^2 \mathbf{n} \mathbf{n}^\top \right) \Sigma^{-1}.
\end{aligned} \tag{15}$$

Table 1. PSNR results for every scene in Mip-NeRF 360 [2] dataset.

Method	bicycle	bonsai	counter	garden	kitchen	room	stump	average
Mip-NeRF [1]	24.37	33.46	29.55	26.98	32.23	31.64	26.40	29.23
3DHGS [7]	25.39	33.30	29.62	27.68	32.17	32.12	26.64	29.56
Scaffold-GS [8]	24.50	32.70	29.34	27.17	31.30	31.93	26.27	28.84
3DGS-MCMC [5]	26.15	32.88	29.51	28.16	32.27	32.48	27.80	29.89
3DGS [4]	25.25	31.98	28.70	27.41	30.32	30.63	26.55	28.69
Ours (3DGS)	25.17	32.18	29.19	27.41	31.69	31.78	26.69	29.16
SSS [9]	25.68	33.50	29.87	28.09	32.43	32.57	27.17	29.90
Ours (SSS)	25.76	33.59	29.89	28.12	32.52	32.65	27.21	29.96

Table 2. SSIM results for every scene in Mip-NeRF 360 [2] dataset.

Method	bicycle	bonsai	counter	garden	kitchen	room	stump	average
Mip-NeRF [1]	0.685	0.941	0.894	0.813	0.920	0.913	0.744	0.844
3DHGS [7]	0.768	0.950	0.909	0.868	0.930	0.921	0.770	0.873
Scaffold-GS [8]	0.705	0.946	0.914	0.842	0.928	0.925	0.784	0.848
3DGS-MCMC [5]	0.810	0.950	0.920	0.890	0.940	0.940	0.820	0.900
3DGS [4]	0.771	0.938	0.905	0.868	0.922	0.914	0.775	0.870
Ours (3DGS)	0.749	0.946	0.915	0.858	0.933	0.928	0.769	0.871
SSS [9]	0.798	0.956	0.926	0.882	0.939	0.938	0.813	0.893
Ours (SSS)	0.798	0.954	0.925	0.883	0.940	0.939	0.813	0.893

Table 3. LPIPS results for every scene in Mip-NeRF 360 [2] dataset.

Method	bicycle	bonsai	counter	garden	kitchen	room	stump	average
Mip-NeRF [1]	0.301	0.176	0.204	0.170	0.127	0.211	0.261	0.207
3DHGS [7]	0.202	0.180	0.201	0.104	0.125	0.220	0.215	0.178
Scaffold-GS [8]	0.306	0.185	0.191	0.146	0.126	0.202	0.284	0.220
3DGS-MCMC [5]	0.180	0.220	0.220	0.100	0.140	0.250	0.190	0.190
3DGS [4]	0.205	0.205	0.204	0.103	0.129	0.220	0.210	0.182
Ours (3DGS)	0.239	0.177	0.181	0.121	0.115	0.193	0.242	0.181
SSS [9]	0.173	0.151	0.156	0.090	0.104	0.167	0.174	0.145
Ours (SSS)	0.173	0.141	0.155	0.091	0.103	0.166	0.174	0.143

C. Additional Experimental Results

C.1. Detailed Results on Each Scene

We report detailed results on every scene in Mip-NeRF 360 [2], Tanks & Temples [6] and Deep Blending [3] for our modulated methods. We choose Mip-NeRF [1], 3DGS [4], Scaffold-GS [8], 3DHGS [7], 3DGS-MCMC [5] and SSS [9] as baselines.

As shown in Tabs. 1 to 6, in general, our modulation strategy outperforms both 3DGS [4] and SSS [9]. While the modulation strategy may not produce positive improvements in every scene, it delivers clear and consistent benefits in the majority of cases.

C.2. Multi-Resolution Analysis

To further validate the correctness of our theoretical insights, we conduct a comprehensive multi-resolution evaluation. As the ideal jinc kernel exhibits slow spatial decay, a practical implementation inevitably requires spatial truncation. We therefore compare the truncated jinc kernel with the standard Gaussian kernel used in 3DGS by training both methods at the native resolution, followed by evaluation at the original resolution, as well as several downsampled resolutions. The experiment is conducted in Tanks & Temples [6].

The results shown in Tab. 7 demonstrate a consistent and theoretically aligned trend. At the native resolution, the truncated jinc kernel exhibits a slight performance drop relative to the Gaussian baseline, which is expected given the truncation. However, as the test resolution decreases,

Table 4. PSNR results for every scene in Tanks & Temples [6] and Deep Blending [3].

Method	Tanks & Temples			Deep Blending		
	train	truck	average	drjohnson	playroom	average
Mip-NeRF [1]	19.52	24.91	22.22	29.14	29.66	29.40
3DHGS [7]	22.95	26.04	24.49	29.32	30.20	29.76
Scaffold-GS [8]	22.15	25.77	23.96	29.80	30.62	30.21
3DGS-MCMC [5]	22.47	26.11	24.29	29.00	30.33	29.67
3DGS [4]	21.09	25.18	23.14	28.77	30.04	29.41
Ours (3DGS)	22.18	25.54	23.86	29.41	30.26	29.84
SSS [9]	23.32	26.41	24.87	29.66	30.47	30.07
Ours (SSS)	23.47	26.42	24.95	29.71	30.62	30.17

Table 5. SSIM results for every scene in Tanks & Temples [6] and Deep Blending [3].

Method	Tanks & Temples			Deep Blending		
	train	truck	average	drjohnson	playroom	average
Mip-NeRF [1]	0.660	0.857	0.759	0.901	0.900	0.901
3DHGS [7]	0.827	0.887	0.857	0.904	0.907	0.905
Scaffold-GS [8]	0.822	0.883	0.853	0.907	0.904	0.906
3DGS-MCMC [5]	0.830	0.890	0.860	0.890	0.900	0.890
3DGS [4]	0.802	0.879	0.841	0.899	0.906	0.903
Ours (3DGS)	0.820	0.885	0.853	0.905	0.908	0.907
SSS [9]	0.850	0.897	0.873	0.905	0.909	0.907
Ours (SSS)	0.851	0.897	0.874	0.908	0.914	0.911

Table 6. LPIPS results for every scene in Tanks & Temples [6] and Deep Blending [3].

Method	Tanks & Temples			Deep Blending		
	train	truck	average	drjohnson	playroom	average
Mip-NeRF [1]	0.354	0.159	0.257	0.237	0.252	0.245
3DHGS [7]	0.197	0.141	0.169	0.240	0.243	0.242
Scaffold-GS [8]	0.206	0.147	0.177	0.250	0.258	0.254
3DGS-MCMC [5]	0.240	0.140	0.190	0.330	0.310	0.320
3DGS [4]	0.218	0.148	0.183	0.244	0.241	0.243
Ours (3DGS)	0.196	0.140	0.168	0.235	0.238	0.237
SSS [9]	0.166	0.109	0.138	0.249	0.245	0.247
Ours (SSS)	0.163	0.107	0.135	0.246	0.239	0.242

Table 7. Quantitative results with different resolutions in Tanks & Temples [6].

Method	Full Resolution			1/2 Resolution			1/4 Resolution			1/8 Resolution		
	PSNR	SSIM	LPIPS	PSNR	SSIM	LPIPS	PSNR	SSIM	LPIPS	PSNR	SSIM	LPIPS
3DGS [4]	23.14	0.841	0.183	22.16	0.838	0.136	18.39	0.686	0.195	15.77	0.538	0.252
Jinc	22.55	0.824	0.186	21.98	0.812	0.143	18.91	0.673	0.169	16.47	0.559	0.204

the spatial support required for accurate reconstruction becomes correspondingly smaller. Under these conditions, the jinc kernel more faithfully preserves the frequency characteristics of the underlying signal and outperforms the Gaus-

sian kernel. This resolution-dependent behavior provides additional evidence for the correctness of our theoretical analysis.

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