Supplemental Material
Gaussian Fusion: Accurate 3D Reconstruction via Geometry-Guided Displacement Interpolation

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1. Real Object Dataset

We illustrate our real object dataset at the end of this supplemental material (depth maps and masks). We place an Intel L515 packing box ($115mm \times 115mm \times 60mm$) under each item as a size reference.

2. Mathematical Definition and Theorem

For self-contained, we give the important definition (*Wasserstein space*) and theorem (*Displacement interpolation*) in the following subsections. It may help readers to understand our unique displacement interpolation framework.

2.1. Wasserstein Space

Definition 1 ***Wasserstein space**[1] Let $(\mathcal{X}, d)$ be a Polish metric space, and let $p \in [1, \infty)$. The Wasserstein space of order p is defined as*

$$P_p(\mathcal{X}) := \left\{ \mu \in P(\mathcal{X}); \int_{\mathcal{X}} d(x_0, x)^p \mu(dx) < +\infty \right\}$$

where μ is probability measures on $\mathcal{X}$, and $x_0 \in \mathcal{X}$ is arbitrary. This space does not depend on the choice of the point x_0. Then W_p defines a (finite) distance on $P_p(\mathcal{X})$.

2.2. Displacement Interpolation

Theorem 2.1 ***Displacement interpolation**[1]*
Let $(\mathcal{X}, d)$ be a Polish space, and $(\mathcal{A})^{0,1}$ a coercive Lagrangian action on $\mathcal{X}$, with continuous cost functions $c^{s,t}$. Whenever $0 \leq s < t \leq 1$, denote by $C^{s,t}(\mu, \nu)$ the optimal transport cost between the probability measures μ and ν for the cost $c^{s,t}$; write $c = c^{0,1}$ and $C = C^{0,1}$. Let μ_0 and μ_1 be any two probability measures on $\mathcal{X}$, such that the optimal transport cost $C(\mu_0, \mu_1))$ is finite. Then, given a continuous path $(\mu_t)_{0 \leq t \leq 1}$, the following properties are equivalent:
(i) For each $t \in [0, 1]$, μ_t is the law of γ_t, where $(\gamma_t)_{0 \leq t \leq 1}$ is a dynamical optimal coupling of (μ_0, μ_1);
(ii) For any of three intermediate times $t_1 < t_2 < t_3$ in $[0, 1]$,

$$C^{t_1,t_2}(\mu_{t_1}, \mu_{t_2}) + C^{t_2,t_3}(\mu_{t_2}, \mu_{t_3}) = C^{t_1,t_3}(\mu_{t_1}, \mu_{t_3});$$

(iii) The path $(\mu_t)_{0 \leq t \leq 1}$ is a minimizing curve for the coercive action functional defined on $P(\mathcal{X})$ by

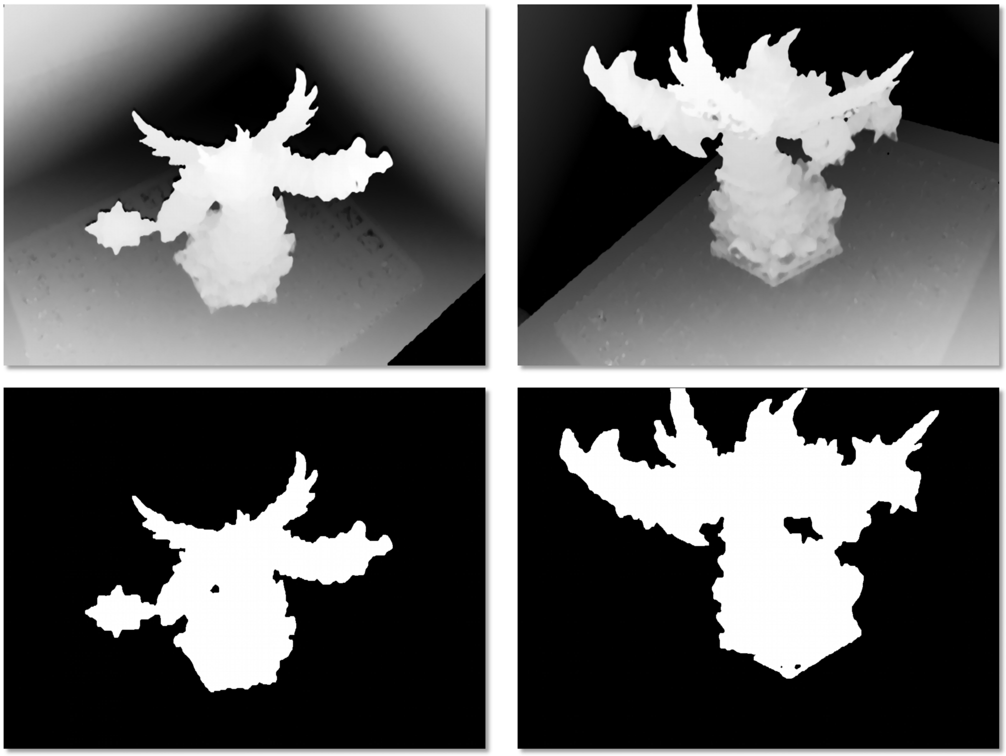
$$\begin{aligned} \mathbb{A}^{s,t}(\mu) &= \sup_{N \in \mathbb{N}} \sup_{s = t_0 < t_1 < \ldots < t_N = t} \sum_{i=0}^{N-1} C^{t_i, t_{i+1}}(\mu_{t_i}, \mu_{t_{i+1}}) \\ &= \inf_{\gamma} \mathbb{E}\mathcal{A}^{s,t}(\gamma), \end{aligned} \tag{1}$$

where the last infimum is over all random curves $\gamma : [s, t] \to \mathcal{X}$ such that law $(\gamma_\tau) = \mu_\tau (s \leq \tau \leq t)$.

Corollary 1 ***Uniqueness of displacement interpolation**[1]*
With the same assumptions as in Theorem Displacement interpolation, if:
(i) there is a unique optimal transport plan π between μ_0 and μ_1;
(ii) $\pi(dx_0, dx_1)-$ almost surely, x_0 and x_1 are joined by a unique minimizing curve;
then there is a unique displacement interpolation $(\mu_t)_{0 \leq t \leq 1}$ joining μ_0 to μ_1.

References

[1] Cédric Villani. *Optimal transport: old and new*, volume 338. Springer Science & Business Media, 2008.



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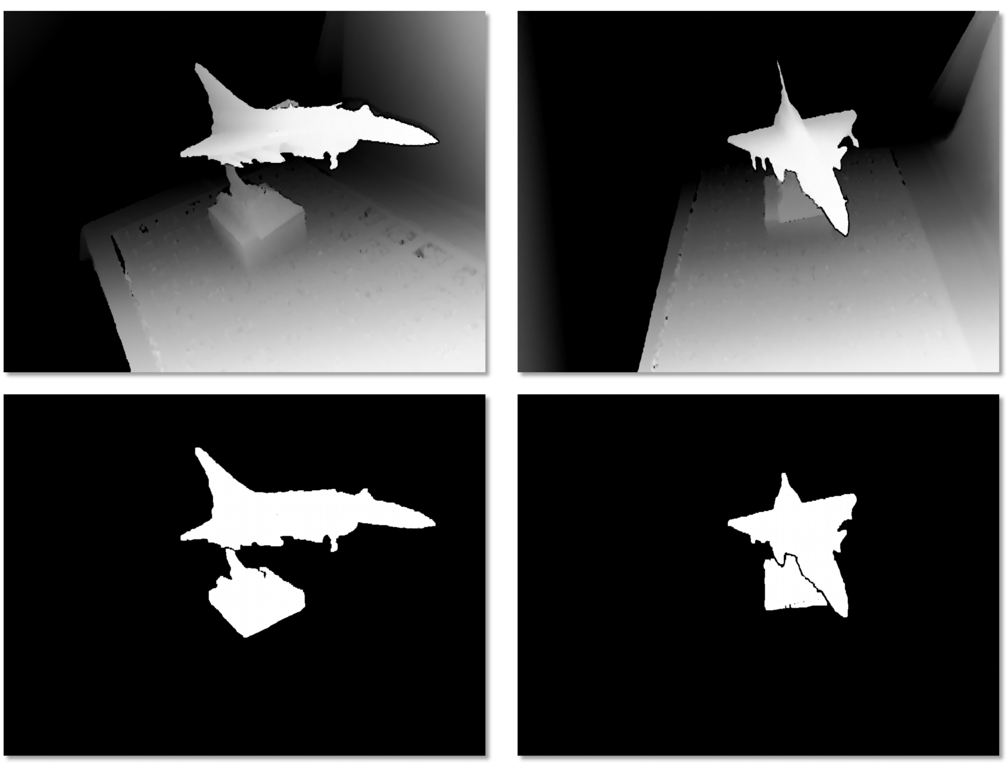

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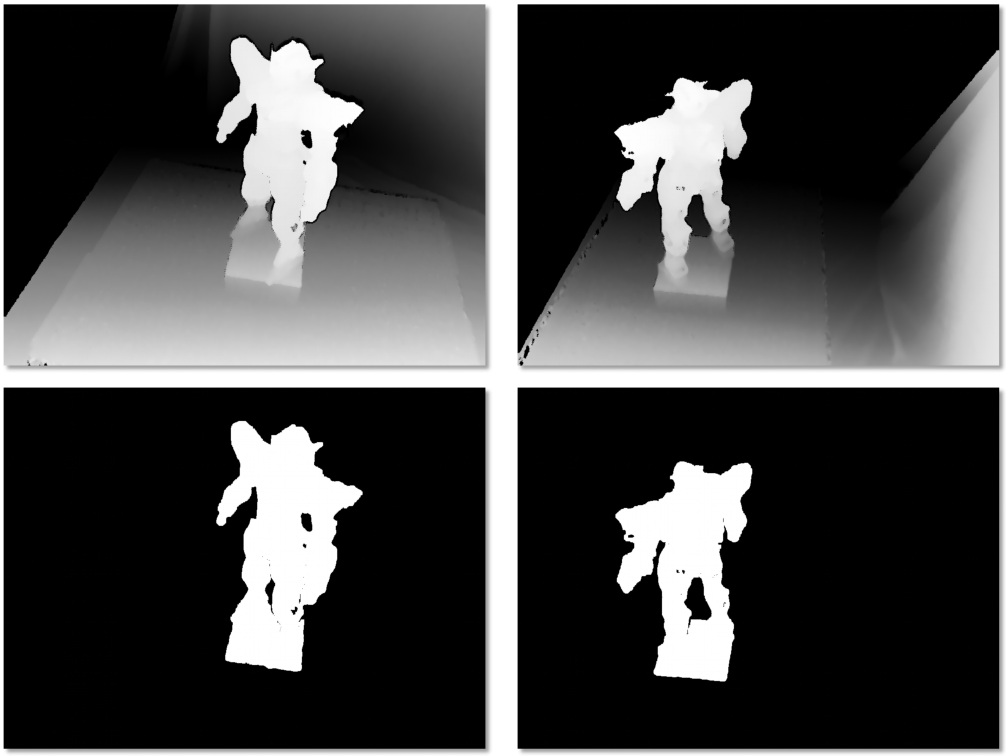

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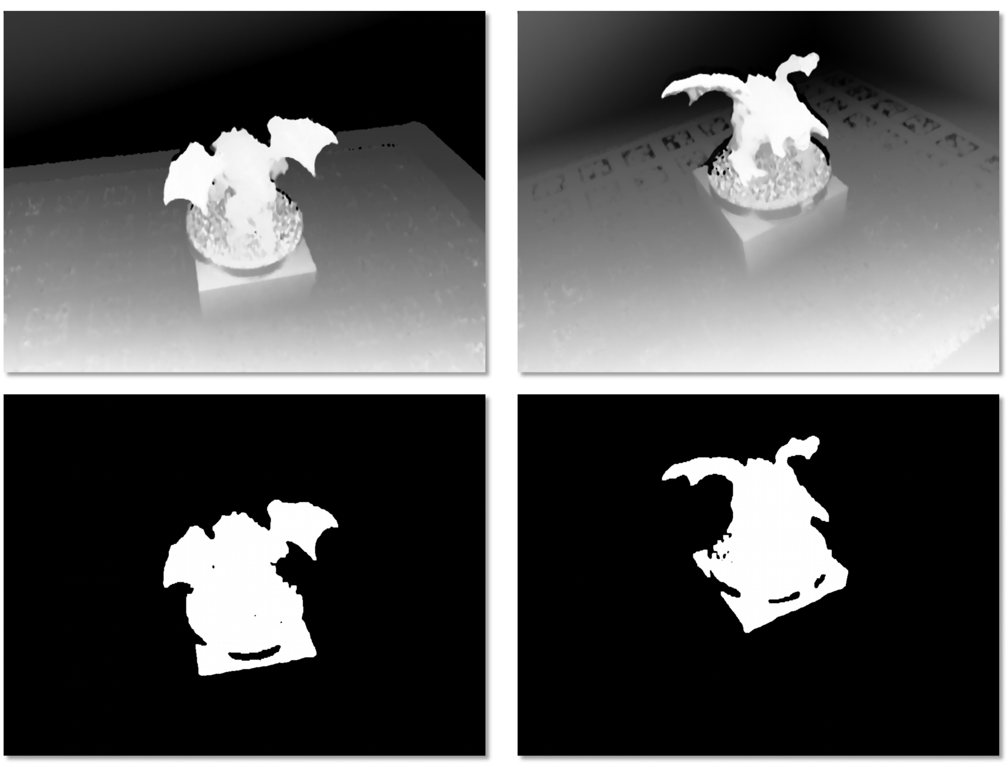

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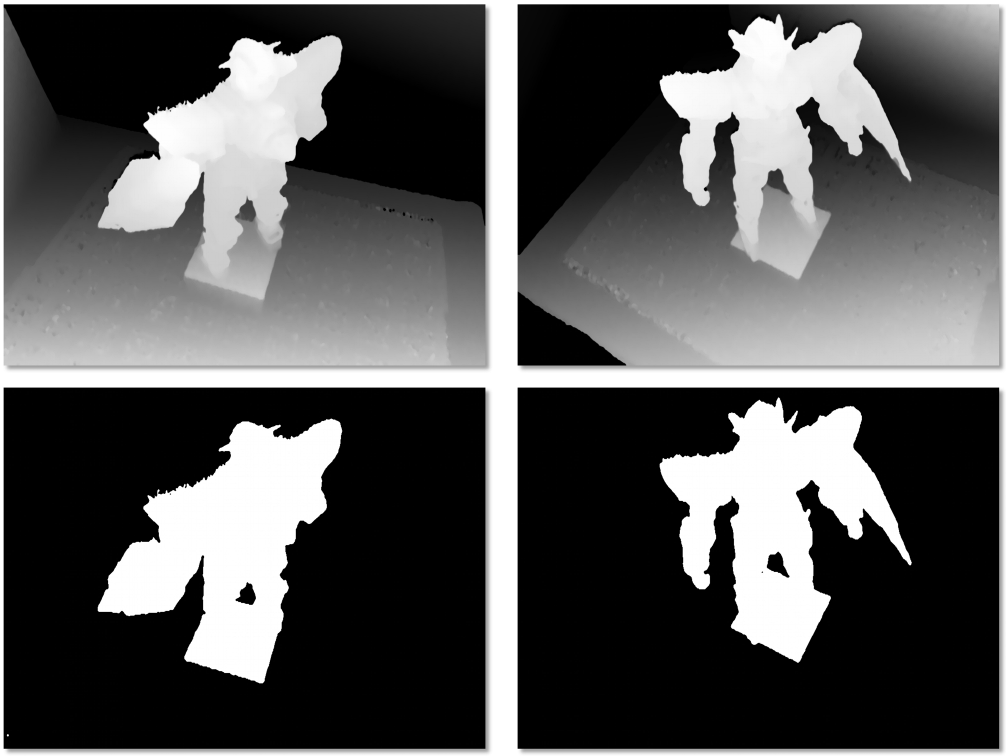

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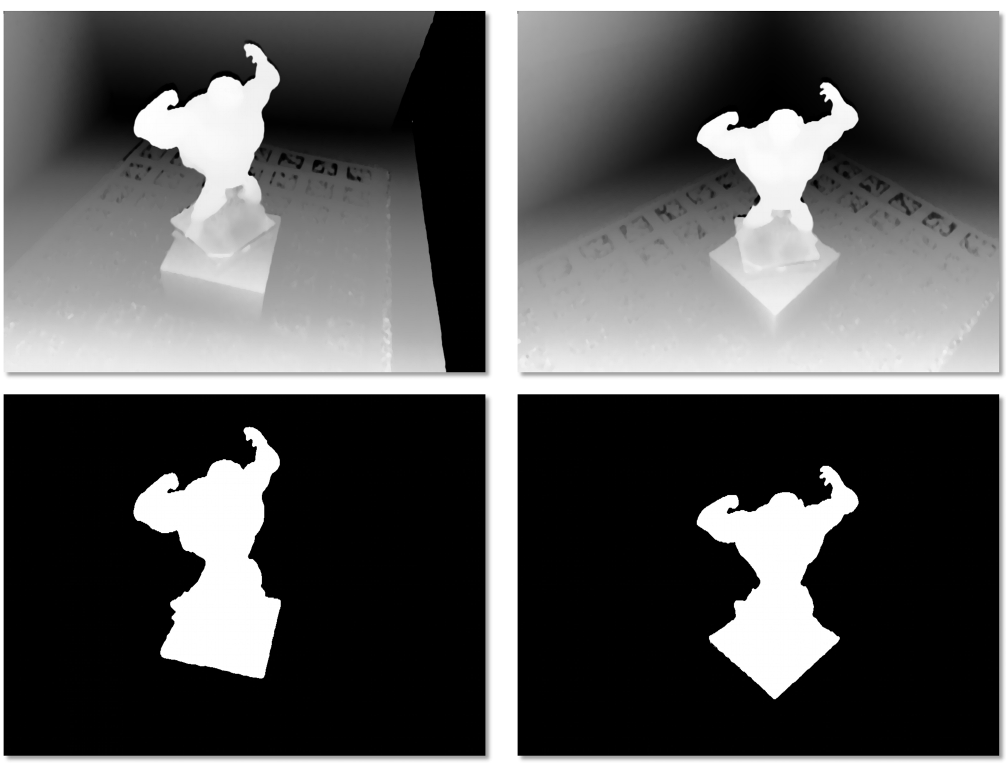

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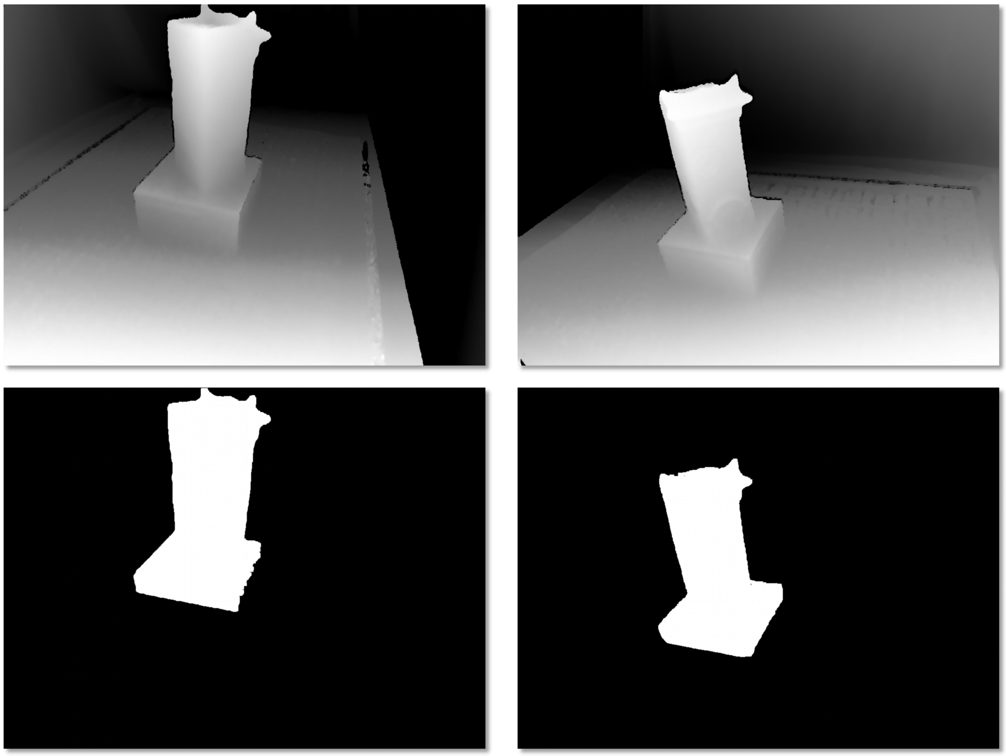
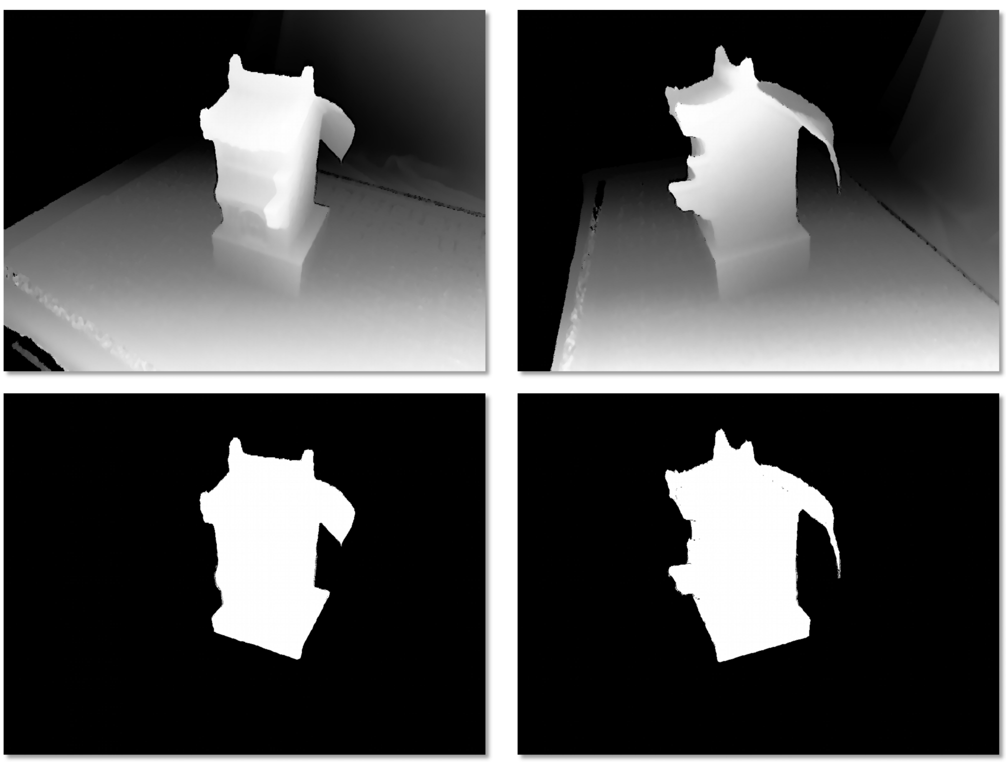
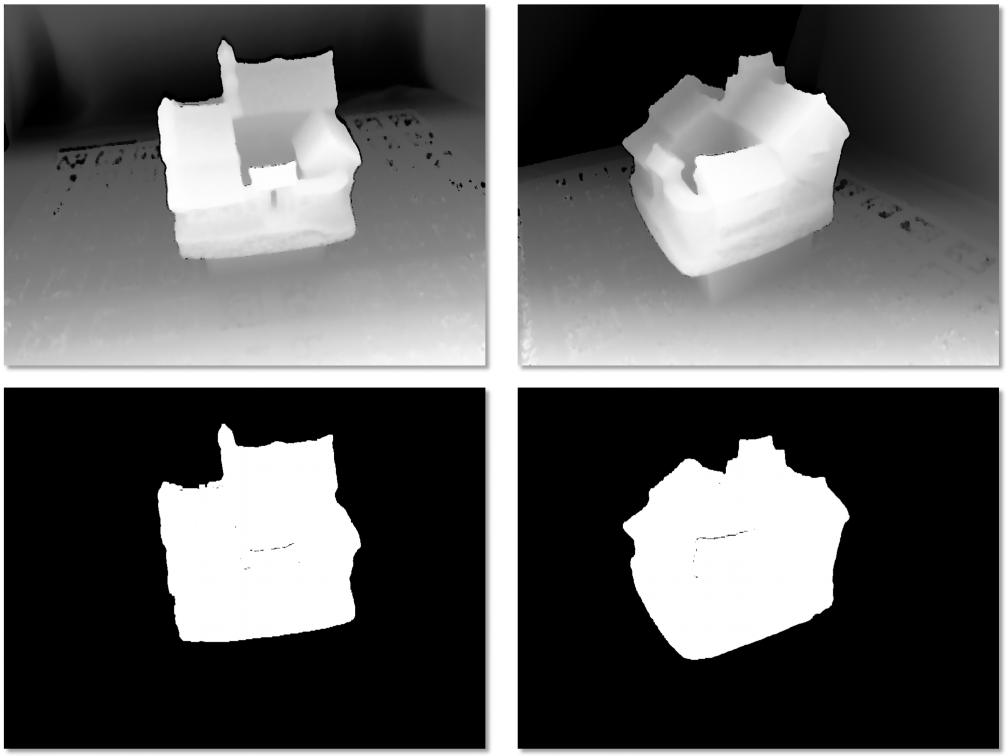


House_0



House_1



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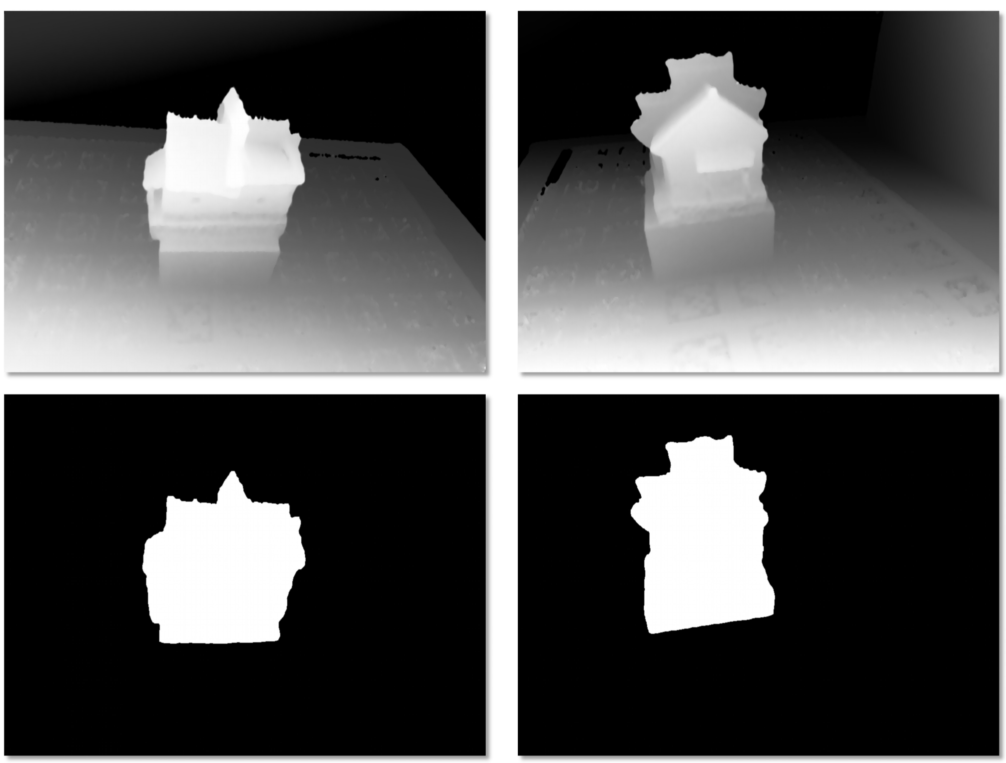

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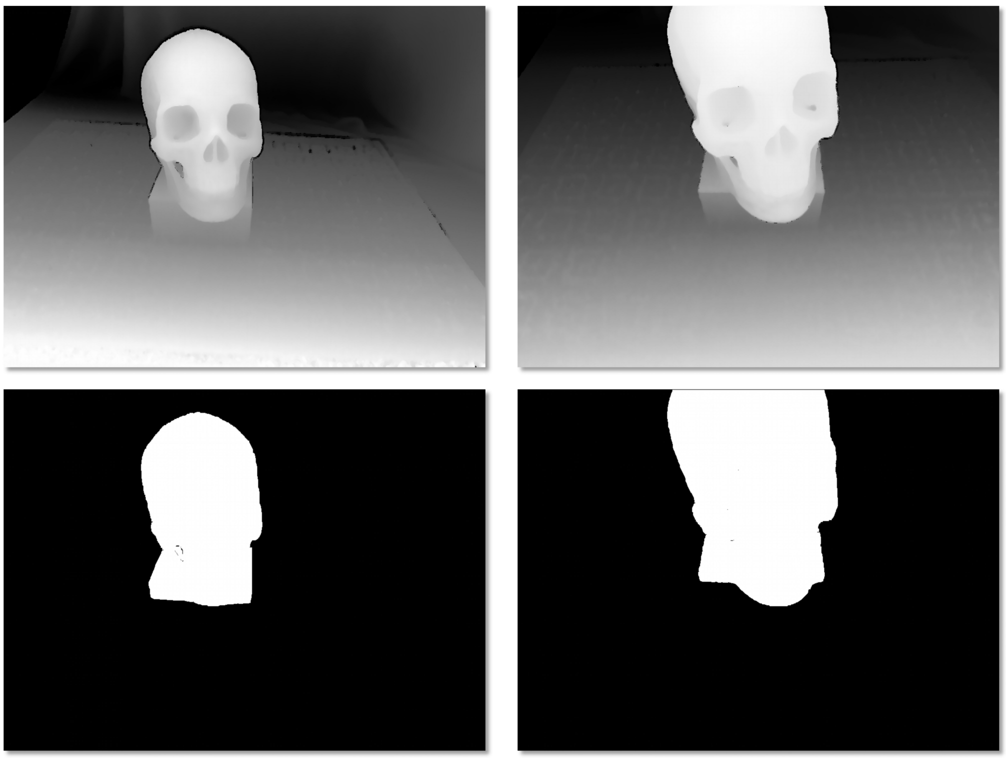

Skeleton