A.1. Convexity and Minimum of $\text{QE}(\tau)$

We first revisit the formulations for the $\tau$ quantile $Q(\tau)$ and scaling factor $\alpha$ in the following:

$$Q(\tau) = -b \ln(2 - 2\tau). \hspace{1cm} (10)$$

$$\alpha = b - (Q(\tau) + b) \exp\left(-\frac{Q(\tau)}{b}\right) + 2Q(\tau)(1 - \tau). \hspace{1cm} (13)$$

Combining Eq. (10) and Eq. (13), we can rewrite $\alpha$ as

$$\alpha = b(2\tau - 1). \hspace{1cm} (A1)$$

Recall that the quantization error under our framework is given as:

$$\text{QE}(\tau) = (\alpha - b)^2(1 + \exp(-\frac{Q(\tau)}{b})) + b^2$$

$$- ((b + Q(\tau))^2 - 2\alpha Q(\tau)) \exp(-\frac{Q(\tau)}{b}) + 2(1 - \tau)(Q(\tau) - \alpha)^2. \hspace{1cm} (14)$$

By putting Eq. (10) and Eq. (A1) into Eq. (14), we reformulate $\text{QE}(\tau)$ as

$$\text{QE}(\tau) = b^2(16\tau^3 + 44\tau^2 - 40\tau - 4(\tau - 1) \ln(2 - 2\tau) + 13). \hspace{1cm} (A2)$$

According to Eq. (A2), the derivative of $\text{QE}(\tau)$ w.r.t. $\tau$ can be derived as

$$\frac{\partial \text{QE}(\tau)}{\partial \tau} = -4b^2(12\tau^2 - 22\tau + \ln(2 - 2\tau) + 11)$$

$$= 4b^2G(\tau), \hspace{1cm} (A3)$$

where

$$G(\tau) = -12\tau^2 + 22\tau - \ln(2 - 2\tau) - 11. \hspace{1cm} (A4)$$

Figure A1. Flipping percentage of “dead weights” with and without our ReCU (ResNet-18 on CIFAR-100).

Note that $b$ is estimated via the maximum likelihood estimation as

$$\hat{b} = \text{Mean}(|W|), \hspace{1cm} (11)$$

which indicates $b \neq 0$. We can know that

$$\frac{\partial \text{QE}(\tau)}{\partial \tau} = 0 \iff G(\tau) = 0. \hspace{1cm} (A5)$$

Thus, the extreme value of $\text{QE}(\tau)$ is irrelevant to $b$. Further, we yield the derivative of $G(\tau)$ w.r.t. $\tau$ as

$$\frac{\partial G(\tau)}{\partial \tau} = -24\tau + \frac{1}{1 - \tau} + 22. \hspace{1cm} (A6)$$

From Eq. (A6), it is easy to know that $\frac{\partial G(\tau)}{\partial \tau} > 0$ if $\tau \leq 1$. Therefore, $G(\tau)$ is monotonically increasing when $\tau \leq 1$. By solving $G(\tau) = 0$, we have $\tau \approx 0.82$. That means when $0.82 < \tau \leq 1$, $G(\tau) > 0$, while when $0.5 < x < 0.82$, $G(\tau) < 0$. That is to say, $\frac{\partial \text{QE}(\tau)}{\partial \tau} \leq 0$ when $0.5 < \tau < 0.82$, and $\frac{\partial \text{QE}(\tau)}{\partial \tau} > 0$ when $0.82 < \tau \leq 1$. Thus $\text{QE}(\tau)$ is a convex function w.r.t. $\tau \in (0.5, 1]$ and reaches the minimum when $\tau \approx 0.82$, which completes the proof.

A.2. Evidence on Reviving Dead Weights

We compare the sign difference (flipping percentage) of “dead weights” within 20% the largest magnitudes at points

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figureA1.png}
\caption{Flipping percentage of “dead weights” with and without our ReCU (ResNet-18 on CIFAR-100).}
\end{figure}
Table A1. Top-1 accuracy of ResNet-18 w.r.t. different training epochs on CIFAR-100.

<table>
<thead>
<tr>
<th>Training epochs</th>
<th>100</th>
<th>300</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vanilla</td>
<td>52.1</td>
<td>59.6</td>
<td>62.0</td>
</tr>
<tr>
<td>Ours</td>
<td>66.3</td>
<td>68.2</td>
<td>69.1</td>
</tr>
</tbody>
</table>

of half and final training epochs in Fig. A1. As can be seen, less than 10% of “dead weights” are updated without ReCU. In contrast, about 13% to 35% are updated with ReCU. Thus, our ReCU greatly revives “dead weights”.

A.3. Training Convergence

As discussed in Sec. 4.1, the “dead weights” introduces an obstacle to the training convergence of BNNs. In Tab. A1, we show the effectiveness of ReCU in overcoming this problem. As seen, ReCU achieves 66.3% top-1 accuracy with only 100 training epochs, while the vanilla BNN obtains 62.0% even when it is trained for 600 epochs.