ReCU: Reviving the Dead Weights in Binary Neural Networks (Supplementary Material)

A.1. Convexity and Minimum of $\mathrm{QE}(\tau)$

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We first revisit the formulations for the τ quantile $Q_{(\tau)}$ and scaling factor α in the following

$$Q_{(\tau)} = -b\ln(2-2\tau). \tag{10}$$

$$\alpha = b - (Q_{(\tau)} + b)\exp(-\frac{Q_{(\tau)}}{b}) + 2Q_{(\tau)}(1-\tau). \tag{13}$$

Combining Eq. (10) and Eq. (13), we can rewrite α as

$$\alpha = b(2\tau - 1). \tag{A1}$$

Recall that the quantization error under our framework is given as

$$\begin{aligned}\mathrm{QE}(\tau) &= (\alpha-b)^2\big(1+\exp(-\frac{Q_{(\tau)}}{b})\big) + b^2 \\ &\quad - \big((b+Q_{(\tau)})^2 - 2\alpha Q_{(\tau)}\big)\exp(-\frac{Q_{(\tau)}}{b}) \\ &\quad + 2(1-\tau)(Q_{(\tau)}-\alpha)^2.\end{aligned} \tag{14}$$

By putting Eq. (10) and Eq. (A1) into Eq. (14), we reformulate $\mathrm{QE}(\tau)$ as

$$\begin{aligned}&\mathrm{QE}(\tau) \\ &= b^2\big(-16\tau^3 + 44\tau^2 - 40\tau - 4(\tau-1)\ln(2-2\tau) + 13\big).\end{aligned} \tag{A2}$$

According to Eq. (A2), the derivative of $\mathrm{QE}(\tau)$ *w.r.t.* τ can be derived as

$$\begin{aligned}\frac{\partial\,\mathrm{QE}(\tau)}{\partial\tau} &= -4b^2(12\tau^2 - 22\tau + \ln(2-2\tau) + 11) \\ &= 4b^2 G(\tau),\end{aligned} \tag{A3}$$

where

$$G(\tau) = -12\tau^2 + 22\tau - \ln(2-2\tau) - 11. \tag{A4}$$

Note that b is estimated via the maximum likelihood estimation as

$$\hat{b} = \mathrm{Mean}(|\mathcal{W}|), \tag{11}$$

which indicates $b \neq 0$. We can know that

$$\frac{\partial\,\mathrm{QE}(\tau)}{\partial\tau} = 0 \iff G(\tau) = 0. \tag{A5}$$

Thus, the extreme value of $\mathrm{QE}(\tau)$ is irrelevant to b. Further, we yield the derivative of $G(\tau)$ *w.r.t.* τ as

$$\frac{\partial G(\tau)}{\partial\tau} = -24\tau + \frac{1}{1-\tau} + 22. \tag{A6}$$

From Eq. (A6), it is easy to know that $\frac{\partial G(\tau)}{\partial\tau} > 0$ if $\tau \le 1$. Therefore, $G(\tau)$ is monotonically increasing when $\tau \le 1$. By solving $G(\tau) = 0$, we have $\tau \approx 0.82$. That means when $0.82 < \tau \le 1$, $G(\tau) > 0$, while when $0.5 < x < 0.82$, $G(\tau) < 0$. That is to say, $\frac{\partial QE(\tau)}{\partial\tau} < 0$ when $0.5 < \tau < 0.82$, and $\frac{\partial\,\mathrm{QE}(\tau)}{\partial\tau} > 0$ when $0.82 < \tau \le 1$. Thus $\mathrm{QE}(\tau)$ is a convex function *w.r.t.* $\tau \in (0.5, 1]$ and reaches the minimum when $\tau \approx 0.82$, which completes the proof. ■

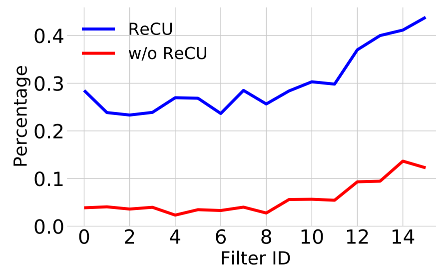

Figure A1. Flipping percentage of "dead weights" with and without our ReCU (ResNet-18 on CIFAR-100).

A.2. Evidence on Reviving Dead Weights

We compare the sign difference (flipping percentage) of "dead weights" within 20% the largest magnitudes at points

Table A1. Top-1 accuracy of ResNet-18 *w.r.t.* different training epochs on CIFAR-100.

Training epochs	100	300	600
Vanilla	52.1	59.6	62.0
Ours	66.3	68.2	69.1

of half and final training epochs in Fig. A1. As can be seen, less than 10% of "dead weights" are updated without ReCU. In contrast, about 13% to 35% are updated with ReCU. Thus, our ReCU greatly revives "dead weights".

A.3. Training Convergence

As discussed in Sec. 4.1, the "dead weights" introduces an obstacle to the training convergence of BNNs. In Tab. A1, we show the effectiveness of ReCU in overcoming this problem. As seen, ReCU achieves 66.3% top-1 accuracy with only 100 training epochs, while the vanilla BNN obtains 62.0% even when it is trained for 600 epochs.