Appendix

Proof of Proposition 3.1.

Proof. Suppose that $\Phi^{\mathcal{M}}$ is a presheaf morphism and $U \in \mathcal{T}_B$ is an open set. Let $F = (f_V)_{V \in \mathcal{T}_B}$ be a consistent assignment. Then for any $V \subseteq U$, $\text{res}^{\mathcal{D}}_{U,V}(f_U) = f_V$. Since $\Phi^{\mathcal{M}}$ is a presheaf morphism, then by definition for any $f_U \in \mathcal{D}(U)$,

$$\text{res}^{\mathcal{M}}_{U,V}(\Phi^{\mathcal{M}}_U(f_U)) = \Phi^{\mathcal{M}}_V(\text{res}^{\mathcal{D}}_{U,V}(f_U)).$$

But then since A is consistent, $\Phi^{\mathcal{M}}_V(\text{res}^{\mathcal{D}}_{U,V}(f_U)) = \Phi^{\mathcal{M}}_V(f_V)$, so $\text{res}^{\mathcal{M}}_{U,V}(\Phi^{\mathcal{M}}_U(f_U)) = \Phi^{\mathcal{M}}_V(f_V)$ and therefore

$$d_V(\text{res}^{\mathcal{M}}_{U,V}(\Phi^{\mathcal{M}}_U(f_U)), \Phi^{\mathcal{M}}_V(f_V)) = 0.$$

Since this is true for all open $V \subseteq U$, then $\text{Incon}(F, U) = 0$.

Now suppose that for any consistent assignment F of $\mathcal{D}$, the local inconsistency of F at any open set U is 0. We need to show that for any $f_U \in \mathcal{D}(U)$ and any open $V \subseteq U$, $\text{res}^{\mathcal{M}}_{U,V}(\Phi^{\mathcal{M}}_U(f_U)) = \Phi^{\mathcal{M}}_V(\text{res}^{\mathcal{D}}_{V,U}(f_U))$.

Choose some $y \in Y$. Let $\widetilde{f}_I : I \to Y$ be the function in $\mathcal{D}(I)$ that maps

$$\widetilde{f}_I(i) := \begin{cases} f_U(i) & \text{if } i \in U \\ y & \text{otherwise.} \end{cases}$$

$\widetilde{f}_I$ defines a consistent assignment $(\text{res}^{\mathcal{D}}_{I,U}(\widetilde{f}_I))_{U \in \mathcal{T}_B}$. In particular $f_U = \text{res}^{\mathcal{D}}_{I,U}(\widetilde{f}_I)$. Then F has inconsistency 0 by assumption and hence

$$\max_{W \in \Lambda_U} d_W(\text{res}^{\mathcal{M}}_{U,W}(\Phi^{\mathcal{M}}_U(f_U)), \Phi^{\mathcal{M}}_W(f_W)) = 0.$$

In particular, this implies that $d_V(\text{res}^{\mathcal{M}}_{U,V}(\Phi^{\mathcal{M}}_U(f_U)), \Phi^{\mathcal{M}}_V(f_V)) = 0$ and $f_V = \text{res}^{\mathcal{D}}_{U,V}(f_U)$ hence

$$\text{res}^{\mathcal{M}}_{U,V}(\Phi^{\mathcal{M}}_U(f_U)) = \Phi^{\mathcal{M}}_V(\text{res}^{\mathcal{D}}_{V,U}(f_U))$$

Since this is true for all open sets U, V with $V \subseteq U$ and all $f_U \in \mathcal{D}(U)$, then $\Phi^{\mathcal{M}}$ is a presheaf morphism. □