Building a Winning Team: Selecting Source Model Ensembles using a Submodular Transferability Estimation Approach

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Abstract

Estimating the transferability of publicly available pre-trained models to a target task has assumed an important place for transfer learning tasks in recent years. Existing efforts propose metrics that allow a user to choose one model from a pool of pre-trained models without having to fine-tune each model individually and identify one explicitly. With the growth in the number of available pre-trained models and the popularity of model ensembles, it also becomes essential to study the transferability of multiple-source models for a given target task. The few existing efforts study transferability in such multi-source ensemble settings using just the outputs of the classification layer and neglect possible domain or task mismatch. Moreover, they overlook the most important factor while selecting the source models, viz., the cohesiveness factor between them, which can impact the performance and confidence in the prediction of the ensemble. To address these gaps, we propose a novel Optimal tranSport-based suBmOdular tRaNsferability metric (OSBORN) to estimate the transferability of an ensemble of models to a downstream task. OSBORN collectively accounts for image domain difference, task difference, and cohesiveness of models in the ensemble to provide reliable estimates of transferability. We gauge the performance of OSBORN on both image classification and semantic segmentation tasks. Our setup includes 28 source datasets, 11 target datasets, 5 model architectures, and 2 pre-training methods. We benchmark our method against current state-of-the-art metrics MS-LEEP and E-LEEP, and outperform them consistently using the proposed approach.

1. Introduction

In computer vision, transfer learning is a go-to strategy to train Deep Neural Networks (DNNs) on newer domains and datasets across tasks such as image classification [30, 21], image segmentation [44, 60] and object detection [16, 42]. This widespread usage is due to the easy availability of a large pool of open-sourced pre-trained models (trained on large-scale datasets such as ImageNet [31, 3]), which, when fine-tuned, achieve faster convergence and better performance than training from scratch. However, every time a user wants to employ transfer learning, the question that has increasingly grown relevant with an increased number of source models is: “Which combination of dataset and architecture should I pick to fine-tune to achieve the best performance on my target dataset?” To solve this, we need a tool that helps us choose a source or set of source models, which require minimal fine-tuning and achieves maximal performance.

Transferability estimation (TE) metrics have been proposed in recent years to tackle this problem [48, 36, 58, 47, 39]. With these metrics, a particular source model can be selected without conducting expensive fine-tuning of all available source models on the target training set. Most efforts in this direction are, however limited by their capability of selecting only a single source model, thus restricting their use in an ensemble learning setting. There has been only one work so far [1] which extends an existing single-source transferability estimation method [36] to an ensemble setting. While this work showed promising results, it did not consider the similarity between source and target datasets in the latent representation space, or account for the relationships between individual models in the ensemble. This problem space remains nascent at this time, necessitating more efforts to estimate transferability reliably in different conditions.

Ensemble models have been popular for a few decades now in machine learning [14, 7, 51]. Ensemble models are known to increase task accuracy, decrease overall predictive variance and increase robustness against out-of-distribution
data samples [15]. Recent efforts have shown the usefulness of ensembles of pre-trained models [52], especially considering the widespread availability of pre-trained models in the community [41]. The problem of estimating transferability for a model ensemble from a large source model pool becomes even more relevant in this context.

In this work, we introduce a novel transferability estimation metric specifically designed for ensemble selection called Optimal Transport-based Submodular Transferability metric (OSBORN). As stated earlier, a recent effort in this direction [1] showed promising results for such a score, but focused on individual model’s performance (via the classifier’s outputs) and did not consider the feature (latent representation) space mismatch, or how these models interact with each other in the ensemble. To address this, OSBORN measures the latent space mismatch between the source and the target datasets (domain difference) in addition to the mismatch in the classifier’s outputs (task difference). Also, to account for the interaction between models in the ensemble, we introduce a novel model cohesion term, which captures the mutual cooperation between models towards forming an ensemble. Cohesion is required to ensure that individual models in an ensemble are in agreement with each other in terms of predictions (and not voting out each other). Thus, in this work, we propose a domain, task and cohesion-aware transferability estimator for ensemble selection from a source pool of multiple models.

Beyond bringing the abovementioned factors into transferability estimation for ensembles, we show that the proposed score can be viewed as a submodular set function [4]. This allows us to follow a greedy maximization strategy, which is known to provide a high-quality solution for the problem based on well-known theoretical guarantees [34]. We thus select cohesive and closely related models for a particular target dataset. To evaluate our metric, we conduct extensive experiments using 28 source datasets, 11 target datasets, and 5 model architectures. In downstream tasks, we consider fully-supervised pre-training-based image classification, self-supervised pre-training-based image classification, semantic segmentation as well as domain adaptation. Table 1 presents an overview of our experiment breadth, as compared to other recent efforts on this problem. In particular, to the best of our knowledge, we are the first to perform transferability estimation of ensembles for image classification and domain adaptation tasks.

To summarize, we make the following contributions: (1) We introduce a novel transferability estimation metric for ensemble selection that considers domain similarity, task similarity and inter-model cohesion in its design; (2) We show that viewing the proposed metric as a submodular set function allows us to use a simple greedy maximization strategy to select a source model ensemble for a given target dataset; (3) We study the performance of our metric across a wide range of downstream tasks and model pools; (4) We evaluate the reliability of our metric using different correlation metrics in our studies, and also carry out additional analysis and ablation studies to study its usefulness. We outperform earlier methods by a margin of 58.62%, 66.06%, and 96.36% in terms of Pearson Correlation Coefficient (PCC), Kendall τ (KT) [25] and Weighted Kendall τ (WKT) [50] for the image classification task. 1

2. Related Work

Transfer Learning: Over the years, transfer learning has been applied and explored across various fields [10, 33, 2, 5], as well as across datasets, model architectures, and pre-training strategies [32, 12, 20]. These efforts have included the study of interesting and practical questions such as which particular layers are more transferable [57] or estimating the correlation between pre-training and fine-tuning performance [27]. Beyond finetuning of source models to target datasets, task transfer methods [59, 11] have also

1Project page: https://vimalkb007.github.io/OSBORN/
studied relationships between visual tasks such as semantic segmentation, depth prediction and vanishing point prediction, or used attribution maps to relate such tasks [45, 46]. In contrast to the aforementioned methods, the objective of our work is dataset transferability estimation.

Transferability Estimation Metrics (Single Source): As stated earlier, gauging transferability reduces the effort in finding an optimal source model for a particular target dataset because it averts the expensive fine-tuning process. In recent years, significant efforts have been made in this problem space, considering the relevance of this problem to practitioners. The H-Score was proposed [6] to measure the usefulness (in terms of discriminativeness) of pre-trained source models for the target task. While this method shows promising results as a pioneer work in this field, it misses considering the scenarios where the source and target data have different distributions. Subsequently, NCE [48], and LEEP [36] developed methods that used the classifier outputs of pre-trained source models when the target dataset is forward-propagated through the model to estimate the loglikelihood of the target dataset. NCE largely focused on estimating transferability in scenarios where the source and target tasks share the same input data (e.g., face recognition and facial attribute classification). Subsequent methods such as LogME [58] also showed that likelihood methods might be prone to over-fitting. To tackle this, LogME [58] estimated the maximum value of label evidence (instead of maximum likelihood) given the feature set extracted by the pre-trained source models. Considering the fact that previous methods largely relied on classifier outputs and their sub-optimal performance in practical scenarios like cross-domain settings, OTCE [47] proposed an optimal transport framework to compute domain difference (based on feature space) and task difference (based on label space) to estimate transferability. This method leveraged the source model’s latent representations in addition to classifier outputs with no explicit assumptions on the source and target datasets. All the above works are, however focused on estimating transferability from a single source model to a target dataset.

Transferability Estimation Metrics (Multi-Source Ensembles): Agostinelli et al [1] recently proposed the first work on extending transferability estimation to select source model ensembles in [1], specifically focused on semantic segmentation. This work extends LEEP [36] to ensembles, and shows promising results in the considered settings. Our work builds on this effort in multiple ways: (i) instead of solely relying on classifier outputs for estimating transferability [36, 1, 48], we also consider the domain mismatch in the latent feature representation space; (ii) beyond looking at the individual model’s outputs in an ensemble, we also consider the interactions and correlation between the model outputs; (iii) we make no assumptions on the source and target data distributions; and (iv) while [1] focused on segmentation, we show our method’s results on classification, segmentation and domain adaptation tasks. We also show results on multiple pre-training strategies while previous works [36, 58, 48, 47] mostly focus on fully-supervised pre-training strategies. Our proposed metric can also be viewed as a submodular function, which allows us to leverage ranking-based greedy optimization strategies to make it efficient in practice.

Ensemble Learning. Learning ensembles of models has been popular in machine learning to increase overall task performance, decrease prediction variance, prevent over-fitting, and increase out-of-distribution robustness [7, 18, 56, 38]. More recent efforts in training ensembles of neural network models have focused on speeding up their training [49, 52], leveraging a single model’s capacity to train multiple subnetworks whose predictions are ensembled to improve robustness [19], or studying mixture-of-experts paradigms which bring together thousands of subnetworks for large language models [43]. We clarify that our work focuses rather on selecting model ensembles from a larger source model pool via estimating transferability without explicitly training ensembles themselves. One can view our work as a step before ensemble learning when there is a larger model pool and only few models can be ensemble.

As stated in [1], this setting is commonly encountered by a practitioner in the real-world across application domains.

3. Background and Preliminaries

Notations: Given M source datasets, we denote the rth source dataset as \( D_s^r = \{ (x_i^r, y_i^r) \}_{i=1}^{N^r} \sim P_s^r(x, y) \) and target dataset as \( D_t = \{ (x_i^t, y_i^t) \}_{i=1}^{N^t} \sim P_t(x, y) \) where, \( x_i^r, x_i^t \in X \), \( y_i^r, y_i^t \in Y \), and \( y_i^t \in \mathcal{Y}_t \). Note that we do not restrict the label spaces \( P(Y_s^r) \) and \( P(Y_t) \) to span the same category set. We base our study on a domain-agnostic and task-agnostic setting.

Transferability Estimation for Ensembles: For every source dataset \( D_s^r \), we assume there exists a pre-trained model on that dataset denoted by \( (\theta_s^r, h_s^r) \) where \( \theta \) is the
In order to minimize the collection of such source models. As stated earlier, we focus on a multiple source model selection setting (i.e. ensembles) where our metric provides a transferability estimation (TE) score \( \alpha_{M,e \rightarrow \ell} \) for a given subset of models \( M_e \) from the source pool \( M \). When correlated to the accuracy \( A_{M,e \rightarrow \ell} \) (i.e. fine-tuned accuracy of the ensemble on the target test set), this TE score provides the reliability of the transferability estimate. Following [1], we calculate the ensemble accuracy by fine-tuning individual models in subset \( M_e \) (both \( \theta \) and \( h \)) on the target train set and averaging their predictions on the target test set.

**Submodularity in TE for Ensembles.** The main idea of TE involves choosing optimal source models for a given target dataset. Apart from performance & computation trade-offs, a crucial motivation to select a subset of models is to mitigate risk of negative transfer. Fig 2 herein shows that opting for all models in the ensemble could lead to a decrease in overall performance compared to selecting a smaller set of models. This can be due to the detrimental impact of weak or non-transferable models in the ensemble, highlighting the importance of carefully combining models to ensure optimal performance. Further, finding an optimal ensemble for a given target dataset requires checking all possible combinations of different source models for a particular ensemble size. This exhaustive process is an NP-hard problem. In this paper, we propose a submodular approach to rank the available models in the source pool according to the performance gain they would yield if added to the subset pool of the ensemble and select the top \( k \) models, where \( k \) is the required size of the ensemble. While submodular subset selection is popular in different machine learning settings [4, 24, 53], to the best of our knowledge, this is the first such use for transferability estimation. To this end, we first formally define submodularity below.

**Definition 3.1.** Let \( \Omega \) be a set and \( \mathcal{P}(\Omega) \) be the power set of \( \Omega \), then a submodular function is a set function \( f: \mathcal{P}(\Omega) \rightarrow \mathbb{R} \). The submodular function follows the property of diminishing returns, i.e. adding a new element to a smaller set produces a larger increase in \( f \) compared to a larger set. Mathematically, if for all \( X, Y \subseteq \Omega \), where \( X \subseteq Y \) and for all \( v \in \Omega \setminus Y \), the property follows:

\[
f(X + v) - f(X) \geq f(Y + v) - f(Y)
\]

A key benefit of posing a problem as one of submodular subset selection is that a greedy approach can be leveraged to efficiently identify a solution of required subset size that is reasonably close to the optimal solution. Nemhauser [34] showed that the quality of the subset chosen greedily cannot be worse than \( 1 - e^{-1} \) of the optimal value. This makes submodularity an attractive approach for usage in the field of TE for ensembles as we can rank the models in the source pool and select an ensemble of desired size. Further details on how to greedily select the models are discussed later in this paper.

**Evaluation Criteria.** As stated earlier, the reliability of a TE method is obtained by measuring the correlation between \( \alpha_{M,e \rightarrow \ell} \) and \( A_{M,e \rightarrow \ell} \). Previous works [58, 36, 1, 47, 48] measure this correlation using different techniques such as Pearson Correlation Coefficient (PCC), Kendall \( \tau \) (KT) [25] and Weighted Kendall \( \tau \) (WKT) [50]. We report results for all these correlation measures to be comprehensive in our analysis.

### 4. OSBORN: Transferability Estimation Metric for Model Ensemble Selection

In order to design a reliable transferability estimation approach for model ensembles, we propose the Optimal Transport-based Submodular Transferability metric (OSBORN), which considers three factors: domain difference, task difference, and inter-model cohesion. Inspired by earlier efforts on single-source transferability estimation [47], we consider both classifier output and distance in the latent representation space in our approach. Besides, since our focus is on model ensembles, we consider inter-model relationships in this metric. We now describe each of these quantities.

**Minimize Domain Difference (\( W_D \).** In order to minimize the latent space mismatch between the source and target datasets, similar to [47], we choose Wasserstein distance and Optimal Transport (OT) to compute this mismatch owing to its advantages in capturing the geometries of underlying data. Mathematically, the \( p \)-Wasserstein distance is given as follows:

\[
W_p(\beta, \gamma) = \left( \inf_{\pi \in H(\beta, \gamma)} \int_D d\pi(x, z) \right)^{1/p} \tag{2}
\]

where, \( p \geq 1 \), \( \beta, \gamma \) are continuous or discrete random variables in a complete and separable space \( S, D(\cdot, \cdot) : S \times S \rightarrow \mathbb{R}^+ \) is a distance or a cost function between two points \( x \) and \( \pi(x, z) \). The coupling matrix which can also be understood as the joint probability distributions with marginals \( \beta \) and \( \gamma \). In particular, in this work, we use the 1-Wasserstein distance, also called the Earth Mover Distance, to calculate the domain difference between source and target latents as:

\[
W_D(\theta_s, x_t) = \sum_{i,j=1}^{m,n} \frac{1}{2} \left( \| \theta_s(x_s^i) - \theta_s(x_t^j) \|_2^2 \right) \pi_{ij}^*, \tag{3}
\]

where \( \| \cdot - \cdot \|_2^2 \) is the distance or cost metric, \( \pi^* \) is the optimal coupling matrix of size \( m \times n \) obtained by solving the optimal transport (OT) problem using the Sinkhorn
In order to measure the ground truth target labels and Minimize Model Disagreement (Cohesiveness) in an agnostic setting, which motivates us to combine this with tally shown that using only CE is insufficient in a domain-specific task. However, in [47], it is experimentally shown that using only CE is insufficient in a domain-agnostic setting, which motivates us to combine this with $W_D$ to account for feature representation space mismatch.

**Minimize Task Difference ($W_T$).** In order to measure the difference between a source task and the given target task, we use the mismatch between the model/classifier’s outputs for source and target data forward-propagated through the source model. We use the conditional entropy (CE) of the predicted labels $\hat{y}_t \in \mathcal{Y}_t$ of the target dataset samples given their ground truth labels $y_t \in \mathcal{Y}_t$. The predicted labels are obtained by forward-propagating the target samples $x_t$ through the corresponding source model $\theta_s$. Let $Y_t$ be a random variable that takes values in the range of $\mathcal{Y}_t$; and $X_t$ be a random variable that takes values in the range of $\mathcal{Y}_t$, then $W_T$ can be calculated as:

$$W_T(\theta_s, x_t) = H(\hat{Y}_t | Y_t)$$

$$= - \sum_{y_t \in \mathcal{Y}_t} \sum_{\hat{y}_t \in \mathcal{Y}_t} \hat{P}(\hat{y}_t, y_t) \log \frac{\hat{P}(\hat{y}_t, y_t)}{\hat{P}(y_t)} \tag{4}$$

where $\hat{P}(\hat{y}_t, y_t)$ is the joint distribution of predicted and ground truth target labels and $\hat{P}(y_t)$ is the marginal distribution of the ground truth labels. These quantities can be easily computed using the optimal coupling matrix (obtained in Eqn 3) as follows:

$$\hat{P}(\hat{y}_t, y_t) = \sum_{i,j:y_t^i = y_t, \hat{y}_t^j = y_t} \pi^*_{ij}, \tag{5}$$

The marginal distribution can be obtained from the joint distribution as follows:

$$\hat{P}(y_t) = \sum_{\hat{y}_t \in \mathcal{Y}_t} \hat{P}(\hat{y}_t, y_t), \tag{6}$$

Intuitively, similar tasks will result in a low $W_T$ value. Using $W_T$ i.e CE alone represents empirical transferability according to [48]. However, in [47], it is experimentally shown that using only CE is insufficient in a domain-agnostic setting, which motivates us to combine this with $W_D$ to account for feature representation space mismatch.

**Minimize Model Disagreement (Cohesiveness $W_C$).** For an ensemble, it is important that the individual models reinforce the predictions of each other and have less disagreement amongst themselves to have overall good performance. To understand the cohesiveness of an ensemble, we use Conditional Entropy to capture the amount of disagreement between models in the subset of models $M_e$. Mathematically, we represent $W_C$ as:

$$W_C(M_e, x_t) = \sum_{m_i, m_j \in M_e} H(m_i(x_t) | m_j(x_t)) \tag{7}$$

A model ensemble that obtains a low OSBORN score will have better transferability to a target dataset. Our experiments show that a simple combination of these three quantities (with no weighting co-efficients) outperforms existing methods in all our experiments. In our ablation studies and analysis, we study the contribution of each OSBORN component as well as the effect of weighting each component differently.

**Submodular Subset Selection in OSBORN.** As stated earlier, we show that the proposed OSBORN metric translates to a submodular optimization problem, which allows us to rank and pick models efficiently from the source pool. While the aforementioned quantities were written from a minimization perspective (for clarity and ease of understanding), to pose this as a submodular maximization problem, we consider the corresponding scoring function to be maximized as:

$$\text{OSBORN} = \sum_{m_i \in M_e} [W_D(m_i, x_t) + W_T(m_i, x_t)] + W_C(M_e, x_t) \tag{8}$$
The scoring function

Theorem 4.1. Without expensive fine-tuning, the value of our set function is a transferability estimate designed such that it is highly correlated to the fine-tune accuracy (see Table 3 & 4), thus enabling us to select models without expensive fine-tuning.

Theorem 4.1. The scoring function \( f(X) \), as defined in Equation 9, is a submodular function.

Proof. Let \( X_1 \) and \( X_2 \) be two sets such that \( X_1 \subseteq X_2 \subseteq M \). If we consider an unselected model instance \( v \in M \setminus X_2 \). The gain in the score is obtained by appending \( v \) to the set \( X_1 \), and this is calculated as:

\[
f(X_1 \cup v) - f(X_1) = -[W_D (v, x_t) + W_T (v, x_t)]
- \sum_{m_i \in X_1} H (m_i (x_t) | v (x_t))
- \sum_{m_j \in X_1} H (v (x_t) | m_j (x_t))
\]

Similarly, the gain obtained by set \( X_2 \) is given by:

\[
f(X_2 \cup v) - f(X_2) = -[W_D (v, x_t) + W_T (v, x_t)]
- \sum_{m_i \in X_2} H (m_i (x_t) | v (x_t))
- \sum_{m_j \in X_2} H (v (x_t) | m_j (x_t))
\]

Equation 10

Equation 11

As we have \( X_1 \subseteq X_2 \), the number of terms in the summation of Equation 11 will be greater than or equal to that of Equation 10. Since entropy is always a non-negative value, we can say that

\[
- \sum_{m_i \in X_1} H (m_i (x_t) | v (x_t)) - \sum_{m_j \in X_1} H (v (x_t) | m_j (x_t)) \geq
- \sum_{m_i \in X_2} H (m_i (x_t) | v (x_t)) - \sum_{m_j \in X_2} H (v (x_t) | m_j (x_t))
\]

This implies that

\[
f(X_1 \cup v) - f(X_1) \geq f(X_2 \cup v) - f(X_2)
\]

Equation 12

We can see that Equation 12 satisfies the condition in Definition 3.1. This completes the proof. 

Submodular Optimization using Greedy Maximization.

Since our set function \( f(M_e) \) (mentioned in Eq. 9) is submodular, it exhibits monotonicity, i.e. the set with maximum gain is always the entire source pool \( M \). However, since we want to select a subset of models i.e. ensemble set from the source pool \( M \), we impose a cardinality constraint.

Formally, we aim to select the set \( M_e \) of size at most \( k \) that maximizes the gain:

\[
\text{max}_{|M_e|=k} f(M_e)
\]

Equation 13

This problem is however NP-hard, but we use the greedy maximization strategy to find a near-optimal set of models \( M_e \) for the target dataset. In practice, we pre-calculate pair-wise domain difference \( W_D \) and task difference \( W_T \) between each source and target datasets. Then, we calculate the model cohesion term \( W_C \) for adding each model \( m_i \) to the set of already selected models \( M_e \). Using these three quantities pertaining to \( m_i \), we calculate the gain achieved by adding it to the set \( M_e \) as \( f(M_e \cup m_i) - f(M_e) \) and greedily pick the model with the highest gain and add it to the set \( M_e \). We continue this iteration till we achieve the ensemble set size of \( k \). Once the target samples are forward-propagated through the source models, the quantities in our metric can be computed independently for each source model, thus making our overall computations parallelizable.

Considering \( M_e^* \) as the optimal ensemble set, it is well-known from [34] that such a greedy approach has a performance guarantee of at least 63% of the optimal ensemble set, i.e.

\[
f(M_e) \geq \left( 1 - \frac{1}{e} \right) f(M_e^*)
\]

Equation 14

In practice, we observe that we see that the avg. accuracy of the ensemble selected by greedy (76.315%) in a fully-supervised setting is, 95.56% of the avg. accuracy of the optimal ensemble(79.857%). Similarly for self-supervised setting, the avg. accuracy of the ensemble selected by greedy (79.857%) is, 93.50% of the avg. accuracy of the optimal ensemble(84.962%), as shown in Table 2. More details on the experiments are presented in Sec 5.

<table>
<thead>
<tr>
<th>Target Dataset</th>
<th>Ensemble Accuracy (Fully Supervised)</th>
<th>Ensemble Accuracy (Self Supervised)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Greedy</td>
<td>Optimal</td>
</tr>
<tr>
<td>Oxford102Flowers</td>
<td>69.540</td>
<td>72.540</td>
</tr>
<tr>
<td>Caltech101</td>
<td>68.533</td>
<td>75.333</td>
</tr>
<tr>
<td>StanfordCars</td>
<td>69.692</td>
<td>72.540</td>
</tr>
<tr>
<td>Average</td>
<td>76.315</td>
<td>79.857</td>
</tr>
</tbody>
</table>

Table 2. Comparison of the target test set accuracies achieved by fine-tuned ensembles selected using the greedy optimization of OSBORN vs the optimal ensembles. We clearly observe that our approach empirically gives significantly stronger performances than the theoretical guarantee.
5. Experiments and Results

Experimental Setup. We follow the same experimental setup as the previous work on source model ensemble selection [1] to evaluate our transferability metric in the multiple source model setting. Given a total of \( M \) models in the source pool, our objective is to select an ensemble model by choosing \( k \) models from the source pool. We follow [1] in setting \( k \) to 3 for fairness of comparison. We also conducted a study to evaluate this on the Oxford-IIIT Pets dataset, and found that maximum accuracy is gained for an ensemble of size 3 (see Fig 4), which further reinforces our choice for conducting experiments.

Classification Datasets. For the classification tasks, we consider 11 widely-used datasets including CIFAR-10 [29], CIFAR-100 [29], Caltech-101 [13], Stanford Cars [28], Oxford 102 Flowers [37], Oxford-IIIT Pets [40], Imagenette [22], CUB200 [54], FashionMNIST [55], SVHN [35], Stanford Dogs [26]. These datasets are popularly used in many transfer learning tasks. Out of these 11 datasets, we set Caltech-101 [13], Stanford Cars [28], Oxford 102 Flowers [37], Oxford-IIIT Pets [40], Stanford Dogs [26] as our target datasets and estimate transferability using OSBORN.

Model Architectures (Fully-supervised). For this setting, we consider 2 source model architectures ResNet-101 [21] and DenseNet-201 [23], keeping in mind the model diversity and capacity. We take these models from the open-sourced PyTorch Library [41]. Initially, both the models are initialized with the fully-supervised ImageNet weights [31], and then we train them on the 11 classification datasets to prepare our source model pool.

Model Architectures (Self-supervised). For this setting, we consider ResNet-50 [21] as our source model architecture but initialize it with weights obtained from two self-supervised pre-training strategies, namely BYOL [17] and MoCov2 [8]. We have two variants of ResNet-50 models to produce enough diversity. And as done in the previous case, we train these two models on the 11 classification datasets to prepare our source model pool. We use multiple pre-trained SSL models to build our pool. However, finetuning is done in a fully-supervised fashion. Our motivation here was to study if OSBORN can estimate transferability reliably across multiple pre-training settings.

Training Setup for Source Models (Classification Tasks). For all classification tasks, we train the source models using a cross-entropy loss and optimize it using Stochastic Gradient Descent (SGD) with momentum. Given these details, the most important hyperparameters are learning rate, batch size and weight decay. We train the models with a grid search of learning rate in \((1e^{-1}, 1e^{-2}, 1e^{-3}, 1e^{-4})\), batch size in \((32, 64, 128)\), and weight decay in \((1e^{-3}, 1e^{-4}, 1e^{-5}, 1e^{-6}, 0)\) to pick the best hyperparameters. All our experiments are written in PyTorch and are conducted on a single Tesla V-100 GPU. For the fully-supervised pre-trained setting, we initialize the models with ImageNet weights. In the case of a self-supervised pre-trained setting, we initialize the models using BYOL or MoCov2 (on ImageNet weights). For our experiments on the multi-domain DomainNet dataset, we initialize our models with ImageNet weights.

Training Setup for Source Models (Semantic Segmentation Tasks). We train the source models using a pixel-wise cross-entropy loss and optimize it using Stochastic Gradient Descent (SGD) with momentum. The most important hyperparameters herein are learning rate, batch size and weight decay. We train the models with a grid search of learning rate in \((1e^{-1}, 1e^{-2}, 1e^{-3}, 1e^{-4})\), batch size in \((32, 64, 128)\), and weight decay in \((1e^{-3}, 1e^{-4}, 1e^{-5}, 1e^{-6}, 0)\), and pick the best hyperparameters. All these experiments are also written in PyTorch and conducted on a single Tesla V-100 GPU. We initialize source models using the COCO pre-trained weights.

Implementation of Source Models and Baselines. We use open-source models available via the PyTorch Library for classification and semantic segmentation tasks. We use the PyTorch Lightning Library to obtain model weights for a self-supervised pre-training setting. We use the code released by the respective papers for calculating OTCE [47], MS-LEEP, E-LEEP, IoU-EEP and SoftIoU-EEP [1] scores.

Evaluating Ensemble Performance. We follow the protocol in [1] for computing ground truth accuracies of ensembles. We fine-tune (both feature extractor and classifier of) all the source models present in the ensemble using the target training set. Then, we individually make predictions using the source models on the target test set and average them to get the final ensemble prediction. We note that no target-trained models are in the source pool. We compare this final prediction with the ground-truth label and calculate the classification accuracy. Note that we need to fine-tune all source models only once and can re-use their predictions on the test set across all ensemble combinations. As stated earlier, we report Pearson Correlation Coefficient (PCC), Kendall \(\tau\) (KT) and Weighted Kendall \(\tau\) (WKT) in our results.

Evaluation on Fully-Supervised Pre-Trained Models. We herein compare our OSBORN with the baseline metrics, i.e. MS-LEEP and E-LEEP, in terms of three correlation.
We Evaluation on Self-Supervised Pre-Trained Models. We can visually see the overall performance of our metric over E-LEEP in terms of WKT; improves 442\% over BORN improves 268\% over MS-LEEP and 96\% over E-LEEP in terms of PCC. We can see that our metric constantly outperforms the baselines across every dataset by a large margin.

Table 3. Comparison of different ensemble transferability estimation metrics for fully-supervised models (classification tasks). The best results are indicated in bold. Note: MS: MS-LEEP, E: E-LEEP, Ours: OSBORN.

<table>
<thead>
<tr>
<th>Target Dataset</th>
<th>Weighted Kendall’s τ</th>
<th>Kendall’s τ</th>
<th>Pearson</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MS</td>
<td>E</td>
<td>Ours</td>
</tr>
<tr>
<td>Oxford102Flowers</td>
<td>0.086</td>
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<tr>
<td>Average</td>
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</table>

Figure 5. Comparison of OSBORN over 5 target datasets in terms of Weighted Kendall’s τ. We can see that our metric constantly outperforms the baselines across every dataset by a large margin.

metrics, WKT, KT, and PCC\(^2\). The correlation values are reported in Table 3. Averaged across five target datasets, OSBORN improves 96.36\% over MS-LEEP and 140\% over E-LEEP in terms of WKT; improves 66.06\% over MS-LEEP and 93.16\% over E-LEEP in terms of KT; improves 58.62\% over MS-LEEP and 75.23\% over E-LEEP in terms of PCC. We can visually see the overall performance of our metric outperforming the existing baselines significantly in Fig 5.

Evaluation on Self-Supervised Pre-Trained Models. We compare the performance of our method with the baseline methods, i.e. MS-LEEP and E-LEEP. We present the experimental results regarding different correlation coefficients in Table 4. Note that we use the Frobenius norm regularizer while solving the OT problem because it gave us better results when compared to using other regularizers. In the appendix, we report results without any regularizers and compare them with the Frobenius norm variant. Table 4 shows that, averaged across five target datasets, OSBORN improves 268.69\% over MS-LEEP and 231.82\% over E-LEEP in terms of WKT; improves 442.10\% over MS-LEEP and 379.07\% over E-LEEP in terms of KT; improves 527.27\% over MS-LEEP and 392.86\% over E-LEEP in terms of PCC.

Performance of Selected Ensembles. Table 2 reports the ensemble accuracy of the models selected through OSBORN. For completeness of this discussion, we also report the same results for OSBORN without greedy maximization as well as for MS-LEEP and E-LEEP in Table 5. Following [1], we first calculate the OSBORN value for every ensemble candidate and pick the ensemble that bags the highest value. We follow a similar strategy with MS-LEEP and E-LEEP to pick the best model according to their values. To compute the ensemble accuracy, we used the individual models fine-tuned on the target train set and got their predictions on the target test set. We average these predictions and compare them with the ground truth labels to obtain overall accuracy. We observe that the ensemble selected by OSBORN achieves the highest test accuracy across all datasets. In the case of both fully supervised and self-supervised settings, the baseline methods, i.e. MS-LEEP and E-LEEP, select the same ensembles (despite having different correlation values) in every case, which is why they obtain the same ensemble accuracy.

Scaling Number of Models in Ensemble. As shown earlier in this section (Fig 4), we found the performance to saturate after an ensemble size of 3 in the datasets considered in this work as well as in [1]. On the other hand, we also observe unsurprisingly that the cost of ensemble selection can go up significantly as the ensemble size increases. We show the cost performance of models selected for the Caltech101 dataset in Fig 6. Despite the increasing trend, we note that the time taken is still in the order of seconds, which makes the proposed OSBORN metric practical and relevant.

Ablation Studies. We conducted additional experiments to understand the influence of each component in OSBORN (included in the Appendix). In general, while simple addition of the three quantities in OSBORN without any weights outperformed previous methods, we observed that these can be finetuned through grid search over a larger range of values to get even better transferability estimates. This however varies with the target dataset. On Caltech101 as the target dataset, we noticed that giving more weightage to \(W_D\)
### Weighted Kendall’s τ

<table>
<thead>
<tr>
<th>Target Dataset</th>
<th>Weighted Kendall’s τ</th>
<th>Kendall’s τ</th>
<th>Pearson</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MS</td>
<td>E</td>
<td>Ours</td>
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<tr>
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<td><strong>0.260</strong></td>
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<td><strong>Average</strong></td>
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<td>0.110</td>
<td><strong>0.365</strong></td>
</tr>
</tbody>
</table>

Table 4. Comparison of different ensemble transferability estimation metrics for self-supervised pre-trained models (classification tasks). The best results are indicated in bold. Note: MS: MS-LEEP, E: E-LEEP and Ours: OSBORN.

### 6. Conclusions

In this paper, we propose a novel optimal transport-based transferability estimation metric, OSBORN, carefully designed for ensembles that consider multiple factors, such as the mismatch in the latent space, label space, and the cohesiveness amongst the individual models in the ensemble. We show that the proposed metric can be treated as a submodular optimization problem, allowing us to leverage a greedy strategy for source model ensemble selection. We show experimentally that our metric outperforms the existing metrics MS-LEEP and E-LEEP across tasks on multiple correlation metrics. Future directions include increasing the computational efficiency of this method, as well as studying its applicability to other tasks and problem settings.

### Acknowledgements

This work was partly supported by KLA and the Department of Science and Technology, India through the DST ICPS Data Science Cluster program. We would like to thank the authors of [1] for insightful discussions. Further, we thank the anonymous reviewers for their valuable feedback that improved the presentation of this paper.

### References


[5] Adrià Puigdomènech Badía, Pablo Sprechmann, Alex Vitvitskyi, Daniel Guo, Bilal Piot, Steven Kapturowski, Olivier Tieleman, Martín Arjovsky, Alexander Pritzel, Andrew Bolt,


Jonathan Krause, Michael Stark, Jia Deng, and Li Fei-Fei. 3d object representations for fine-grained categorization. In 4th International IEEE Workshop on 3D Representation and Recognition (3DRR-13), Sydney, Australia, 2013. 

Alex Krizhevsky. Learning multiple layers of features from tiny images. University of Toronto, 05 2012. 


Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton. Imagenet classification with deep convolutional neural net-


[56] Yongquan Yang, Haijun Lv, and Ning Chen. A survey on ensemble learning under the era of deep learning. Artificial Intelligence Review, nov 2022. 3


