

FLatten Transformer: Vision Transformer with Focused Linear Attention

Supplementary Material

A. Proof of Proposition 1

As mentioned in the main paper, with the aim to restore the sharp distribution in linear attention, we present our **Focused Function** f_p :

$$\text{Sim}(Q_i, K_j) = \phi_p(Q_i) \phi_p(K_j)^T, \quad (1)$$

$$\text{where } \phi_p(x) = f_p(\text{ReLU}(x)), \quad f_p(x) = \frac{\|x\|}{\|x^{**p}\|} x^{**p}, \quad (2)$$

and x^{**p} represents the power p of x bit by bit. We follow previous linear attention modules to use the ReLU function first to ensure the non-negativity of input. Therefore, when proving the effects of f_p , we only consider $x, y \geq 0$.

Proposition 1 (Feature direction adjustment with f_p) *Let $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in \mathbb{R}^n, x_i, y_j \geq 0$. Assume $0 < \langle x, y \rangle < \|x\| \|y\|$ and x, y have the **single** largest value x_m, y_n respectively.*

For a pair of feature $\{x, y\}$ with $m = n$:

$$\exists p > 1, \text{ s.t. } \langle \phi_p(x), \phi_p(y) \rangle > \langle x, y \rangle. \quad (3)$$

For a pair of feature $\{x, y\}$ with $m \neq n$:

$$\exists p > 1, \text{ s.t. } \langle \phi_p(x), \phi_p(y) \rangle < \langle x, y \rangle. \quad (4)$$

Proof.

$$\begin{aligned} \phi_p(x) &= f_p(\text{ReLU}(x)) = f_p(x), \\ \phi_p(y) &= f_p(\text{ReLU}(y)) = f_p(y). \end{aligned} \quad (5)$$

$$\|f_p(x)\| = \frac{\|x\|}{\|x^{**p}\|} \|x^{**p}\| = \|x\|, \quad \|f_p(y)\| = \|y\|. \quad (6)$$

Therefore, we have:

$$\begin{aligned} \langle \phi_p(x), \phi_p(y) \rangle &= \langle f_p(x), f_p(y) \rangle \\ &= \|f_p(x)\| \|f_p(y)\| \langle u, v \rangle \\ &= \|x\| \|y\| \langle u, v \rangle, \end{aligned} \quad (7)$$

where

$$\begin{aligned} \langle u, v \rangle &= \left\langle \frac{f_p(x)}{\|f_p(x)\|}, \frac{f_p(y)}{\|f_p(y)\|} \right\rangle \\ &= \frac{\sum_{i=1}^n x_i^p y_i^p}{\sqrt{\left(\sum_{i=1}^n x_i^{2p}\right) \left(\sum_{i=1}^n y_i^{2p}\right)}} \\ &= \frac{\sum_{i=1}^n a_i^p b_i^p}{\sqrt{\left(\sum_{i=1}^n a_i^{2p}\right) \left(\sum_{i=1}^n b_i^{2p}\right)}}, \end{aligned} \quad (8)$$

and

$$\begin{aligned} a_i &= x_i / \max_{1 \leq i \leq n} (x_i), \quad b_i = y_i / \max_{1 \leq i \leq n} (y_i), \\ a_i, b_i &\in [0, 1]. \end{aligned} \quad (9)$$

Based on our assumption, we have:

$$\exists! m, \text{ s.t. } a_m = 1, \quad \exists! n, \text{ s.t. } b_n = 1. \quad (10)$$

Therefore,

$$\lim_{p \rightarrow \infty} a_i^p = \begin{cases} 1, & i = m \\ 0, & i \neq m \end{cases}, \quad \lim_{p \rightarrow \infty} b_j^p = \begin{cases} 1, & j = n \\ 0, & j \neq n \end{cases}. \quad (11)$$

Then we consider the following two cases:

(1) $m = n$:

$$\begin{aligned} \lim_{p \rightarrow \infty} \langle u, v \rangle &= \lim_{p \rightarrow \infty} \frac{\sum_{i=1}^n a_i^p b_i^p}{\sqrt{\left(\sum_{i=1}^n a_i^{2p}\right) \left(\sum_{i=1}^n b_i^{2p}\right)}} \\ &= \frac{1 \times 1}{\sqrt{1 \times 1}} = 1. \end{aligned} \quad (12)$$

Eq. (7), Eq. (12) \Rightarrow

$$\begin{aligned} \lim_{p \rightarrow \infty} \langle \phi_p(x), \phi_p(y) \rangle &= \lim_{p \rightarrow \infty} \|x\| \|y\| \langle u, v \rangle \\ &= \|x\| \|y\| > \langle x, y \rangle. \end{aligned} \quad (13)$$

Thus we have,

$$\exists p > 1, \text{ s.t. } \langle \phi_p(x), \phi_p(y) \rangle > \langle x, y \rangle. \quad (14)$$

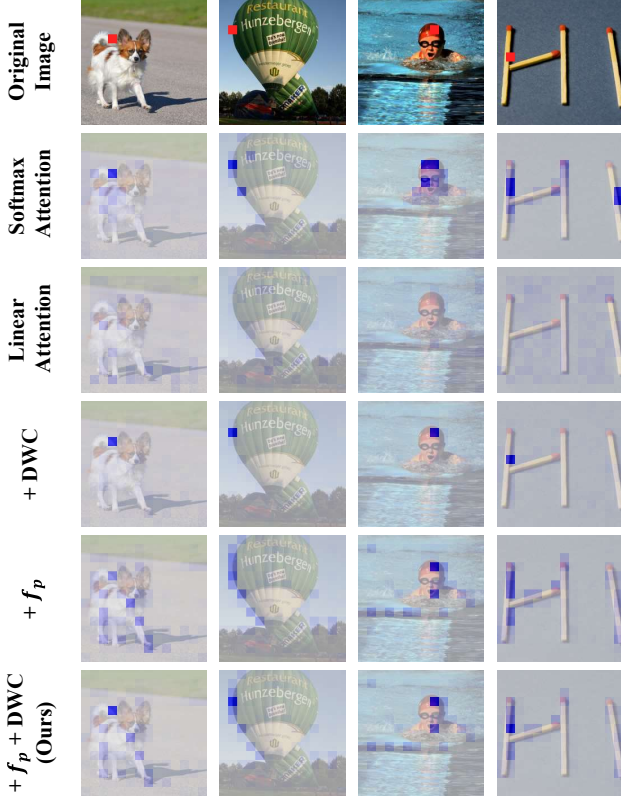


Figure 1. The distribution of attention weights from DeiT-tiny. Feature corresponding to the red block is used as query.

(2) $m \neq n$:

$$\begin{aligned} \lim_{p \rightarrow \infty} \langle u, v \rangle &= \lim_{p \rightarrow \infty} \frac{\sum_{i=1}^n a_i^{2p} b_i^{2p}}{\sqrt{\left(\sum_{i=1}^n a_i^{2p}\right) \left(\sum_{i=1}^n b_i^{2p}\right)}} \\ &= \frac{1 \times 0 + 0 \times 1}{\sqrt{1 \times 1}} = 0. \end{aligned} \quad (15)$$

Eq. (7), Eq. (15) \Rightarrow

$$\begin{aligned} \lim_{p \rightarrow \infty} \langle \phi_p(x), \phi_p(y) \rangle &= \lim_{p \rightarrow \infty} \|x\| \|y\| \langle u, v \rangle \\ &= 0 < \langle x, y \rangle. \end{aligned} \quad (16)$$

Thus we have,

$$\exists p > 1, \text{ s.t. } \langle \phi_p(x), \phi_p(y) \rangle < \langle x, y \rangle. \quad (17)$$

□

Therefore, with a proper p , our focused function $f_p(\cdot)$ practically achieves a more distinguished difference between similar query-key pairs (Eq. (3)) and dissimilar query-key pairs (Eq. (4)). Actually, f_p divides the features into several groups according to their nearest axes, improving the similarity within each group while reducing the similarity between the groups, thus restoring the sharp attention distribution as the original Softmax function.

Method	Reso	#Params	Flops	Top-1
DeiT-T [3]	224 ²	5.7M	1.2G	72.2
FLatten-DeiT-T	224 ²	6.1M	1.1G	74.1 (+1.9)
PVT-T [4]	224 ²	13.2M	1.9G	75.1
FLatten-PVT-T	224 ²	12.2M	2.0G	77.8 (+2.7)
PVT-S	224 ²	24.5M	3.8G	79.8
FLatten-PVT-S	224 ²	21.7M	4.0G	81.7 (+1.9)
PVT-M	224 ²	44.2M	6.7G	81.2
FLatten-PVT-M	224 ²	37.2M	7.0G	83.0 (+1.8)
PVT-L	224 ²	61.4M	9.8G	81.7
FLatten-PVT-L	224 ²	50.6M	10.4G	83.4 (+1.7)
PVTv2-B0 [5]	224 ²	3.4M	0.6G	70.5
FLatten-PVTv2-B0	224 ²	3.6M	0.6G	71.1 (+0.6)
PVTv2-B1	224 ²	13.1M	2.1G	78.7
FLatten-PVTv2-B1	224 ²	12.9M	2.2G	79.5 (+0.7)
PVTv2-B2	224 ²	25.4M	4.0G	82.0
FLatten-PVTv2-B2	224 ²	22.6M	4.3G	82.5 (+0.5)
PVTv2-B3	224 ²	45.2M	6.9G	83.2
FLatten-PVTv2-B3	224 ²	38.3M	7.3G	83.7 (+0.5)
PVTv2-B4	224 ²	62.6M	10.1G	83.6
FLatten-PVTv2-B4	224 ²	51.8M	10.7G	84.0 (+0.4)
Swin-T [2]	224 ²	29M	4.5G	81.3
FLatten-Swin-T	224 ²	29M	4.5G	82.1 (+0.8)
Swin-S	224 ²	50M	8.7G	83.0
FLatten-Swin-S	224 ²	51M	8.7G	83.5 (+0.5)
Swin-B	224 ²	88M	15.4G	83.5
FLatten-Swin-B	224 ²	89M	15.4G	83.8 (+0.3)
Swin-B	384 ²	88M	47.0G	84.5
FLatten-Swin-B	384 ²	91M	46.5G	85.0 (+0.5)
CSwin-T [1]	224 ²	23M	4.3G	82.7
FLatten-CSwin-T	224 ²	21M	4.3G	83.1 (+0.4)
CSwin-S	224 ²	35M	6.9G	83.6
FLatten-CSwin-S	224 ²	35M	6.9G	83.8 (+0.2)
CSwin-B	224 ²	78M	15.0G	84.2
FLatten-CSwin-B	224 ²	75M	15.0G	84.5 (+0.3)
CSwin-B	384 ²	78M	47.0G	85.4
FLatten-CSwin-B	384 ²	78M	46.4G	85.5 (+0.1)

Table 1. Comparisons of focused linear attention with other vision transformer backbones on the ImageNet-1K classification task.

B. More Visualizations

We visualize more examples of attention weights in Fig. 1. To better show the contribution of our focused function and DWC, we start from the vanilla linear attention and introduce f_p and DWC separately. As demonstrated in the last three rows, DWC improves local focus ability but cannot focus on any position, while f_p practically enhances model’s focus ability, helping model focus on more informative regions. Combining f_p and DWC, our focused linear attention module restores the sharp distribution as the original Softmax attention.

C. Full Classification Results

Due to the page limit, we only present representative ImageNet classification results in Figure 6 of main paper. Here, we give all the classification results when applying our focused linear attention module on various sizes of the five baseline models in Tab.1.

D. Model Architectures

We summarize the architectures of five Transformer models adopted in the main paper, including DeiT [3], PVT [4], PVTv2 [5], Swin Transformer [2], CSwin Transformer [1] in Tab.2-8. In practice, we substitute the original self-attention blocks at all stages of the DeiT, PVT and PVTv2 with the focused linear attention block, but only adopt our module at early stages of Swin and CSwin. The model structure (width and depth) are kept unchanged, except for CSwin-T and CSwin-B, where we increase the depth of the first and second stages and correspondingly reduce the depth of the third stage to better reflect our module’s advantage of enlarged receptive field.

References

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stage	output	FLatten-DeiT-T	
		FLatten	DeiT Block
res1	14×14	$\begin{bmatrix} \text{win } 14 \times 14 \\ \text{dim } 192 \\ \text{head } 3 \end{bmatrix} \times 12$	None

Table 2. Architectures of FLatten-DeiT models.

stage	output	FLatten-PVT-M		FLatten-PVT-L	
		FLatten	PVT Block	FLatten	PVT Block
res1	56×56	Conv 1×1 , stride=4, 64, LN			
		$\begin{bmatrix} \text{win } 56 \times 56 \\ \text{dim } 64 \\ \text{head } 1 \end{bmatrix} \times 2$	None	$\begin{bmatrix} \text{win } 56 \times 56 \\ \text{dim } 64 \\ \text{head } 1 \end{bmatrix} \times 3$	None
res2	28×28	Conv 1×1 , stride=2, 128, LN			
		$\begin{bmatrix} \text{win } 28 \times 28 \\ \text{dim } 128 \\ \text{head } 2 \end{bmatrix} \times 2$	None	$\begin{bmatrix} \text{win } 28 \times 28 \\ \text{dim } 128 \\ \text{head } 2 \end{bmatrix} \times 3$	None
res3	14×14	Conv 1×1 , stride=2, 320, LN			
		$\begin{bmatrix} \text{win } 14 \times 14 \\ \text{dim } 320 \\ \text{head } 5 \end{bmatrix} \times 2$	None	$\begin{bmatrix} \text{win } 14 \times 14 \\ \text{dim } 320 \\ \text{head } 5 \end{bmatrix} \times 6$	None
res4	7×7	Conv 1×1 , stride=2, 512, LN			
		$\begin{bmatrix} \text{win } 7 \times 7 \\ \text{dim } 512 \\ \text{head } 8 \end{bmatrix} \times 2$	None	$\begin{bmatrix} \text{win } 7 \times 7 \\ \text{dim } 512 \\ \text{head } 8 \end{bmatrix} \times 3$	None

Table 3. Architectures of FLatten-PVT models (Part1).

stage	output	FLatten-PVT-M		FLatten-PVT-L	
		FLatten	PVT Block	FLatten	PVT Block
res1	56×56	Conv 1×1 , stride=4, 64, LN			
		$\begin{bmatrix} \text{win } 56 \times 56 \\ \text{dim } 64 \\ \text{head } 1 \end{bmatrix} \times 3$	None	$\begin{bmatrix} \text{win } 56 \times 56 \\ \text{dim } 64 \\ \text{head } 1 \end{bmatrix} \times 3$	None
res2	28×28	Conv 1×1 , stride=2, 128, LN			
		$\begin{bmatrix} \text{win } 28 \times 28 \\ \text{dim } 128 \\ \text{head } 2 \end{bmatrix} \times 3$	None	$\begin{bmatrix} \text{win } 28 \times 28 \\ \text{dim } 128 \\ \text{head } 2 \end{bmatrix} \times 8$	None
res3	14×14	Conv 1×1 , stride=2, 320, LN			
		$\begin{bmatrix} \text{win } 14 \times 14 \\ \text{dim } 320 \\ \text{head } 5 \end{bmatrix} \times 18$	None	$\begin{bmatrix} \text{win } 14 \times 14 \\ \text{dim } 320 \\ \text{head } 5 \end{bmatrix} \times 27$	None
res4	7×7	Conv 1×1 , stride=2, 512, LN			
		$\begin{bmatrix} \text{win } 7 \times 7 \\ \text{dim } 512 \\ \text{head } 8 \end{bmatrix} \times 3$	None	$\begin{bmatrix} \text{win } 7 \times 7 \\ \text{dim } 512 \\ \text{head } 8 \end{bmatrix} \times 3$	None

Table 4. Architectures of FLatten-PVT models (Part2).

stage	output	FLatten-PVTv2-B0				FLatten-PVTv2-B1				FLatten-PVTv2-B2			
		FLatten		PVTv2 Block		FLatten		PVTv2 Block		FLatten		PVTv2 Block	
res1	56×56	Conv4x4, stride=4, 32, LN				Conv4x4, stride=4, 64, LN							
		win 56×56 dim 32 head 1	$\times 2$	None		win 56×56 dim 64 head 1	$\times 2$	None		win 56×56 dim 64 head 1	$\times 3$	None	
res2	28×28	Conv1x1, stride=2, 64, LN				Conv1x1, stride=2, 128, LN							
		win 28×28 dim 64 head 2	$\times 2$	None		win 28×28 dim 128 head 2	$\times 2$	None		win 28×28 dim 128 head 2	$\times 3$	None	
res3	14×14	Conv2x2, stride=2, 160, LN				Conv2x2, stride=2, 320, LN							
		win 14×14 dim 160 head 5	$\times 2$	None		win 14×14 dim 320 head 5	$\times 2$	None		win 14×14 dim 320 head 5	$\times 6$	None	
res4	7×7	Conv2x2, stride=2, 256, LN				Conv2x2, stride=2, 512, LN							
		win 7×7 dim 512 head 8	$\times 2$	None		win 7×7 dim 512 head 8	$\times 2$	None		win 7×7 dim 512 head 8	$\times 3$	None	

Table 5. Architectures of FLatten-PVTv2 models (Part1).

stage	output	FLatten-PVTv2-B3				FLatten-PVTv2-B4			
		FLatten		PVTv2 Block		FLatten		PVTv2 Block	
res1	56×56	Conv4x4, stride=4, 64, LN							
		win 56×56 dim 64 head 1	$\times 3$	None		win 56×56 dim 64 head 1	$\times 3$	None	
res2	28×28	Conv2x2, stride=2, 128, LN							
		win 28×28 dim 128 head 2	$\times 3$	None		win 28×28 dim 128 head 2	$\times 8$	None	
res3	14×14	Conv2x2, stride=2, 320, LN							
		win 14×14 dim 320 head 5	$\times 18$	None		win 14×14 dim 320 head 5	$\times 27$	None	
res4	7×7	Conv1x1, stride=2, 512, LN							
		win 7×7 dim 512 head 8	$\times 3$	None		win 7×7 dim 512 head 8	$\times 3$	None	

Table 6. Architectures of FLatten-PVTv2 models (Part2).

stage	output	FLatten-Swin-T			FLatten-Swin-S			FLatten-Swin-B		
		FLatten		Swin Block	FLatten		Swin Block	FLatten		Swin Block
res1	56×56	concat 4×4 , 96, LN			concat 4×4 , 96, LN			concat 4×4 , 128, LN		
		win 56×56 dim 96 head 3	$\times 2$	None	win 56×56 dim 96 head 3	$\times 2$	None	win 56×56 dim 128 head 3	$\times 2$	None
res2	28×28	concat 4×4 , 192, LN			concat 4×4 , 192, LN			concat 4×4 , 256, LN		
		win 28×28 dim 192 head 6	$\times 2$	None	win 28×28 dim 192 head 6	$\times 2$	None	win 28×28 dim 256 head 6	$\times 2$	None
res3	14×14	concat 4×4 , 384, LN			concat 4×4 , 384, LN			concat 4×4 , 512, LN		
		None		win 7×7 dim 384 head 12	$\times 6$	None		win 7×7 dim 384 head 12	$\times 18$	None
res4	7×7	concat 4×4 , 768, LN			concat 4×4 , 768, LN			concat 4×4 , 1024, LN		
		None		win 7×7 dim 768 head 24	$\times 2$	None		win 7×7 dim 768 head 24	$\times 2$	None

Table 7. Architectures of FLatten-Swin models.

stage	output	FLatten-CSwin-T			FLatten-CSwin-S			FLatten-CSwin-B		
		FLatten		CSwin Block	FLatten		CSwin Block	FLatten		CSwin Block
res1	56×56	Conv 7×7 , stride=4, 64, LN			Conv 7×7 , stride=4, 96, LN			Conv 7×7 , stride=4, 128, LN		
		win 3×3 dim 64 head 2	$\times 2$	None	win 3×3 dim 64 head 2	$\times 2$	None	win 3×3 dim 96 head 4	$\times 3$	None
res2	28×28	Conv 7×7 , stride=4, 128, LN			Conv 7×7 , stride=4, 192, LN			Conv 7×7 , stride=4, 256, LN		
		win 3×3 dim 128 head 4	$\times 4$	None	win 3×3 dim 128 head 4	$\times 4$	None	win 3×3 dim 192 head 8	$\times 6$	None
res3	14×14	Conv 7×7 , stride=4, 256, LN			Conv 7×7 , stride=384, LN			Conv 7×7 , stride=512, LN		
		None		win 3×3 dim 256 head 8	$\times 18$	None		win 3×3 dim 256 head 8	$\times 32$	None
res4	7×7	Conv 7×7 , stride=4, 512, LN			Conv 7×7 , stride=4, 768, LN			Conv 7×7 , stride=4, 1024, LN		
		None		win 7×7 dim 512 head 16	$\times 1$	None		win 7×7 dim 512 head 16	$\times 2$	None

Table 8. Architectures of FLatten-CSwin models.