# Supplementary Material

## A. Additional algorithms

### A.1. Training TPDM

TPDM can be trained with a three-dimensional volume dataset and Algorithm 2.

**Algorithm 2** Training TPDM

**Require:** $\{\boldsymbol{X}_i \in \mathbb{N}^{d_1 \times d_2 \times d_3}\}_1^M$, $\{\sigma_i\}_0^1$
  $D_{prim} \leftarrow \{\}$, $D_{aux} \leftarrow \{\}$ ▷ Create 2D datasets
  **for** $i$ **in** $1 : M$ **do**
    **for** $j$ **in** $1 : d_3$ **do**
      $D_{prim}$.add($\boldsymbol{X}_i[:, :, j]$)
    **end for**
    **for** $j$ **in** $1 : d_1$ **do**
      $D_{aux}$.add($\boldsymbol{X}_i[j, :, :]$)
    **end for**
  **end for**
  $\boldsymbol{s}_{\boldsymbol{\theta}^*} \leftarrow$ train_2D_DPM($D_{prim}$, $\{\sigma_i\}_0^1$) ▷ Train DPMs
  $\boldsymbol{s}_{\boldsymbol{\phi}^*} \leftarrow$ train_2D_DPM($D_{aux}$, $\{\sigma_i\}_0^1$)
  **return** $\boldsymbol{s}_{\boldsymbol{\theta}^*}$, $\boldsymbol{s}_{\boldsymbol{\phi}^*}$

### A.2. Sampling with real value $K$

When $K$ is a real number, select the primary model and the auxiliary model in a stochastic way through sampling from the Bernoulli distribution with $p = 1 - 1/K$ (Algorithm 3).

### A.3. 3D voxel volume generation with TPDM

TPDM's unconditional sampling can be performed by removing the DPS step of the primary model from the conditional sampling algorithm of TPDM (Algorithm 4).

## B. Dataset

### B.1. BMR-ZSR-5mm

We generated a 1mm volumetric dataset (*i.e.* `BMR-ZSR-1mm`) using structural brain 3T T1-weighted images from the Alzheimer's Disease Neuroimaging Initiative (ADNI) dataset (271 subjects with probable dementia and 211 subjects with normal cognition) and data from a university hospital's voluntary health screening program (441 normal). Evaluation was performed on 1 subject from the ADNI dataset which has normal cognition with the retrospective slice thickness degradation or the CS-MRI sub-sampling simulation.

The prospective 5mm volumetric dataset (*i.e.* `BMR-ZSR-5mm`) is also structural brain 3T T1-weighted images, which is composed of seven patients with ischemic stroke. The clinical information of the subjects is in Table 4. Five out of seven patients had 3D volumetric 1mm

**Algorithm 3** Solving 3D Inverse Problem with TPDM

**Require:** $\boldsymbol{Y} \in \mathbb{N}^{d_1' \times d_2' \times d_3}$, $\boldsymbol{A}(\cdot) : \mathbb{N}^{d_1 \times d_2} \rightarrow \mathbb{N}^{d_1' \times d_2'}$, $\boldsymbol{s}_{\boldsymbol{\theta}^*}$, $\boldsymbol{s}_{\boldsymbol{\phi}^*}$, $\{\sigma_i\}_0^1$, $N$, $K$, $\lambda$

  $\boldsymbol{X}_N \sim \mathcal{N}(\boldsymbol{0}, \sigma_1^2 \boldsymbol{I}) \in \mathbb{N}^{d_1 \times d_2 \times d_3}$
  **for** $i$ **in** $N - 1 : 0$ **do**
    is_primary $\sim Bernoulli(1 - \frac{1}{K})$
    $t \leftarrow \frac{i}{N}$
    $\boldsymbol{X}_i \leftarrow$ torch.empty_like($\boldsymbol{X}_N$)
    **if** is_primary **then**
      **for** $j$ **in** $1 : d_3$ **do**
        $\boldsymbol{x} \leftarrow \boldsymbol{X}_{i+1}[:, :, j]$
        $\boldsymbol{y} \leftarrow \boldsymbol{Y}[:, :, j]$
        $\hat{\boldsymbol{x}}_0 \leftarrow \boldsymbol{x} + \sigma_t^2 \cdot \boldsymbol{s}_{\boldsymbol{\theta}^*}(\boldsymbol{x}, t)$
        $\boldsymbol{x}' \leftarrow$ step_2D_DPM($\boldsymbol{x}$, $\boldsymbol{s}_{\boldsymbol{\theta}^*}$, $\sigma_t$, $t$)
        $\boldsymbol{x}'' \leftarrow \boldsymbol{x}' - \lambda \nabla_{\boldsymbol{x}} \|\boldsymbol{A}(\hat{\boldsymbol{x}}_0) - \boldsymbol{y}\|_2^2$
        $\boldsymbol{X}_i[:, :, j] \leftarrow \boldsymbol{x}''$
      **end for**
    **else**
      **for** $j$ **in** $1 : d_1$ **do**
        $\boldsymbol{x} \leftarrow \boldsymbol{X}_{i+1}[j, :, :]$
        $\boldsymbol{x}' \leftarrow$ step_2D_DPM($\boldsymbol{x}$, $\boldsymbol{s}_{\boldsymbol{\phi}^*}$, $\sigma_t$, $t$)
        $\boldsymbol{X}_i[j, :, :] \leftarrow \boldsymbol{x}'$
      **end for**
    **end if**
  **end for**
  **return** $\boldsymbol{X}_0$

**Algorithm 4** Unconditional Sampling with TPDM

**Require:** $\boldsymbol{s}_{\boldsymbol{\theta}^*}$, $\boldsymbol{s}_{\boldsymbol{\phi}^*}$, $\{\sigma_i\}_0^1$, $N$, $K$
  $\boldsymbol{X}_N \sim \mathcal{N}(\boldsymbol{0}, \sigma_1^2 \boldsymbol{I}) \in \mathbb{N}^{d_1 \times d_2 \times d_3}$
  **for** $i$ **in** $N - 1 : 0$ **do**
    $t \leftarrow \frac{i}{N}$
    $\boldsymbol{X}_i \leftarrow$ torch.empty_like($\boldsymbol{X}_N$)
    **if** $\mod(i, K) \neq 0$ **then**
      **for** $j$ **in** $1 : d_3$ **do**
        $\boldsymbol{x} \leftarrow \boldsymbol{X}_{i+1}[:, :, j]$
        $\boldsymbol{x}' \leftarrow$ step_2D_DPM($\boldsymbol{x}$, $\boldsymbol{s}_{\boldsymbol{\theta}^*}$, $\sigma_t$, $t$)
        $\boldsymbol{X}_i[:, :, j] \leftarrow \boldsymbol{x}'$
      **end for**
    **else**
      **for** $j$ **in** $1 : d_1$ **do**
        $\boldsymbol{x} \leftarrow \boldsymbol{X}_{i+1}[j, :, :]$
        $\boldsymbol{x}' \leftarrow$ step_2D_DPM($\boldsymbol{x}$, $\boldsymbol{s}_{\boldsymbol{\phi}^*}$, $\sigma_t$, $t$)
        $\boldsymbol{X}_i[j, :, :] \leftarrow \boldsymbol{x}'$
      **end for**
    **end if**
  **end for**
  **return** $\boldsymbol{X}_0$

T1-weighted images acquired simultaneously with a 5mm

T1-weighted image.

### B.2. LDCT-CUBE

The `LDCT-CUBE` dataset was built based on the contrast-enhanced abdominal CT presented in the AAPM 2016 CT low-dose grand challenge [25]. The data set was converted into 10 volumes with 256×256 slices in the axial slice direction through the same method as in [6] (`LDCT`). Since `LDCT` has different lengths in the vertical axis direction, a common part of volumes was manually selected and 256 consecutive slices were cropped to generate 256×256×256 cube-shaped volumes. Zero padding was added if the original slice was less than 256 slices. See Table 5 for the detailed cropping parameters.

## C. Additonal results

### C.1. MRI Z-axis super-resolution (MR-ZSR)

Additional results of the prospective clinical evaluation of a slice thickness of 5mm to 1mm MR-ZSR are shown in Fig. 8, Fig. 9, and Table 6. It was shown that MR-ZSR using TPDM works well for various GCA scales, especially in the presence of lesions. Although the `BMR-ZSR-5mm` had slightly different MRI sequence parameters from the `BMR-ZSR-1mm` used for training, TPDM was well adapted without any additional model modification. These reconstructed 1 mm images were evaluated as suitable for use as an input for a conventional cortical mask segmentation algorithm designed to operate only on images acquired with actual 1mm slice thickness.

Table 7 shows the results of the retrospective quantitative evaluation of MR-ZSR with a slice thickness of 3mm to 1mm.

### C.2. Compressed-sensing MRI (CS-MRI)

We further attempted reconstruction on ×8 and ×24 accelerated Poisson sub-sampled CS-MRI volumes. The results are Table 8 and Table 9, respectively. If the problem is straightforward (×8 acceleration), each 2D image can be restored with a high degree of accuracy, leading to near-perfect outcomes even with only the 2D solving method (DPS [5]). Nevertheless, as the complexity of the challenge increases (×24, ×48), we can only get better results in 3D with the assistance of a 3D prior.

## D. Sampling hyperparameters

Sampling hyperparameters utilized for TPDM, MCG [7], DPS [5], DiffusionMBIR [6], score-MRI [8], and score-CT [39] are presented for each experiment. The sampling hyperparameters for all comparative experiments were configured to match the specific hyperparameters that yielded optimal results from the models identified during the experimental phase. Common to all experiments in TPDM, an integer value $K$=2 was used for the MRI model, and a real number value $K$=2.7 was used for the CT model. All diffusion models were sampled with N=2000 sampling steps, regardless of the problem.

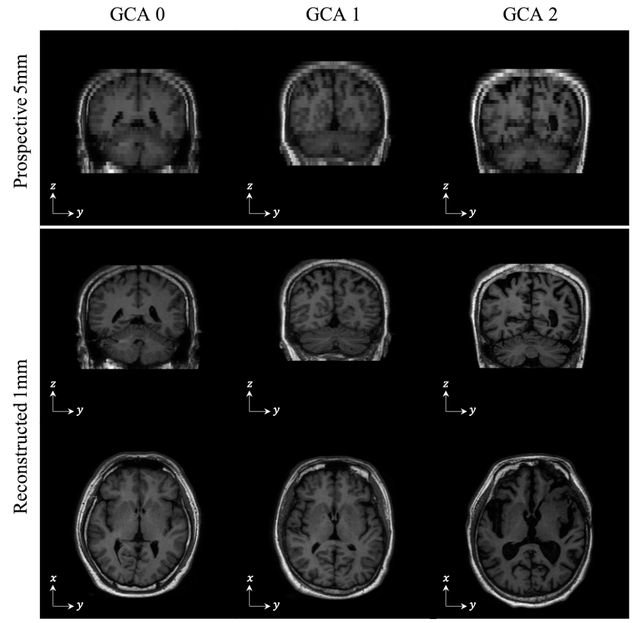


Figure 8. Result of the prospective 5mm→1mm (×5) MR-ZSR for different GCA scales. first/second row: primary plane, third row: auxiliary plane.

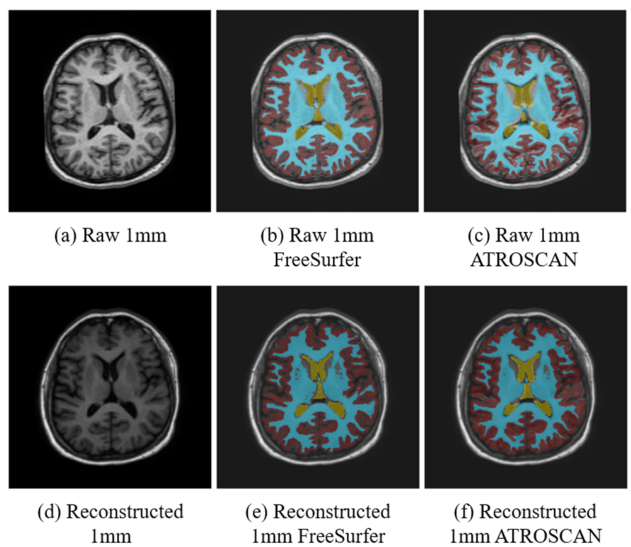


Figure 9. Comparison of estimated cortical mask between raw 1mm image and 5mm→1mm (×5) image from the prospective test volume.

| # | Age | Gender | Cortical infarct | GCA scale | WMH grade | Previous stroke | Hypertension | Diabetes | Hyperlipidemia | Current smoking |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 74 | M | No | 1 | 0 | No | Yes | No | No | No |
| 2 | 71 | M | Yes | 2 | 2 | No | Yes | Yes | No | Yes |
| 3 | 33 | M | No | 0 | 0 | No | No | No | No | No |
| 4 | 48 | F | No | 1 | 2 | Yes | No | Yes | No | No |
| 5 | 39 | M | Yes | 1 | 1 | No | No | No | No | No |
| 6 | 58 | F | No | 1 | 2 | Yes | No | Yes | Yes | No |
| 7 | 68 | M | No | 2 | 1 | Yes | Yes | No | No | Yes |

Table 4. Subject information of the prospective clinical evaluation of MR-ZSR.

| Patient ID | # of raw slices | Cropped slices range |
|---|---|---|
| L096 | 658 | 224:480 |
| L109 | 254 | 000:254 |
| L143 | 468 | 212:468 |
| L192 | 480 | 064:320 |
| L286 | 420 | 000:256 |
| L291 | 685 | 249:505 |
| L310 | 426 | 030:286 |
| L333 | 488 | 049:305 |
| L506 | 421 | 000:256 |
| L067 (test) | 448 | 004:260 |

Table 5. Cropping information of the LDCT-CUBE dataset. The range shown includes the start point and does not include the end-point. The indexes start at 0.

| # | Mean cortical thickness | | | |
|---|---|---|---|---|
| | Raw 1mm | | TPDM 5mm $\rightarrow$ 1mm | |
| | FreeSurfer | Astroscan | FreeSurfer | Astroscan |
| 1 | 2.21±0.92 | 2.21±0.78 | 2.26±0.98 | 2.52±0.84 |
| 2 | 2.07±0.93 | 1.90±0.72 | 2.31±1.12 | 2.38±0.84 |
| 3 | 2.37±1.00 | 2.40±0.81 | 2.30±1.06 | 2.67±0.85 |
| 4 | 2.31±1.00 | 2.36±0.82 | 2.38±1.06 | 2.68±0.86 |
| 5 | 2.23±0.95 | 2.24±0.79 | 2.25±1.02 | 2.54±0.84 |
| 6 | N/A | N/A | 2.06±1.00 | 2.27±0.87 |
| 7 | N/A | N/A | 2.26±1.06 | 2.40±0.86 |

Table 6. Result of the mean cortical thickness measurement of prospective ground truth 1mm MRI volume and upscaled 1mm MRI volume from 5mm by TPDM.

| Method | PSNR $\uparrow$ | SSIM $\uparrow$ | | |
|---|---|---|---|---|
| | | Axial$^+$ | Coronal$^*$ | Sagittal |
| TPDM (ours) | **38.76** | **0.982** | **0.979** | **0.978** |
| DiffusionMBIR [6] | N/W | | | |

Table 7. Quantitative evaluation (PSNR, SSIM) of MR-ZSR (3mm$\rightarrow$1mm; $\times$3) on the BMR-ZSR-1mm test set. N/W: Not Working. $^*$: primary plane, $^+$: auxiliary plane.

| Method | PSNR $\uparrow$ | SSIM $\uparrow$ | | |
|---|---|---|---|---|
| | | Axial$^*$ | Coronal$^+$ | Sagittal |
| TPDM (ours) | 44.96 | 0.988 | 0.989 | 0.988 |
| DiffusionMBIR [6] | 41.21 | 0.934 | 0.934 | 0.934 |
| DPS [5] | **47.10** | **0.991** | **0.991** | **0.991** |
| score-MRI [8] | 39.90 | 0.914 | 0.914 | 0.913 |

Table 8. Quantitative evaluation (PSNR, SSIM) of CS-MRI (Poisson, $\times$8 acc) on the BMR-ZSR-1mm test set. $^*$: primary plane, $^+$: auxiliary plane.

| Method | PSNR $\uparrow$ | SSIM $\uparrow$ | | |
|---|---|---|---|---|
| | | Axial$^*$ | Coronal$^+$ | Sagittal |
| TPDM (ours) | **40.34** | **0.979** | **0.978** | **0.978** |
| DiffusionMBIR [6] | 37.48 | 0.895 | 0.899 | 0.897 |
| DPS [5] | 39.06 | 0.965 | 0.967 | 0.965 |
| score-MRI [8] | 35.54 | 0.843 | 0.845 | 0.844 |

Table 9. Quantitative evaluation (PSNR, SSIM) of CS-MRI (Poisson, $\times$24 acc) on the BMR-ZSR-1mm test set. $^*$: primary plane, $^+$: auxiliary plane.

### D.1. MR-ZSR

Retrospective 5mm to 1mm used $\lambda$=4 for TPDM/DPS, $\lambda$=0.1 for MCG. Prospective 5mm to 1mm uses $\lambda$=1 for TPDM. Retrospective 3mm to 1mm was performed with TPDM by $\lambda$=2.

### D.2. CS-MRI

For Poisson sub-sampled $\times$48 acceleration, $\lambda$=0.01 was used by TPDM/DPS. DiffusionMBIR used $\lambda$=0.0001 and $\rho$=0.1. For the $\times$24 acceleration, TPDM/DPS uses $\lambda$=0.007, and DiffusionMBIR uses $\lambda$=0.0001 and $\rho$=0.1. For $\times$8 acceleration, $\lambda$=0.002 for TPDM/DPS, and $\lambda$=0.0005 and $\rho$=0.1 for DiffusionMBIR. Score-MRI has no hyperparameters configuring the sampling stage.

### D.3. SV-CT

For the 36-view SV-CT problem, $\lambda$=0.025 was used for TPDM/DPS, and $\lambda$=0.01 and $\rho$=40 were used for Diffusion-MBIR. Score-CT used $\lambda$=0.8.

## E. Computational resources

Both the training and sampling processes of the TPDM were executed utilizing two NVIDIA GeForce RTX 3090 GPUs. Employing the settings expounded upon in the text, the training duration for the MRI and CT models amounted to approximately 3 days and 1 day, respectively, for each 2D model, be it primary or auxiliary. The process of TPDM sampling necessitated an approximate timeframe of 24 to 36 hours per volume, contingent upon the specific problem type. Adopting a batch size of 6 during sampling, TPDM consumption of VRAM totaled around 48GB.

## F. Code Availability

The official implementation of TPDM and pre-trained MRI model checkpoint can be accessed at https://github.com/hyn2028/tpdm. This repository provides the necessary resources and instructions to replicate the experiments and utilize the TPDM.