E3Sym: Leveraging E(3) Invariance for Unsupervised 3D Planar Reflective Symmetry Detection

Supplementary Material

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1. Validation on Real-World Data

To validate the ability of our method to detect symmetries in real-world scenarios, we test our method on reconstructed or raw scanned models. The visualization results are displayed in Figure 1, the left is a sofa reconstructed by NeuS [7], and the right is a raw scanned chair in ScanNet [3]. Despite the presence of artifacts on the sofa and the chair is incomplete (the right leg connection is missing), and the shapes are not completely symmetric, our method can detect acceptable symmetries, demonstrating its effectiveness and robustness.

![Figure 1: Results on reconstructed or raw scanned models. (a) a sofa reconstructed by NeuS [7], (b) a raw scanned chair in ScanNet [3].](image)

2. Rotational Symmetry Reasoning

As Chertok et al. proposed in their 2D image symmetry detection work [2], two distinct reflection transforms can determine a unique corresponding rotational transform. For a 3D shape with multiple reflective symmetries detected, the shape might be rotational symmetric. To eliminate detecting excessive planar reflective symmetries for rotational symmetric shapes, we can recover the corresponding rotational symmetry from detected symmetry planes. To be specific, for each pair of distinct reflective transforms from detected planar reflective symmetries $i$ and $j$: $T_{i\text{ref}}^r$ and $T_{j\text{ref}}^r$, the corresponding rotational transform can be recovered as $T_{rot}^{ij} = T_{i\text{ref}}^r \cdot T_{j\text{ref}}^r$. Figure 2 provides examples for rotational symmetry reasoning, the left one is an octahedron with 13 rotation axes and the right are four shapes sharing continuous rotational symmetry, the recovered rotational axes are highlighted in red for visualization. Note that all rotational axes of octahedron are reasoned because our method is able to detect an arbitrary number of reflective symmetry planes, showing the effectiveness of the proposed approach.

![Figure 2: Our method can detect reasonable rotational symmetries.](image)

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3. Application to Shape Completion

As useful high-level information, symmetry can be useful in a wide variety of applications. We apply symmetry detection to shape completion in this case. After obtaining the symmetries of an incomplete shape, we can reasonably restore its missing parts. Figure 3 shows a comparison of our method against baselines, the input incomplete point cloud is a randomly rotated airplane (for better visualization, we rotate it back in Figure 3), the symmetry parts are mirrored according to the detected symmetry planes and are highlighted in light blue. The Chamfer distances ($\times 10^{-4}$) between the completed point cloud and ground truth point cloud of different methods are displayed on the bottom as well, our method achieve the lowest Chamfer distance. Please zoom in to see more details.

4. Additional Results

4.1. Quantitative results

Following PRS-Net [5], GTE (Ground Truth Error) is also adopted as a metric, which measures how close the predicted symmetry is to the corresponding ground truth. Due to the directional ambiguity of a plane, the error is computed as:

$$GTE(S, S_{gt}) = \min(MSE(S, S_{gt}), MSE(-S, S_{gt})),$$

(1)

where $S_{gt}$ is the ground truth symmetry of $S$ and MSE is the mean squared error. Quantitative results compared with baseline methods are listed in Table 1, which shows that our method outperforms other methods in GTE as well.

4.2. Qualitative results

Additional qualitative reflective symmetry detection results in ShapeNet compared with different methods are shown in Fig. 4. Our method is able to detect all valid planar reflective symmetries, while other methods detect redundant or inaccurate planes, showing the effectiveness of our method. In Figs. 5, 6, 7, additional visualization results in 12 categories of ShapeNet are displayed to further demonstrate the effectiveness to detect symmetries for different shapes.

4.3. Comparison with Vector Neurons

We evaluate our method compared with the novel Vector Neurons (VN) [4] that can produce rotation invariant features from point clouds on MVP [6]. We replace the encoder in our framework with VN-PointNet for feature extraction. The method of Vector Neurons [4] gets $10.17$ for SDE and $12.54$ for SDE*, where SDE* measures the performance when the incomplete shape is randomly rotated. Apart from that the extracted features of our method perform better than Vector Neurons [4] (see Table 3 in the main paper), benefiting from the end-to-end unsupervised framework we proposed, the performance of our framework with the encoder replaced by VN-PointNet still outperforms the previous state-of-the-arts by a large margin, e.g., PRS-Net [5] that achieves $34.10$ for SDE and $46.13$ for SDE* respectively.

References

Figure 4: Qualitative reflective symmetry detection results in ShapeNet comparing different methods.
Figure 5: Qualitative reflective symmetry detection results in ShapeNet, including (a) Cap, (b) Airplane, (c) Basket, (d) Bathtub.
Figure 6: Qualitative reflective symmetry detection results in ShapeNet, including (a) Bed, (b) Bench, (c) Bird House, (d) Book Shelf.
Figure 7: Qualitative reflective symmetry detection results in ShapeNet, including (a) Camera, (b) Car, (c) Chair, (d) Earphone.


