Supplementary material for FCCNs: Fully Complex-valued Convolutional Networks using Complex-valued Color Model and Loss Function

1. Complex-valued Cross Entropy Loss

Recall how we derived our complex-valued loss $f_{comp} : \mathbb{C}^{H\times W\times 3} \rightarrow \mathbb{C}^c$ from the original cross-entropy loss, which is given as:

$$\mathcal{L}_{comp} = -\frac{1}{n}\sum_{j=1}^{n}\sum_{k=1}^{c} y_{jk}\log z_{jk} \tag{1}$$

where z_{jk} is the complex-valued output, and y_{jk} is the ground truth label (with phase equal to 0). To establish the differentiability of this loss, we must ensure the differentiability of the function $\log z_{jk}$, which is the logarithm of a complex number.

Let $f(z) = \log(z)$, where $z = a + ib$. Note that if $f(z)$ can be expanded to a complex form $u(a, b) + iv(a, b)$, for it to be differentiable, it must satisfy Cauchy-Riemann equations[2], which are given as:

$$\frac{\partial u}{\partial a} = \frac{\partial v}{\partial b} \qquad \text{and} \qquad \frac{\partial v}{\partial a} = -\frac{\partial u}{\partial b} \tag{2}$$

Let's prove that logarithm of a complex number satisfies these equations.

Proof: Given function $f(z) = \log(z)$, where $z \in \mathbb{C}$, i.e., $z = a + ib$, we can compute the magnitude $|z|$ and phase θ of z as follows:

$$|z| = \sqrt{a^2 + b^2} \qquad \text{and} \qquad \theta = \tan^{-1}(b/a)$$

Since $z = |z|e^{i\theta}$, using above equations, we can expand $\log(z)$ into a complex form in the following manner:

$$\log(z) = (1/2)\log(a^2 + b^2) + i\tan^{-1}(b/a) \tag{3}$$

Thus, $u = (1/2)\log(a^2 + b^2)$ and $v = \tan^{-1}(b/a)$. Now, let's compute the four partial derivatives required in Cauchy-Riemann equations 2:

$$\frac{\partial u}{\partial a} = \frac{\partial}{\partial a}(1/2)\log(a^2 + b^2) = \frac{a}{a^2 + b^2} \tag{4}$$

$$\frac{\partial v}{\partial b} = \frac{\partial}{\partial b}\tan^{-1}(b/a) = \frac{a}{a^2 + b^2} \tag{5}$$

$$\frac{\partial v}{\partial a} = \frac{\partial}{\partial a}\tan^{-1}(b/a) = \frac{-b}{a^2 + b^2} \tag{6}$$

$$\frac{\partial u}{\partial b} = \frac{\partial}{\partial b}(1/2)\log(a^2 + b^2) = \frac{b}{a^2 + b^2} \tag{7}$$

From the above equations, we can conclude that the logarithm of a complex number indeed satisfies Cauchy-Riemann equations 2. Hence, $\log(z)$ is differentiable, and so is our proposed loss $\mathcal{L}_{comp}$.

2. Addition Observations

Table 1. Transfer learning experiment on Modern Office-31 dataset

Modern Office-31	DCN[3]	CNN[1]	FCCN
Accuracy	93.71	96.61	**97.23**

For observing the generalization capability of our method, we show results on Finetuned ResNet152, which was pre-trained on ImageNet on webcam subset of Modern Office-31 dataset. We show our results for different methods in Table 1.

References

[1] Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images. 2009. 1

[2] Elias M Stein and Rami Shakarchi. *Complex analysis*, volume 2. Princeton University Press, 2010. 1

[3] Chiheb Trabelsi, Olexa Bilaniuk, Ying Zhang, Dmitriy Serdyuk, Sandeep Subramanian, João Felipe Santos, Soroush Mehri, Negar Rostamzadeh, Yoshua Bengio, and Christopher J. Pal. Deep complex networks. In *6th International Conference on Learning Representations, ICLR 2018, Vancouver, BC, Canada, April 30 - May 3, 2018, Conference Track Proceedings*. OpenReview.net, 2018. 1