Supplementary Material — A Linear N-Point Solver for Structure and Motion from Asynchronous Tracks

1. Appendix

1.1. Explicit Matrix Formulas

In the main text, we discuss the use of matrices $\mathbf{M}_A, \mathbf{M}_B, \mathbf{M}_C$ defined via

$$\underbrace{\mathbf{A}^\intercal\mathbf{A}}_{\doteq\mathbf{M}}\mathbf{x} = \begin{bmatrix}\mathbf{M}_A & \mathbf{M}_B \\ \mathbf{M}_B^\intercal & \mathbf{M}_D\end{bmatrix}\begin{bmatrix}\mathbf{P}_{1:M} \\ \mathbf{v}\end{bmatrix} = \mathbf{0}_{(3M+3)\times 1}, \tag{1}$$

where $\mathbf{A}$ is defined via

$$\underbrace{\begin{bmatrix}\mathbf{F}_1 & & & & \mathbf{G}_1 \\ & \mathbf{F}_2 & & & \mathbf{G}_2 \\ & & \ddots & & \vdots \\ & & & \mathbf{F}_M & \mathbf{G}_M\end{bmatrix}}_{\doteq\mathbf{A}\in\mathbb{R}^{3N\times(3M+3)}}\underbrace{\begin{bmatrix}\mathbf{P}_1 \\ \mathbf{P}_2 \\ \vdots \\ \mathbf{P}_M \\ \mathbf{v}\end{bmatrix}}_{\doteq\mathbf{x}\in\mathbb{R}^{3M+3}} = \mathbf{0}_{3N\times 1} \tag{2}$$

and each $\mathbf{F}_i, \mathbf{G}_i$ is defined as

$$\underbrace{\begin{bmatrix}\left[\mathbf{f}'_{ij}\right]_\times & -t'_{ij}\left[\mathbf{f}'_{ij}\right]_\times \\ \vdots & \vdots \\ \left[\mathbf{f}'_{iN_i}\right]_\times & -t'_{iN_i}\left[\mathbf{f}'_{iN_i}\right]_\times\end{bmatrix}}_{\doteq\left[\mathbf{F}_i \quad \mathbf{G}_i\right]\in\mathbb{R}^{3N_i\times 6}}\begin{bmatrix}\mathbf{P}_i \\ \mathbf{v}\end{bmatrix} = \mathbf{0}_{3N_i\times 1} \tag{3}$$

We thus see that

$$\mathbf{M}_A = \begin{bmatrix}\mathbf{F}_1^\intercal\mathbf{F}_1 & & & \\ & \mathbf{F}_2^\intercal\mathbf{F}_2 & & \\ & & \ddots & \\ & & & \mathbf{F}_M^\intercal\mathbf{F}_M\end{bmatrix} \tag{4}$$

$$\mathbf{M}_B = \begin{bmatrix}\mathbf{F}_1^\intercal\mathbf{G}_1 \\ \mathbf{F}_2^\intercal\mathbf{G}_2 \\ \vdots \\ \mathbf{F}_M^\intercal\mathbf{G}_M\end{bmatrix} \tag{5}$$

$$\mathbf{M}_D = \sum_{i=1}^{M}\mathbf{G}_i^\intercal\mathbf{G}_i \tag{6}$$

where

$$\mathbf{F}_i^\intercal\mathbf{F}_i = -\sum_{j=1}^{N_i}[\mathbf{f}'_{ij}]_\times^2 \tag{7}$$

$$\mathbf{F}_i^\intercal\mathbf{G}_i = \sum_{j=1}^{N_i}t'_{ij}[\mathbf{f}'_{ij}]_\times^2 \tag{8}$$

$$\mathbf{G}_i^\intercal\mathbf{G}_i - = \sum_{j=1}^{N_i}{t'_{ij}}^2[\mathbf{f}'_{ij}]_\times^2 \tag{9}$$

share significant computation due to the products $[\mathbf{f}'_{ij}]_\times^2$. Finally, computing the Shur complement

$$\mathbf{B} = \mathbf{M}_D - \mathbf{M}_B^\intercal\mathbf{M}_A^{-1}\mathbf{M}_B \tag{10}$$

we find that

$$\mathbf{B} = \sum_{i=1}^{M}\mathbf{G}_i^\intercal\mathbf{G}_i - \mathbf{G}_i^\intercal\mathbf{F}_i(\mathbf{F}_i^\intercal\mathbf{F}_i)^{-1}\mathbf{F}_i^\intercal\mathbf{G}_i \tag{11}$$

1.2. Rank of $\mathbf{B}$

Assuming a full rank of $\mathbf{F}_i$, we know that each $\mathbf{G}_i$ has full rank, since $\mathbf{G}_i = \text{diag}(t'_{i1}\mathbf{I}_{3\times 3}, ..., t'_{iN_i}\mathbf{I}_{3\times 3})\mathbf{F}_i$, and the diagonal matrix has full rank for $t'_{ij} \neq 0$. Thus, assume given the SVD of $\mathbf{F}_i = \mathbf{U}_i\mathbf{\Sigma}_i\mathbf{V}_i^\intercal$. Due to the full rank we have that $\mathbf{\Sigma}_i^{-1}$ exists.

Thus, the above formula for $\mathbf{B}$ becomes

$$\mathbf{B} = \sum_{i=1}^{M}\mathbf{G}_i^\intercal\mathbf{G}_i - \mathbf{G}_i^\intercal\mathbf{U}_i\mathbf{U}_i^\intercal\mathbf{G}_i \tag{12}$$

which can be factored into

$$\mathbf{B} = \mathbf{G}^\intercal\left(\mathbf{I}_{3N\times 3N} - \mathbf{U}\mathbf{U}^\intercal\right)\mathbf{G} \tag{13}$$

where

$$\mathbf{G} = \begin{bmatrix}\mathbf{G}_1 \\ \vdots \\ \mathbf{G}_M\end{bmatrix} \quad \mathbf{U} = \begin{bmatrix}\mathbf{U}_1 & & \\ & \ddots & \\ & & \mathbf{U}_M\end{bmatrix} \tag{14}$$

To check that $\mathbf{B}$ has full rank we assume a solution $\hat{\mathbf{v}}$ to $\mathbf{B}\hat{\mathbf{v}} = \mathbf{0}_{3\times 1}$, and then need to enforce that it must be zero. Then

$$\mathbf{G}^\intercal \left(\mathbf{I}_{3N\times 3N} - \mathbf{U}\mathbf{U}^\intercal\right) \mathbf{G}\hat{\mathbf{v}} = \mathbf{0}_{3\times 1} \tag{15}$$

However, since $\mathbf{G}^\intercal$ has full rank, this implies

$$\left(\mathbf{I}_{3N\times 3N} - \mathbf{U}\mathbf{U}^\intercal\right) \mathbf{G}\hat{\mathbf{v}} = \mathbf{0}_{3\times 1} \tag{16}$$

in other words $\mathbf{G}\hat{\mathbf{v}}$ must be in the null space of $\mathbf{I}_{3N\times 3N} - \mathbf{U}\mathbf{U}^\intercal$. This null space is exactly the column space of $\mathbf{U}$, *i.e.* vectors of the form $\mathbf{U}\boldsymbol{\lambda}$ with $\boldsymbol{\lambda} \in \mathbb{R}^{3M}$. So

$$\mathbf{G}\hat{\mathbf{v}} = \mathbf{U}\boldsymbol{\lambda} \tag{17}$$

or equivalently for each block

$$\mathbf{G}_i\hat{\mathbf{v}} = \mathbf{U}_i\boldsymbol{\lambda}_i \tag{18}$$

which can be converted into the equation

$$\begin{bmatrix}\mathbf{G} & -\mathbf{U}_i\end{bmatrix} \begin{bmatrix}\hat{\mathbf{v}} \\ \boldsymbol{\lambda}_i\end{bmatrix} = \mathbf{0}_{3N\times 1} \tag{19}$$

If any of the matrices on the left-hand side have full rank, this implies that $\hat{\mathbf{v}} = \mathbf{0}_{3\times 1}$, and thus that $\mathbf{B}$ has full rank.

1.3. Connection to Epipolar Constraint

Here we show that the point incidence relation is related to the epipolar constraint used in the well-known 5-point or 8-point algorithms. This constraint is formed from point correspondences $\mathbf{x}_{ij}$ across two views at times t_1, t_0. This means in our setting, for each 3D point we have $N_i = 2$, and $j = 0, 1$. Moreover, all point in one view are synchronized, *i.e.* $t_{i0} = t_0$ and $t_{i1} = t_1$ for all $i = 1, ..., M$. We are left with two sets of equations

$$[\mathbf{f}'_{i0}]_\times \mathbf{P}_i - t'_0 [\mathbf{f}'_{i0}]_\times \mathbf{v} = \mathbf{0}_{3\times 1} \tag{20}$$
$$[\mathbf{f}'_{i1}]_\times \mathbf{P}_i - t'_1 [\mathbf{f}'_{i1}]_\times \mathbf{v} = \mathbf{0}_{3\times 1} \tag{21}$$

where $t'_0 = t_0 - t_s$ and $t'_1 = t_1 - t_s$. Without loss of generality we will set $t_s = t_0$ so that $t'_0 = 0$, $t'_1 = t_1 - t_0$. Introducing translation $\mathbf{t} \doteq \mathbf{v}(t_1 - t_0)$ yields

$$[\mathbf{f}'_{i0}]_\times \mathbf{P}_i = \mathbf{0}_{3\times 1} \tag{22}$$
$$[\mathbf{f}'_{i1}]_\times \mathbf{P}_i - [\mathbf{f}'_{i1}]_\times \mathbf{t} = \mathbf{0}_{3\times 1} \tag{23}$$

Next we left-multiply the second set of equations by ${\mathbf{f}'}^\intercal_{i0}$ yielding

$${\mathbf{f}'}^\intercal_{i0} [\mathbf{f}'_{i1}]_\times \mathbf{P}_i - {\mathbf{f}'}^\intercal_{i0} [\mathbf{f}'_{i1}]_\times \mathbf{t} = 0 \tag{24}$$

We cycle both triple products to get

$$\underbrace{{\mathbf{f}'}^\intercal_{i1} [\mathbf{P}_i]_\times \mathbf{f}'_{i0}}_{=0} - {\mathbf{f}'}^\intercal_{i1} [\mathbf{t}]_\times \mathbf{f}'_{i0} = 0 \tag{25}$$

The first term is 0 due to $[\mathbf{f}'_{i0}]_\times \mathbf{P}_i = \mathbf{0}_{3\times 1}$ and we are thus left with

$${\mathbf{f}'}^\intercal_{i1} [\mathbf{t}]_\times \mathbf{f}'_{i0} = 0 \tag{26}$$

Next we substitute $\mathbf{f}'_{i0} = \mathbf{I}_{3\times 3}\mathbf{f}_{i0}$ and $\mathbf{f}'_{i1} = \mathbf{R}(t_1)\mathbf{f}_{i0}$. We call $\mathbf{R} \doteq \mathbf{R}(t_1)$, then

$$\mathbf{f}^\intercal_{i1} \underbrace{\mathbf{R}^\intercal [\mathbf{t}]_\times}_{\doteq \mathbf{E}} \mathbf{f}_{i0} = 0 \tag{27}$$

The underlined matrix is also called the essential matrix, and the above constraint is exactly the epipolar constraint.

1.4. Connection to Line Solver

The line solver in [1] makes the assumption that points $\mathbf{P}_i$ lie on a line positioned at unit depth. We can thus write the position of $\mathbf{P}_i$ as a linear combination of unit vectors $\mathbf{e}^\ell_1$ and $\mathbf{e}^\ell_3$ as

$$\mathbf{P}_i = p_{i,1}\mathbf{e}^\ell_1 - \mathbf{e}^\ell_3. \tag{28}$$

Here $\mathbf{e}^\ell_3$ points from the closest point on the line to the origin, and $\mathbf{e}^\ell_1$ points in the direction of the line. Further define the unit vector $\mathbf{e}^\ell_2 = \mathbf{e}^\ell_3 \times \mathbf{e}^\ell_1$. We start off by multiplying the original incidence relation from the left with ${\mathbf{e}^\ell_1}^\intercal$ yielding

$${\mathbf{e}^\ell_1}^\intercal [\mathbf{f}_{ij}]_\times \mathbf{P}_i - t'_{ij} {\mathbf{e}^\ell_1}^\intercal [\mathbf{f}_{ij}]_\times \mathbf{v} = 0. \tag{29}$$

Next, we use $\mathbf{a}^\intercal [\mathbf{b}]_\times \mathbf{c} = \mathbf{c}^\intercal [\mathbf{a}]_\times \mathbf{b}$ to arrive at

$$\mathbf{f}^\intercal_{ij}\mathbf{e}^\ell_2 - t'_{ij}\mathbf{f}^\intercal_{ij}(\mathbf{e}^\ell_1 \times \mathbf{v}) = 0. \tag{30}$$

Finally, we express the linear velocity in the basis $\mathbf{e}^\ell_1, \mathbf{e}^\ell_2, \mathbf{e}^\ell_3$, *i.e.*

$$\mathbf{v} = u_x\mathbf{e}^\ell_1 + u_y\mathbf{e}^\ell_2 + u_z\mathbf{e}^\ell_3 \tag{31}$$

inserting above, and expanding the cross product we have that

$$\mathbf{f}^\intercal_{ij}\mathbf{e}^\ell_2 + t'_{ij}\mathbf{f}^\intercal_{ij}(u_z\mathbf{e}^\ell_2 - u_y\mathbf{e}^\ell_3) = 0. \tag{32}$$

This is exactly Eq. 6 in [1], which demonstrates that the line solver is a special case of the point solver described here.

1.5. Arbitrary Taylor Expansions

In what follows, we will expand the camera motion as an S order Taylor Series:

$$\mathbf{R}(t_{ij}) \approx \exp\left(\left[\sum_{s=1}^{S} \frac{\boldsymbol{\omega}^{(s)} {t'}^s_{ij}}{s!}\right]_\times\right) \tag{33}$$

$$\mathbf{p}(t_{ij}) \approx \sum_{s=1}^{S} \frac{\mathbf{v}^{(s)} {t'}^s_{ij}}{s!}. \tag{34}$$

Here we denote $\boldsymbol{\omega}^{(s)}$ the s order angular rate, and $\mathbf{v}^{(s)}$ the s order linear rate. Again we will focus on recovering the linear rates, and leave the angular rates as given. Inserting these definitions into the incidence relations yields the linear system

$$\left[\mathbf{f}'_{ij}\right]_\times \mathbf{P}_i - \sum_{s=1}^{S} \frac{t'^{s}_{ij}}{s!} \left[\mathbf{f}'_{ij}\right]_\times \mathbf{v}^{(s)} = \mathbf{0}_{3\times 1} \tag{35}$$

We stack all such constraints that involve the point $\mathbf{P}_i$ into a single system of equations.

$$\underbrace{\begin{bmatrix} \left[\mathbf{f}'_{i1}\right]_\times & -t'_{i1}\left[\mathbf{f}'_{i1}\right]_\times & \dots & -\frac{t'^{S}_{i1}}{S!}\left[\mathbf{f}'_{i1}\right]_\times \\ \vdots & \vdots & \ddots & \vdots \\ \left[\mathbf{f}'_{iN_i}\right]_\times & -t'_{iN_i}\left[\mathbf{f}'_{iN_i}\right]_\times & \dots & -\frac{t'^{S}_{iN_i}}{S!}\left[\mathbf{f}'_{iN_i}\right]_\times \end{bmatrix}}_{\doteq\left[\mathbf{F}_i \quad \mathbf{G}_\mathbf{i}^{(\mathbf{1})} \quad \dots \quad \mathbf{G}_\mathbf{i}^{(\mathbf{S})}\right] \in \mathbb{R}^{3N_i\times(3+3S)}} \begin{bmatrix} \mathbf{P}_i \\ \mathbf{v}^{(1)} \\ \vdots \\ \mathbf{v}^{(S)} \end{bmatrix} = \mathbf{0}_{3N_i\times 1} \tag{36}$$

where $\mathbf{G}_i^{(s)} \in \mathbb{R}^{3N_i\times 3}$. For $S=1$ we recover the old case. Denoting

$$\mathbf{v} \doteq \begin{bmatrix} \mathbf{v}^{(1)} \\ \vdots \\ \mathbf{v}^{(S)} \end{bmatrix} \quad \mathbf{G}_i \doteq \left[\mathbf{G}_\mathbf{i}^{(\mathbf{1})} \quad \dots \quad \mathbf{G}_\mathbf{i}^{(\mathbf{S})}\right] \tag{37}$$

we arrive at the same algorithm as before. However, crucially, $\mathbf{A} \in \mathbb{R}^{3N\times(3M+3S)}$ and the Shur complement reduces $\mathbf{A}^\intercal\mathbf{A}$ to a matrix $\mathbf{B} \in \mathbb{R}^{3S\times 3S}$. The solution duality and degeneracy remains the same. However, the minimality discussion needs to be adjusted since more unknowns are now introduced.

Minimality: The total system has $2N$ constraints, with $3M+3S-1$ unknowns (including scale ambiguity). This means that

$$N \geq \left\lceil \frac{3M+3S-1}{2} \right\rceil \tag{38}$$

At the same time, stability in the solver requires

$$N \geq 2M \tag{39}$$

we will try to find out when a minimal system arises, *i.e.* when $\left\lceil \frac{3M+3S-1}{2} \right\rceil = 2M$. We will treat two cases:

Case 1: Let $h \doteq 3M+3S-1$ be odd with $h = 2k+1$ and $k = \frac{h-1}{2}$ for some k, then the left side of the above equation becomes

$$\left\lceil \frac{h}{2} \right\rceil = k+1 \tag{40}$$

$$= \frac{3M+3S-2}{2} + 1 \tag{41}$$

$$= \frac{3M+3S}{2} \tag{42}$$

Setting this equal to $2M$ yields

$$M = 3S \tag{43}$$

We thus find that configurations $(S,M) = (s,3s)$ for $s = 1,2,...$ yield minimal systems. Setting $N = 2M = 6s$ with $N_i = 2$ yields a total of $12s$ equations with $12s$ unknowns coming from $3s$ 3D points and $6s$ observations.

Case 2: Let $h \doteq 3M+3S-1$ be even, so that $h = 2k$ and

$$\left\lceil \frac{h}{2} \right\rceil = k \tag{44}$$

$$= \frac{3M+3S-1}{2}. \tag{45}$$

Setting this equal to $2M$ we have that

$$M = 3S-1. \tag{46}$$

As a result, configurations $(S,M) = (s,3s-1)$ for $s = 1,2,...$ also yield minimal systems. In particular, setting $N = 2M$ with $N_i = 2$ we have a system of $12s-4$ equations with $12s-4$ unknowns coming from $3s-1$ 3D points, and $6s-2$ observations.

Checking the above equations for $S = 1$ recovers the minimal 4 and 6 point algorithms discussed in the main text.

1.5.1. Experiments with Acceleration Aware Solver

To extend our methodology to acceleration-aware motion estimation, we conducted simulation experiments as described in Sec.4.1 with a 1-second time window. The results shown in Fig. 2 and Fig. 3 reveal that introducing acceleration parameters (adding three degrees of freedom) systematically amplifies the solver's noise sensitivity. Notably, acceleration estimation exhibits 3-4 times higher error susceptibility than velocity under identical noise conditions. This effect arises because the acceleration term's coefficient scales quadratically with the timestamp in the motion model. Nevertheless, it can be mitigated by fusing IMU acceleration data to bootstrap initial velocity estimation.

1.6. Extending the Acceleration-aware Solver

The above formulation for $S = 2$ expands the camera motion up to acceleration yielding the incidence relation

$$\left[\mathbf{f}'_{ij}\right]_\times \mathbf{P}_i - t'_{ij}\left[\mathbf{f}'_{ij}\right]_\times \mathbf{v} - \frac{1}{2}t'^{2}_{ij}\left[\mathbf{f}'_{ij}\right]_\times \mathbf{a} = \mathbf{0}_{3\times 1} \tag{47}$$

Instead of solving for the unknown $\mathbf{a}$ we can assume it given by an IMU (which we already use to estimate angular velocity). For simplicity, we assume it to be constant over the time interval. We thus find that our system transforms into an in-homogeneous system :

$$\left[\mathbf{f}'_{ij}\right]_\times \mathbf{P}_i - t'_{ij}\left[\mathbf{f}'_{ij}\right]_\times \mathbf{v} = \frac{1}{2}t'^{2}_{ij}\left[\mathbf{f}'_{ij}\right]_\times \mathbf{a} \tag{48}$$

stacking these equations for one track yields

$$\underbrace{\begin{bmatrix} [\mathbf{f}'_{i1}]_\times & -t'_{i1}[\mathbf{f}'_{i1}]_\times \\ \vdots & \vdots \\ [\mathbf{f}'_{iN_i}]_\times & -t'_{iN_i}[\mathbf{f}'_{iN_i}]_\times \end{bmatrix}}_{\doteq[\mathbf{F}_i \quad \mathbf{G_i}]\in\mathbb{R}^{3N_i\times 6}} \begin{bmatrix}\mathbf{P}_i \\ \mathbf{v}\end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{2}{t'}^2_{i1}[\mathbf{f}'_{i1}]_\times \mathbf{a} \\ \vdots \\ \frac{1}{2}{t'}^2_{iN_i}[\mathbf{f}'_{iN_i}]_\times \mathbf{a} \end{bmatrix}}_{\doteq\mathbf{b}_i\in\mathbb{R}^{3N_i\times 1}} \tag{49}$$

and stacking these equations for each point we get

$$\underbrace{\begin{bmatrix} \mathbf{F}_1 & & & & \mathbf{G}_1 \\ & \mathbf{F}_2 & & & \mathbf{G}_2 \\ & & \ddots & & \vdots \\ & & & \mathbf{F}_M & \mathbf{G}_M \end{bmatrix}}_{\doteq\mathbf{A}\in\mathbb{R}^{3N\times(3M+3)}} \underbrace{\begin{bmatrix}\mathbf{P}_1 \\ \mathbf{P}_2 \\ \vdots \\ \mathbf{P}_M \\ \mathbf{v}\end{bmatrix}}_{\doteq\mathbf{x}\in\mathbb{R}^{3M+3}} = \underbrace{\begin{bmatrix}\mathbf{b}_1 \\ \vdots \\ \mathbf{b}_M\end{bmatrix}}_{\doteq\mathbf{b}\in\mathbb{R}^{3N\times 1}} \tag{50}$$

multiplying from the left with $\mathbf{A}^\intercal$ we have the system

$$\underbrace{\mathbf{A}^\intercal\mathbf{A}}_{\doteq\mathbf{M}}\mathbf{x} = \begin{bmatrix}\mathbf{M}_A & \mathbf{M}_B \\ \mathbf{M}_B^\intercal & \mathbf{M}_D\end{bmatrix}\begin{bmatrix}\mathbf{P}_{1:M} \\ \mathbf{v}\end{bmatrix} = \underbrace{\mathbf{A}^\intercal\mathbf{b}}_{\doteq\mathbf{c}} = \begin{bmatrix}\mathbf{c}_A \\ \mathbf{c}_B\end{bmatrix}, \tag{51}$$

where we now define $\mathbf{c} \in \mathbb{R}^{(3M+3)\times 1}$ with $\mathbf{c}_A \in \mathbb{R}^{3M\times 1}$ and $\mathbf{c}_B \in \mathbb{R}^{3\times 1}$. Explicit formulas for these are

$$\mathbf{c}_A = \begin{bmatrix}\mathbf{F}_1^\intercal\mathbf{b}_1 \\ \vdots \\ \mathbf{F}_M^\intercal\mathbf{b}_M\end{bmatrix} \quad \mathbf{c}_B = \sum_{i=1}^{M}\mathbf{G}_i^\intercal\mathbf{b}_i \tag{52}$$

where

$$\mathbf{F}_i^\intercal\mathbf{b}_i = -\frac{1}{2}\sum_{j=1}^{N_i}{t'}^2_{ij}\left[\mathbf{f}'_{iN_i}\right]^2_\times\mathbf{a} \tag{53}$$

and

$$\mathbf{G}_i^\intercal\mathbf{b}_i = \frac{1}{2}\sum_{j=1}^{N_i}{t'}^3_{ij}\left[\mathbf{f}'_{iN_i}\right]^2_\times\mathbf{a} \tag{54}$$

Applying the Schur-complement trick to this system yields the following system for $\mathbf{v}$

$$\mathbf{B}\mathbf{v} = \underbrace{\mathbf{c}_B - \mathbf{M}_B^\intercal\mathbf{M}_A^{-1}\mathbf{c}_A}_{\doteq\mathbf{d}} \tag{55}$$

where we have defined $\mathbf{B}$ previously, and we now have introduced $\mathbf{d} \in \mathbb{R}^{3\times 1}$ with the following explicit formula

$$\mathbf{d} = \sum_{j=1}^{M}\mathbf{G}_i^\intercal\mathbf{b}_i - \mathbf{G}_i^\intercal\mathbf{F}_i(\mathbf{F}_i^\intercal\mathbf{F}_i)^{-1}\mathbf{F}_i^\intercal\mathbf{b}_i \tag{56}$$

Solving for $\mathbf{v}$ requires simply inverting the matrix, yielding

$$\hat{\mathbf{v}} = \mathbf{B}^{-1}\mathbf{d} \tag{57}$$

and back substitution to get the 3D points yields

$$\hat{\mathbf{P}}_i = (\mathbf{F}_i^\intercal\mathbf{F}_i)^{-1}\mathbf{F}_i^\intercal(\mathbf{b}_i - \mathbf{G}_i\hat{\mathbf{v}}) \tag{58}$$

It is important to note that, having taken the acceleration into account adds two qualitative differences between our solution to the previous cases: First, absolute scale suddenly becomes observable, meaning that the recovered $\hat{\mathbf{v}}, \hat{\mathbf{P}}_i$ are in meters. Second, our method no longer has multiple solutions, making the depth check unnecessary.

References

[1] Ling Gao, Daniel Gehrig, Hang Su, Davide Scaramuzza, and Laurent Kneip. A linear n-point solver for line and motion estimation with event cameras. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2024. 2

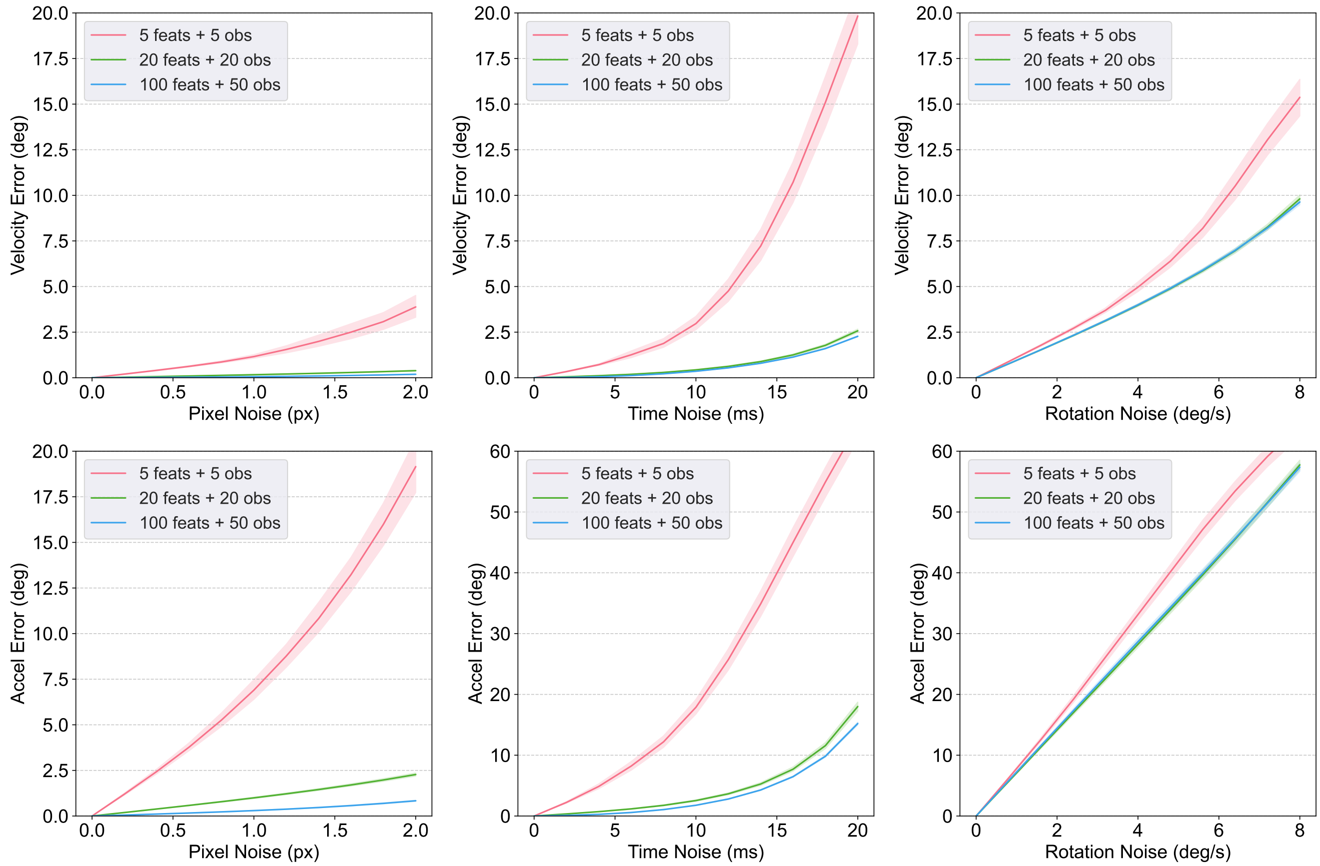

Figure 1. The robustness of acceleration-aware point solver against pixel noise (left) and timestamp jitter (middle) and rotation (right)
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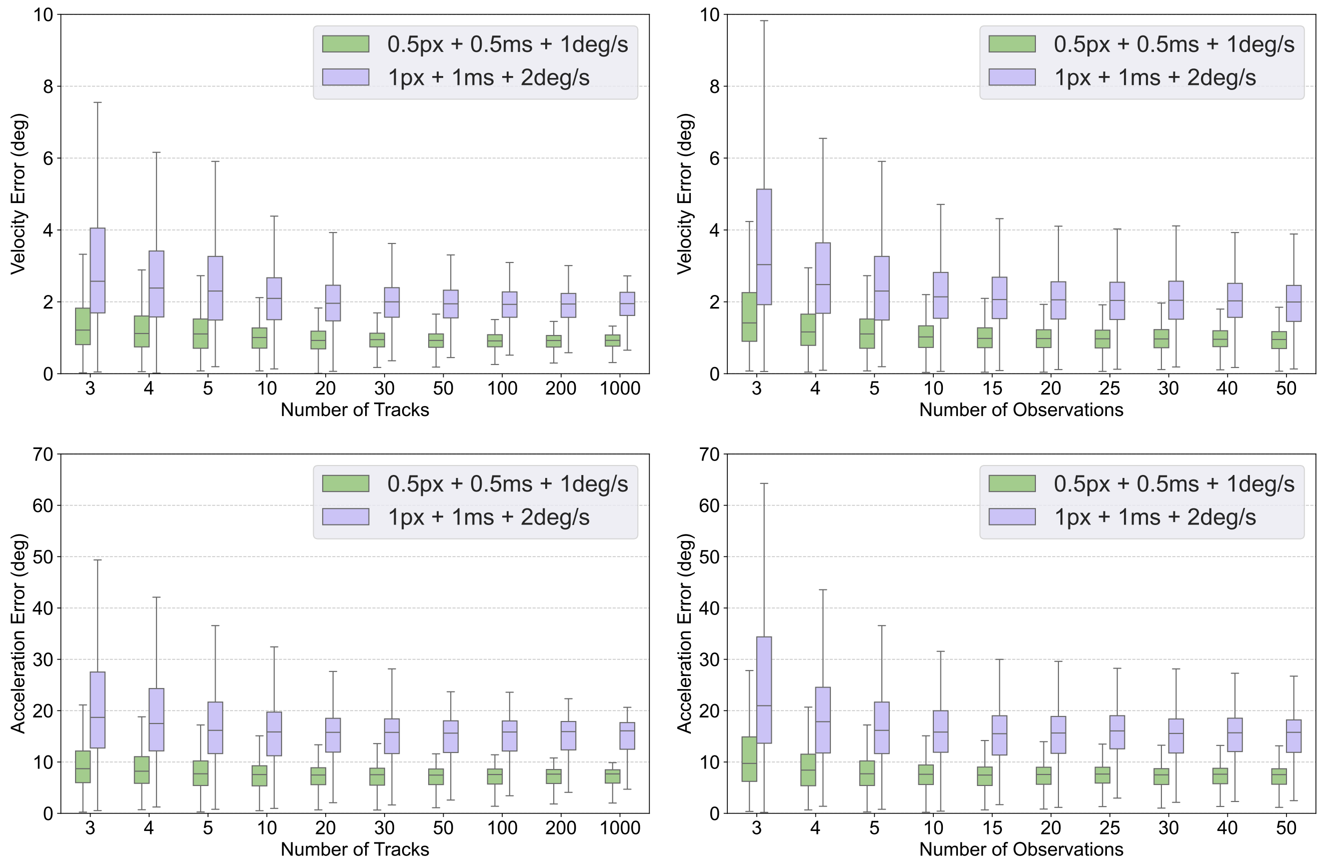

Figure 2. Analysis of acceleration-aware point solver on the number of feature tracks (left) and observations (right) under different combined noise level.

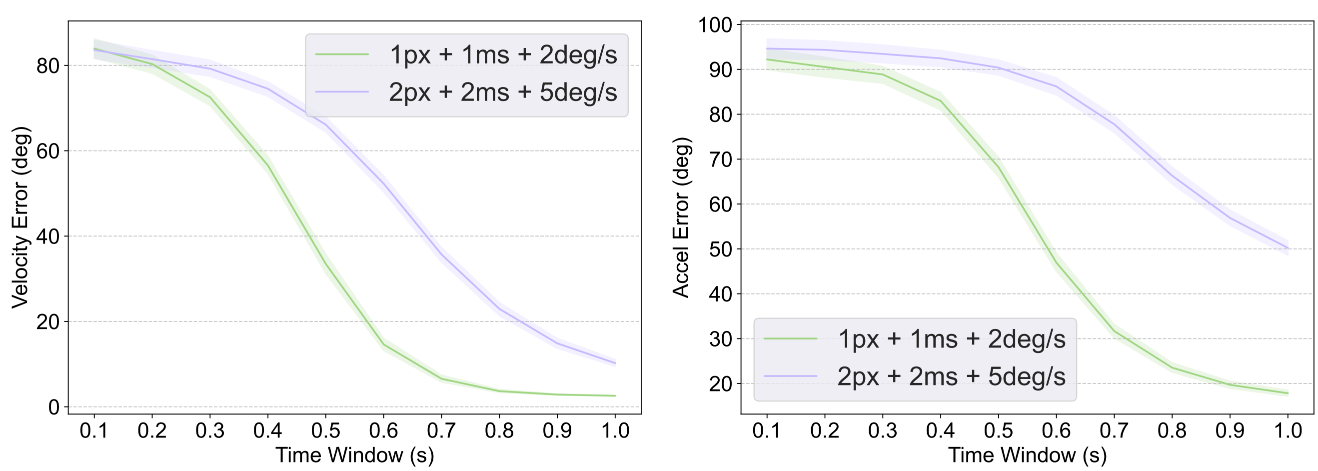

Figure 3. Analysis of the impact of time window on acceleration-aware solver under different noise level.