FVGen: Accelerating Novel-View Synthesis with Adversarial Video Diffusion Distillation

Supplementary Material

In this supplementary material, we mainly present the mathematical proof of Equation (8) in our main paper.

Starting from Equation (6):

$$\mathbf{D}_{\texttt{Soften-RKL}}(p_{\text{fake},t}\|p_{\text{real},t}) = \mathbf{D}_{KL}\left(\frac{1}{2}p_{\text{real},t} + \frac{1}{2}p_{\text{fake},t}\middle\|p_{\text{real},t}\right).$$

For notation simplicity purpose, we define $p_t(\boldsymbol{x}) := p_{\text{real},t}$ and $q_t(\boldsymbol{x}) := p_{\text{fake},t}$; here $\boldsymbol{x} = F(G_\theta(z), t)$ defined in the main paper. Then, we have:

$$\begin{aligned}
&\mathbf{D}_{\texttt{Soften-RKL}}(p_t(\boldsymbol{x})\|q_t(\boldsymbol{x})) \\
&= \mathbf{D}_{KL}\left(\frac{1}{2}p_t(\boldsymbol{x}) + \frac{1}{2}q_t(\boldsymbol{x})\|p_t(\boldsymbol{x})\right) \\
&= \int q_t(\boldsymbol{x})\left(\frac{p_t(\boldsymbol{x})}{q_t(\boldsymbol{x})}+1\right)\log\left(\frac{1}{2}+\frac{q_t(\boldsymbol{x})}{2p_t(\boldsymbol{x})}\right)d\boldsymbol{x}.
\end{aligned}$$

Taking derivative w.r.t to model parameter θ, we have the soften reverse KL divergence loss:

$$\begin{aligned}
&\nabla_\theta \mathcal{L}_{\text{DMD}}^{\texttt{Soften-RKL}} \\
&= \nabla_\theta \int q_t(\boldsymbol{x})\left(\frac{p_t(\boldsymbol{x})}{q_t(\boldsymbol{x})}+1\right)\log\left(\frac{1}{2}+\frac{q_t(\boldsymbol{x})}{2p_t(\boldsymbol{x})}\right)d\boldsymbol{x} \\
&= \int \nabla_\theta \left[q_t(\boldsymbol{x})\left(\frac{p_t(\boldsymbol{x})}{q_t(\boldsymbol{x})}+1\right)\log\left(\frac{1}{2}+\frac{q_t(\boldsymbol{x})}{2p_t(\boldsymbol{x})}\right)\right]d\boldsymbol{x} \\
&= \underbrace{\int \nabla_\theta q_t(\boldsymbol{x})\left[\left(\frac{p_t(\boldsymbol{x})}{q_t(\boldsymbol{x})}+1\right)\log\left(\frac{1}{2}+\frac{q_t(\boldsymbol{x})}{2p_t(\boldsymbol{x})}\right)\right]d\boldsymbol{x}}_{A} \\
&+ \underbrace{\int q_t(\boldsymbol{x})\nabla_\theta \left[\left(\frac{p_t(\boldsymbol{x})}{q_t(\boldsymbol{x})}+1\right)\log\left(\frac{1}{2}+\frac{q_t(\boldsymbol{x})}{2p_t(\boldsymbol{x})}\right)\right]d\boldsymbol{x}}_{B}.
\end{aligned}$$

As $p_t(\boldsymbol{x})$ is the real distribution, which is constant w.r.t θ. According to the chain rule, we have:

$$\begin{aligned}
B &= \int q_t(\boldsymbol{x})\left[-\frac{p_t(\boldsymbol{x})}{q_t^2(\boldsymbol{x})}\log\left(\frac{1}{2}+\frac{q_t(\boldsymbol{x})}{2p_t(\boldsymbol{x})}\right)\right. \\
&\quad \left.+\left(\frac{p_t(\boldsymbol{x})}{q_t(\boldsymbol{x})}+1\right)\frac{1}{p_t(\boldsymbol{x})+q_t(\boldsymbol{x})}\right]\nabla_\theta q_t(\boldsymbol{x})d\boldsymbol{x} \\
&= \int \nabla_\theta q_t(\boldsymbol{x})\left[-\frac{p_t(\boldsymbol{x})}{q_t(\boldsymbol{x})}\log\left(\frac{1}{2}+\frac{q_t(\boldsymbol{x})}{2p_t(\boldsymbol{x})}\right)+1\right].
\end{aligned}$$

Define $r_t(\boldsymbol{x}) = p_t(\boldsymbol{x})/q_t(\boldsymbol{x})$. Combining the derived B and previous A, we have:

$$\begin{aligned}
&\nabla_\theta \mathcal{L}_{\text{DMD}}^{\texttt{Soften-RKL}} \\
&= \int \nabla_\theta q_t(\boldsymbol{x})\left[\log\left(\frac{1}{2}+\frac{1}{2r_t(\boldsymbol{x})}\right)+1\right]d\boldsymbol{x} \\
&= \int q_t(\boldsymbol{x})\frac{2r_t(\boldsymbol{x})}{r_t(\boldsymbol{x})+1}\left(-\frac{1}{2r_t^2(\boldsymbol{x})}\right)\frac{\partial r_t(\boldsymbol{x})}{\partial \boldsymbol{x}}\frac{\partial G_\theta(\boldsymbol{z})}{\partial \theta}d\boldsymbol{x} \\
&= -\int q_t(\boldsymbol{x})\frac{1}{r_t(\boldsymbol{x})(r_t(\boldsymbol{x})+1)}\frac{\partial r_t(\boldsymbol{x})}{\partial \boldsymbol{x}}\frac{\partial G_\theta(\boldsymbol{z})}{\partial \theta}d\boldsymbol{x}.
\end{aligned}$$

According to log-derivative trick, we have:

$$\begin{aligned}
\frac{\partial r_t(\boldsymbol{x})}{\partial \boldsymbol{x}} &= \frac{\frac{\partial p_t(\boldsymbol{x})}{\partial \boldsymbol{x}}\, q_t(\boldsymbol{x}) - \frac{\partial q_t(\boldsymbol{x})}{\partial \boldsymbol{x}}\, p_t(\boldsymbol{x})}{\left(q_t(\boldsymbol{x})\right)^2} \\
&= \frac{\frac{\nabla_{\boldsymbol{x}}\, p_t(\boldsymbol{x})}{p_t(\boldsymbol{x})} - \frac{\nabla_{\boldsymbol{x}}\, q_t(\boldsymbol{x})}{q_t(\boldsymbol{x})}}{\frac{q_t(\boldsymbol{x})}{p_t(\boldsymbol{x})}} \\
&= r_t(\boldsymbol{x})\big(\nabla_{\boldsymbol{x}}\log p_t(\boldsymbol{x}) - \nabla_{\boldsymbol{x}}\log q_t(\boldsymbol{x})\big) \\
&= r_t(\boldsymbol{x})\,\left[s_{\text{real},t}(\boldsymbol{x}) - s_{\text{fake},t}(\boldsymbol{x})\right].
\end{aligned}$$

Therefore:

$$\begin{aligned}
&\nabla_\theta \mathcal{L}_{\text{DMD}}^{\texttt{Soften-RKL}} \\
&= -\int q_t(\boldsymbol{x})\frac{1}{r_t(\boldsymbol{x})+1}\left[s_{\text{real},t}(\boldsymbol{x}) - s_{\text{fake},t}(\boldsymbol{x})\right]\frac{\partial G_\theta(\boldsymbol{z})}{\partial \theta}d\boldsymbol{x} \\
&= -\mathbb{E}\left[\frac{1}{r_t(\boldsymbol{x})+1}\big(s_{\text{real},t}(\boldsymbol{x}) - s_{\text{fake},t}(\boldsymbol{x})\big)\frac{\partial G_\theta(\boldsymbol{z})}{\partial \theta}\right]
\end{aligned}$$

This concludes the proof of Equation (8) in the main paper.