# Any-SSR: How Recursive Least Squares Works in Continual Learning of Large Language Models

Supplementary Material

## A. Proof of Equation (17)

Proof. According to Eq. (15):

$$\hat{\boldsymbol{W}}^{\mathrm{r}}_{k+1} = \boldsymbol{R}_{k+1}\boldsymbol{Q}_{k+1} = \boldsymbol{R}_{k+1}\boldsymbol{Q}_{k} + \boldsymbol{R}_{k+1}\tilde{h}^{\top}_{k+1}\boldsymbol{Y}_{k+1}. \tag{26}$$

Noticed that $\boldsymbol{R}_{k+1}\boldsymbol{Q}_{k+1}$ can be written as:

$$\begin{aligned}\boldsymbol{R}_{k+1}\boldsymbol{Q}_{k} &= \boldsymbol{R}_{k}\boldsymbol{Q}_{k} - \boldsymbol{R}_{k}\tilde{h}^{\top}_{k+1}\boldsymbol{K}\tilde{h}_{k+1}\boldsymbol{R}_{k}\boldsymbol{Q}_{k} \\ &= (\boldsymbol{I} - \boldsymbol{R}_{k}\tilde{h}^{\top}_{k+1}\boldsymbol{K}_{k+1}\tilde{h}_{k+1})\hat{\boldsymbol{W}}_{k},\end{aligned}$$

Where $\boldsymbol{K} = (\boldsymbol{I} + \tilde{h}_{k+1}\boldsymbol{R}_{k}\tilde{h}^{\top}_{k})^{-1}$ and $\boldsymbol{K} \in \mathbb{R}^{n\times n}$.
Since

$$\boldsymbol{K}\boldsymbol{K}^{-1} = \boldsymbol{K}(\boldsymbol{I} + \tilde{h}_{k+1}\boldsymbol{R}_{k}\tilde{h}^{\top}_{k+1}) = \boldsymbol{I}, \tag{27}$$

we have

$$\boldsymbol{K} = \boldsymbol{I} - \boldsymbol{K}\tilde{h}_{k+1}\boldsymbol{R}_{k}\tilde{h}^{\top}_{k+1}. \tag{28}$$

Therefore

$$\boldsymbol{R}_{k+1}\boldsymbol{Q}_{k} = (\boldsymbol{I} - \boldsymbol{R}_{k+1}\tilde{h}^{\top}_{k+1}\tilde{h}_{k+1})\hat{\boldsymbol{W}}^{\mathrm{R}}_{k}. \tag{29}$$

Instantiate Eq. (29) into Eq. (15), we have

$$\hat{\boldsymbol{W}}^{\mathrm{r}}_{k+1} = (\boldsymbol{I} - \boldsymbol{R}_{k+1}\tilde{h}^{\top}_{k+1}\tilde{h}_{k+1})\hat{\boldsymbol{W}}^{\mathrm{R}}_{k} + \boldsymbol{R}_{k+1}\tilde{h}^{\top}_{k+1}\boldsymbol{Y}_{k+1}. \tag{30}$$

which completes the proof.

## B. Additional Model Validation

We have supplemented experiments on TRACE with GLM-4-9B and Qwen3-8B using Any-SSR. Results remain consistent and confirms ASR's generalizability across architectures:

| **Model** | **OP (%)** | **BWT (%)** |
|---|---|---|
| GLM-4-9B | 54.6 | -0.3 |
| Qwen3-8B | 55.1 | -0.1 |