Appendix

A. Algebraic Analysis: Full Table

In this section, we analyse the algebraic complexity metrics reported in Table 4 by grouping the calibration patterns into semantically meaningful categories. These categories reflect different levels of prior knowledge or assumptions about the camera's internal parameters.

A.1. Autocalibration / Self-Calibration

Goal: Recover internal parameters (e.g., focal lengths f, g, skew s, and principal point u, v) from image observations, typically vanishing points from orthogonal directions.

Patterns: `fguvs`, `fguv0`, `fgu0s`, `fgu00`, `ffuvs`

Observations:

- These patterns often result in **higher Jacobian ranks** (up to 5) and **moderate solution degrees** (commonly 2–4), reflecting the overparameterised nature of the symbolic system.
- For example, `fguvs` yields $(\mathrm{dim}, \mathrm{rank}, \mathrm{deg}) = (3, 5, 2)$, while `fguv0` and `fgu00` reduce some complexity by fixing parts of the principal point.
- `ffuvs`, which assumes $f = g$, still exhibits high complexity due to retained skew and principal point unknowns.

A.2. Focal Length Estimation

Goal: Estimate one or both focal lengths under the assumption of known or fixed skew and principal point.

Patterns: `ffuv0`, `f1uv0`, `1guv0`

Observations:

- These cases tend to show **lower dimensions** (mostly 2 or 3) and moderate degrees, e.g., `ffuv0` gives $(4, 3, 2)$.
- The simplification $f = g$ and $s = 0$ reduces parameter entanglement, making these formulations attractive for efficient solvers.
- `f1uv0` and `1guv0` benefit from known g or f respectively and yield Jacobian ranks of 3.

A.3. Zero-Skew with Known Principal Point

Goal: Estimate focal length(s) when the principal point (u, v) and skew are assumed known or fixed to 0.

Patterns: `fgu00`, `fg000`, `f1000`, `ff000`

Observations:

- These are among the **simplest patterns** algebraically: degrees are low (1–4), dimensions are typically 3 or 4.
- `f1000` and `ff000` only keep f as an unknown and yield low Jacobian ranks (1), making them ideal for focal-only estimation.
- `fg000` is an exception, with a high degree (8), possibly due to retained asymmetry in focal lengths.

A.4. Known Focal Ratio

Goal: Use a fixed focal ratio f/g (often $f = g$) to reduce degrees of freedom while keeping some internal parameters symbolic.

Patterns: `ffuvs`, `ffuv0`, `ffu0s`, `ff00s`

Observations:

- These often produce **moderate complexity** results. For instance, `ffuvs` yields $(3, 4, 3)$.
- `ff00s` keeps only skew as unknown and shows a relatively high degree (5) despite a small dimension.
- The $f = g$ constraint simplifies but does not eliminate symbolic interaction, especially when skew or principal point are retained.

A.5. Skew-Only Estimation

Goal: Solve only for the skew parameter s, assuming all other intrinsics are known.

Patterns: `1100s`, `ff00s`

Observations:

- These systems are **low-dimensional** and generally algebraically simple.
- `1100s` yields $(4, 1, 1)$ — ideal conditions for efficient closed-form solutions.
- In contrast, `ff00s` is more complex due to different symbolic structure, with degree 5.

B. Proposed Algorithm

This section presents the full procedure for solving the **`fguvs`** calibration task using our proposed polynomial formulation and a two-stage homotopy continuation method.

Algorithm 1 Direct Calibration for **`fguvs`**

Input: Pairs of orthogonal VPs $\mathcal{P} = \{(u_i, v_i)\}_{i=1}^{5}$
Output: Estimated intrinsic matrix K

Define IAC ω w/h unknowns $f, g, u, v, s \in \mathbb{C}$
Get $\mathcal{F} = \left\{ v_i^\top \omega(f, g, u, v, s) v_j = 0 \;\middle|\; (v_i, v_j) \in \mathcal{P} \right\}$

=== Offline phase
Sample a random instance of VP pairs $\{(v_i, v_j)\}_{i=1}^{5}$
Get start solutions $\{x_j^\star\} = \texttt{MonodromySolve}(\mathcal{F})$

=== Online phase
Construct $\mathcal{F}_{\text{new}}$ given a new scene $\{(v_i, v_j)\}$
Track $\{x_j^\star\}$ to $\mathcal{F}_{\text{new}}$ using PHC
Select the solution with the lowest error

return K

Table 4. Algebraic complexity metrics aggregated over 100 random seeds for each calibration task ('pattern').

pattern	dimension	degree	jacobian-rank
1100s	0.0 ± 0.0	2.0 ± 0.0	1.0 ± 0.0
110v0	0.0 ± 0.0	2.0 ± 0.0	1.0 ± 0.0
110vs	0.0 ± 0.0	5.98 ± 0.14	2.0 ± 0.0
11u00	0.0 ± 0.0	2.0 ± 0.0	1.0 ± 0.0
11u0s	0.0 ± 0.0	3.98 ± 0.14	2.0 ± 0.0
11uv0	0.0 ± 0.0	2.0 ± 0.0	2.0 ± 0.0
11uvs	0.0 ± 0.0	6.0 ± 0.0	3.0 ± 0.0
1g000	0.0 ± 0.0	2.0 ± 0.0	1.0 ± 0.0
1g00s	0.0 ± 0.0	4.0 ± 0.0	2.0 ± 0.0
1g0v0	0.0 ± 0.0	3.96 ± 0.28	2.0 ± 0.0
1g0vs	1.0 ± 0.0	2.0 ± 0.0	3.0 ± 0.0
1gu00	0.0 ± 0.0	3.96 ± 0.28	2.0 ± 0.0
1gu0s	1.0 ± 0.0	2.0 ± 0.0	3.0 ± 0.0
1guv0	0.0 ± 0.0	6.0 ± 0.0	3.0 ± 0.0
1guvs	2.0 ± 0.0	2.0 ± 0.0	4.0 ± 0.0
f1000	0.0 ± 0.0	2.0 ± 0.0	1.0 ± 0.0
f100s	0.0 ± 0.0	3.96 ± 0.28	2.0 ± 0.0
f10v0	0.0 ± 0.0	3.96 ± 0.28	2.0 ± 0.0
f10vs	0.0 ± 0.0	10.0 ± 0.0	3.0 ± 0.0
f1u00	0.0 ± 0.0	2.0 ± 0.0	2.0 ± 0.0
f1u0s	0.0 ± 0.0	8.0 ± 0.0	3.0 ± 0.0
f1uv0	0.0 ± 0.0	6.0 ± 0.0	3.0 ± 0.0
f1uvs	0.0 ± 0.0	8.0 ± 0.0	4.0 ± 0.0
ff000	0.0 ± 0.0	4.0 ± 0.0	1.0 ± 0.0
ff00s	0.0 ± 0.0	7.96 ± 0.28	2.0 ± 0.0
ff0v0	1.0 ± 0.0	2.0 ± 0.0	2.0 ± 0.0
ff0vs	1.0 ± 0.0	3.0 ± 0.0	3.0 ± 0.0
ffu00	1.0 ± 0.0	2.0 ± 0.0	2.0 ± 0.0
ffu0s	1.0 ± 0.0	3.0 ± 0.0	3.0 ± 0.0
ffuv0	2.0 ± 0.0	2.0 ± 0.0	3.0 ± 0.0
ffuvs	2.0 ± 0.0	3.0 ± 0.0	4.0 ± 0.0
fg000	0.02 ± 0.14	7.92 ± 0.56	1.98 ± 0.14
fg00s	1.0 ± 0.0	2.0 ± 0.0	3.0 ± 0.0
fg0v0	1.0 ± 0.0	4.0 ± 0.0	3.0 ± 0.0
fg0vs	2.0 ± 0.0	2.0 ± 0.0	4.0 ± 0.0
fgu00	1.0 ± 0.0	4.0 ± 0.0	3.0 ± 0.0
fgu0s	2.0 ± 0.0	2.0 ± 0.0	4.0 ± 0.0
fguv0	2.0 ± 0.0	4.0 ± 0.0	4.0 ± 0.0
fguvs	3.0 ± 0.0	2.0 ± 0.0	5.0 ± 0.0

C. Real-World Images: PHC-HS v/s SVD

C.1. Quantitative Evaluation

This section presents per-scene calibration error statistics and SVD success rates across six benchmark scenes. For each method, we report the mean absolute error on intrinsic parameters ($\Delta f, \Delta g, \Delta u, \Delta v, \Delta s$) aggregated over all valid frames. Errors are computed only over successful runs, and SVD success rates are reported separately due to its sensitivity to the positive-definiteness of the estimated IAC.

Table 5. Calibration summary for `Herz-Jesus-P25`. Mean error shown per method; lower is better.

Method	Δf	Δg	Δs	Δu	Δv
GeoCalib	0.04743	0.04769	–	0.01007	0.01707
SVD (Stratified)	∞	0.90912	∞	0.81362	0.84688
PHC-HS (Direct)	1.00944	2.88491	0.8405	6.67768	10.84512

Table 6. Calibration summary for `Herz-Jesus-P8`.

Method	Δf	Δg	Δs	Δu	Δv
GeoCalib	0.02899	0.02936	–	0.01007	0.01707
SVD (Stratified)	∞	1.00000	–	1.00000	1.00000
PHC-HS (Direct)	0.97811	1.18281	0.19566	6.01424	11.93725

Table 7. Calibration summary for `castle-P19`.

Method	Δf	Δg	Δs	Δu	Δv
GeoCalib	0.04645	0.04788	–	0.01007	0.01707
SVD (Stratified)	∞	0.95380	0.14167	0.82360	0.80140
PHC-HS (Direct)	0.99504	2.02073	0.41161	5.94341	12.46008

Table 8. Calibration summary for `castle-P30`.

Method	Δf	Δg	Δs	Δu	Δv
GeoCalib	0.04642	0.04769	–	0.01007	0.01707
SVD (Stratified)	∞	0.77300	0.12237	0.95925	1.14962
PHC-HS (Direct)	0.97953	2.89064	0.72931	6.72517	13.13400

Table 9. Calibration summary for `entry-P10`.

Method	Δf	Δg	Δs	Δu	Δv
GeoCalib	0.03223	0.03353	–	0.01007	0.01707
SVD (Stratified)	∞	0.77760	0.05414	0.75122	1.04580
PHC-HS (Direct)	0.95845	5.17297	1.70423	6.87751	14.25480

Table 10. Calibration summary for `fountain-P11`.

Method	Δf	Δg	Δs	Δu	Δv
GeoCalib	0.14040	0.14185	–	0.01007	0.01707
SVD (Stratified)	∞	1.00000	–	1.00000	1.00000
PHC-HS (Direct)	0.93634	1.07×10^{11}	2.22×10^{10}	6.93×10^{10}	2.53×10^{11}

C.2. Numerical Performances of our PHC-HS (Direct)

We evaluate the numerical accuracy and solution behaviour of our proposed PHC-HS (Direct) solver across various calibration patterns. For each setting, we compute the mean of key performance metrics over 100 randomised trials. Table 11 reports the average number of solutions (`rootCnts`) and relative errors in the recovered intrinsic parameters. High error values (e.g., 10^{18}) indicate cases where the solver either diverges or converges to numerically unstable roots, often due to underconstrained or degenerate configurations.

C.3. Runtime Analysis

We compare the average runtime of the SVD solver and our proposed method, PHC-HS (Direct), using a subset of images from **castle-P19**. While the SVD-based method is extremely fast, averaging *0.0088* seconds per run, it frequently fails under minimal or noisy configurations due to the breakdown of Cholesky decomposition. In contrast, our monoHC-based approach consistently yields reliable and accurate results, with an average runtime of *0.192* seconds. Despite being slower, its robustness makes it well-suited for real-time applications where calibration reliability is critical. Full hardware specifications used in these experiments are provided in Sec. D.1.

C.4. Image Validity Analysis

We report the number of valid images per scene, where an image is considered *valid* if it contains more than enough orthogonal vanishing points to support our calibration solvers. The total number refers to all available images in each dataset. Table 12 summarises the ratio of valid to total images for each benchmark scene. Datasets such as *entry-P10* and *Herz-Jesus-P8* achieve full coverage (100%), indicating consistently strong geometric structure across all images. Others like *fountain-P11* show lower validity (54.55%), likely due to a lack of sufficient orthogonal features or line clutter that hinders vanishing point detection. Overall, the majority of datasets exhibit a high proportion of valid images, confirming the applicability of our method across diverse scenes.

D. Implementations

D.1. Implementation Details

Our experimental pipeline combines symbolic computation, numerical algebraic geometry, and real-image preprocessing. We use `Python` for synthetic scene generation, orthogonal vanishing point preprocessing, and integration with external solvers. Real-world vanishing points are extracted using OpenCV. All algebraic computations—including polynomial system construction, dimension analysis, and ideal manipulation—are performed in `Macaulay2`, while numerical solution of polynomial systems is conducted via homotopy continuation using the `MonodromySolver` library.

All experiments were conducted on a consumer laptop equipped with an Intel(R) Core(TM) Ultra 7 165U processor (2.10 GHz), 16.0 GB RAM, and a 64-bit Windows 11 Pro operating system. No GPU acceleration was used; all symbolic and numerical computations were performed on CPU.

To ensure reproducibility, we fix random seeds for both scene generation and solver routines. All pipeline components—including solver wrappers, symbolic preprocessing, and real-image analysis—will be released publicly upon publication.

D.2. Hyperparameter Tuning Procedure for PHC

To ensure robust and accurate solving of our minimal camera calibration system, we performed an extensive hyperparameter sweep over the configuration space of the polyhedral homotopy continuation (PHC) method implemented via `MonodromySolver` in `Macaulay2`. The purpose of this tuning phase was to identify solver settings that produce numerically stable and geometrically accurate intrinsic parameter estimates under noise.

Configuration Space. We considered a total of 8 internal solver parameters controlling step size adaptation, corrector precision, and divergence detection. Each was assigned a discrete set of values, forming a Cartesian product of 1,944 unique configurations. These include, for example, the initial step size (`tStep`), minimum step threshold (`tStepMin`), and the Newton corrector tolerance (`CorrectorTolerance`).

Parameter	**Candidate Values**
`CorrectorTolerance`	{1e–8, 1e–10, 1e–12}
`EndZoneFactor`	{0.10, 0.05, 0.02}
`InfinityThreshold`	{1e9, 1e10}
`maxCorrSteps`	{5, 10}
`numberBeforeIncrease`	{3, 5}
`stepIncreaseFactor`	{1.25, 1.5, 2.0}
`tStep`	{0.005, 0.01, 0.05}
`tStepMin`	{1e–14, 1e–13, 1e–12}

Table 13. Grid of hyperparameters used for PHC tuning.

Selection Criterion. For each configuration, we ran for 10 random seeds. Each output was compared to the ground truth using the mean aggregate deviation over all intrinsic parameters:

$$\delta_{\text{total}} = \mathbb{E}\left[|\delta f| + |\delta g| + |\delta u| + |\delta v| + |\delta s|\right],$$

Table 11. Mean calibration errors across patterns using PHC-HS (Direct) over 100 random seeds

Pattern	root-counts	Δf	Δg	Δv	Δu	Δs
1g00s	1.73	–	1.2e+19	–	–	1.2e+19
1g0vs	1.86	–	8e+18	8e+18	–	8e+18
1gu0s	0.94	–	6e+18	–	6e+18	6e+18
1guvs	1	–	0.999	0.336	1.8	0.326
ff000	3.96	0.02	1.58e-16	–	–	–
ff00s	3.93	0.04	4.28e-16	–	–	0.00814
ff0v0	3.99	3.1e-16	4.14e-16	1	–	–
ff0vs	3.99	3.82e-16	5.16e-16	1	–	0.163
ffu00	4	3.78e-16	2.31e-16	–	1.2e-15	–
ffu0s	3.94	1e+18	1e+18	–	1e+18	1e+18
ffuv0	4	4.82e-16	5.79e-16	2.31e-15	1.45e-15	–
ffuvs	3.9	1e+18	1e+18	1e+18	1e+18	1e+18
fg000	3.96	1.76e-16	1.57e-16	–	–	–
fg00s	3.94	0.02	0.02	–	–	0.00777
fg0v0	4	5.5e-16	7.8e-16	1	–	–
fg0vs	3.92	0.02	4.56e-16	1	–	0.149
fgu00	4	4.95e-16	2.88e-16	–	1.23e-15	–
fgu0s	3.97	1e+18	1e+18	–	1e+18	1e+18
fguv0	4	5.11e-16	6.11e-16	2.02e-15	1.6e-15	–
fguvs	3.93	6.37e-16	0.02	2.23e-15	1.99e-15	4.28e-05

Table 12. Number of valid images per scene and their corresponding proportions. An image is valid if it contains sufficient orthogonal vanishing points for calibration.

Scene	**Valid / Total**	**Fraction**	**Percentage**
castle-P19	16 / 19	0.8421	84.21%
castle-P30	27 / 30	0.9000	90.00%
entry-P10	10 / 10	1.0000	100.00%
fountain-P11	6 / 11	0.5455	54.55%
Herz-Jesus-P25	21 / 25	0.8400	84.00%
Herz-Jesus-P8	8 / 8	1.0000	100.00%

where each δ represents the metrics used in the main experiments 5.

The optimal hyperparameter set was selected as the configuration that minimised this mean total deviation across all trials. This metric reflects both geometric fidelity and numerical consistency, offering a principled surrogate for physical plausibility in the recovered camera parameters.

Outcome. The selected configuration was used uniformly for all homotopy-based experiments presented in the main paper. This tuning procedure ensures that our solver not only adheres to completeness guarantees but also achieves reliable accuracy under real-world noise conditions.

E. Expressions of Calibration Polynomial Systems

E.1. Direct Approach

fguvs We replaced the coefficients with a constant notation $c \in \mathbb{Q}$ to simplify the equation.

$$\left\{\begin{array}{l} f^2g^2+g^2u^2+f^2v^2-2guvs+v^2s^2-c\,g^2u+c\,f^2v-c\,gus+c\,gvs+c\,vs^2-c\,f^2-c\,g^2+c\,gs-c\,s^2, \\ f^2g^2+g^2u^2+f^2v^2-2guvs+v^2s^2-c\,g^2u-c\,f^2v+c\,gus+c\,gvs-c\,vs^2-c\,f^2+c\,g^2-c\,gs-c\,s^2, \\ f^2g^2+g^2u^2+f^2v^2-2guvs+v^2s^2-c\,g^2u+c\,f^2v-c\,gus+c\,gvs+c\,vs^2-c\,f^2+c\,g^2+c\,gs-c\,s^2, \\ f^2g^2+g^2u^2+f^2v^2-2guvs+v^2s^2-c\,g^2u-c\,f^2v+c\,gus+c\,gvs-c\,vs^2-c\,f^2-c\,g^2+c\,gs-c\,s^2, \\ f^2g^2+g^2u^2+f^2v^2-2guvs+v^2s^2-c\,g^2u-c\,f^2v+c\,gus+c\,gvs-c\,vs^2+c\,f^2-c\,g^2+c\,gs+c\,s^2 \end{array}\right\} \tag{3}$$

fgu00

$$\left\{\begin{array}{l} f^2g^2+g^2u^2-\frac{12\,535}{22}g^2u-\frac{16\,875\,000}{11}f^2-\frac{8\,369\,400}{11}g^2, \\ f^2g^2+g^2u^2-\frac{5\,893\,120}{2\,379}g^2u-\frac{1\,640\,250\,000}{793}f^2+\frac{2\,365\,158\,400}{2\,379}g^2, \\ f^2g^2+g^2u^2-5\,905\,g^2u-375\,000\,f^2+1\,494\,600\,g^2, \\ f^2g^2+g^2u^2-\frac{646\,615}{308}g^2u-\frac{7\,875\,000}{11}f^2-\frac{46\,205\,800}{77}g^2, \\ f^2g^2+g^2u^2-\frac{95\,610}{187}g^2u-\frac{90\,000\,000}{187}f^2-\frac{346\,154\,800}{187}g^2 \end{array}\right\} \tag{4}$$

f1000

$$\left\{\frac{7}{22}f^2-\frac{7\,875\,000}{11}\right\} \tag{5}$$

11uvs

$$\left\{\begin{array}{l} v^2s^2-2uvs-\frac{147\,661}{154}vs^2+u^2+v^2+\frac{147\,661}{154}us+\frac{337\,795}{264}vs+\frac{35\,395\,575}{154}s^2 \\ \quad -\frac{337\,795}{264}u-\frac{147\,661}{154}v-\frac{755\,743\,137}{1\,232}s+\frac{13\,421\,920}{21}, \\ v^2s^2-2uvs-\frac{3\,050\,961}{3\,172}vs^2+u^2+v^2+\frac{3\,050\,961}{3\,172}us+\frac{9\,141\,056}{7\,137}vs+\frac{183\,407\,391}{793}s^2 \\ \quad -\frac{9\,141\,056}{7\,137}u-\frac{3\,050\,961}{3\,172}v-\frac{1\,465\,371\,290}{2\,379}s+\frac{4\,577\,633\,720}{7\,137}, \\ v^2s^2-2uvs-\frac{23\,011}{24}vs^2+u^2+v^2+\frac{23\,011}{24}us+\frac{15\,397}{12}vs+\frac{1\,378\,919}{6}s^2 \\ \quad -\frac{15\,397}{12}u-\frac{23\,011}{24}v-\frac{177\,149\,431}{288}s+\frac{1\,924\,180}{3} \end{array}\right\} \tag{6}$$

1100s

$$\left\{ -\frac{15}{22}s^2 + \frac{169}{176}s \right\} \tag{7}$$

ffuv0

$$\begin{Bmatrix} f^4 + f^2u^2 + f^2v^2 - \frac{12\,535}{22}f^2u + \frac{60\,330}{77}f^2v - \frac{20\,310\,000}{7}f^2, \\ f^4 + f^2u^2 + f^2v^2 - \frac{5\,893\,120}{2\,379}f^2u - \frac{2\,951\,655}{793}f^2v + \frac{1\,146\,670\,000}{2\,379}f^2, \\ f^4 + f^2u^2 + f^2v^2 - 5\,905\,f^2u + \frac{1\,705}{2}f^2v + 480\,000\,f^2 \end{Bmatrix} \tag{8}$$

E.2. Stratified Approach

fguvs

$$\left\{\begin{array}{l}
-\dfrac{8\,369\,400}{11}w_1 - \dfrac{231\,220\,575}{77}w_2 + \dfrac{12\,535}{22}w_3 - \dfrac{164\,824\,200}{77}w_4 - \dfrac{60\,330}{77}w_5 + w_6, \\
\dfrac{2\,365\,158\,400}{2\,379}w_1 + \dfrac{1\,332\,239\,200}{793}w_2 + \dfrac{5\,893\,120}{2\,379}w_3 - \dfrac{406\,162\,800}{793}w_4 + \dfrac{2\,951\,655}{793}w_5 + w_6, \\
1\,494\,600\,w_1 - \dfrac{16\,791\,825}{2}w_2 + 5\,905\,w_3 - 1\,014\,600\,w_4 - \dfrac{1\,705}{2}w_5 + w_6, \\
-\dfrac{46\,205\,800}{77}w_1 - \dfrac{71\,812\,450}{77}w_2 + \dfrac{646\,615}{308}w_3 - \dfrac{3\,840\,600}{11}w_4 + \dfrac{13\,685}{11}w_5 + w_6, \\
-\dfrac{346\,154\,800}{187}w_1 - \dfrac{1\,183\,351\,600}{561}w_2 + \dfrac{95\,610}{187}w_3 + \dfrac{76\,764\,800}{187}w_4 + \dfrac{1\,311\,560}{561}w_5 + w_6, \\
-\dfrac{98\,185\,600}{89}w_1 - \dfrac{190\,238\,400}{89}w_2 + \dfrac{184\,795}{89}w_3 - 489\,600\,w_4 - \dfrac{11\,745}{178}w_5 + w_6
\end{array}\right\} \tag{9}$$

fgu00

$$\left\{\begin{array}{l}
-\dfrac{8\,369\,400}{11}w_1 - \dfrac{252\,279\,375}{77}w_2 - \dfrac{16\,875\,000}{11}w_3 + 1, \\
\dfrac{2\,365\,158\,400}{2\,379}w_1 + \dfrac{389\,340\,000}{793}w_2 - \dfrac{1\,640\,250\,000}{793}w_3 + 1, \\
1\,494\,600\,w_1 - \dfrac{22\,460\,625}{2}w_2 - 375\,000\,w_3 + 1
\end{array}\right\} \tag{10}$$

f1000

$$\left\{\begin{array}{l}
-\dfrac{8\,369\,400}{11}w_1 + \dfrac{12\,535}{22}w_2 - \dfrac{164\,824\,200}{77}w_3 - \dfrac{60\,330}{77}w_4 + w_5, \\
\dfrac{2\,365\,158\,400}{2\,379}w_1 + \dfrac{5\,893\,120}{2\,379}w_2 - \dfrac{406\,162\,800}{793}w_3 + \dfrac{2\,951\,655}{793}w_4 + w_5, \\
1\,494\,600\,w_1 + 5\,905\,w_2 - 1\,014\,600\,w_3 - \dfrac{1\,705}{2}w_4 + w_5, \\
-\dfrac{46\,205\,800}{77}w_1 + \dfrac{646\,615}{308}w_2 - \dfrac{3\,840\,600}{11}w_3 + \dfrac{13\,685}{11}w_4 + w_5, \\
-\dfrac{346\,154\,800}{187}w_1 + \dfrac{95\,610}{187}w_2 + \dfrac{76\,764\,800}{187}w_3 + \dfrac{1\,311\,560}{561}w_4 + w_5
\end{array}\right\} \tag{11}$$

11uvs

$$\left\{\begin{array}{l}
-\dfrac{7\,875\,000}{11}w_1 + \dfrac{7}{22}, \\
-\dfrac{144\,000\,000}{793}w_1 + \dfrac{64}{793}, \\
-1\,875\,000\,w_1 + \dfrac{5}{6}
\end{array}\right\} \tag{12}$$

1100s

$$\left\{\begin{aligned} & -\frac{7}{22}w_1-\frac{169}{176}w_2-\frac{15}{22}w_3+1, \\ & -\frac{64}{793}w_1-\frac{450}{793}w_2-\frac{729}{793}w_3+1, \\ & -\frac{5}{6}w_1-\frac{1\,289}{288}w_2-\frac{1}{6}w_3+1 \end{aligned}\right\} \tag{13}$$

ffuv0

$$\left\{\begin{aligned} & \frac{755\,743\,137}{1\,232}w_1+\frac{337\,795}{264}w_2+\frac{35\,395\,575}{154}w_3+\frac{147\,661}{154}w_4+w_5+\frac{27\,013\,579}{66}, \\ & \frac{1\,465\,371\,290}{2\,379}w_1+\frac{9\,141\,056}{7\,137}w_2+\frac{183\,407\,391}{793}w_3+\frac{3\,050\,961}{3\,172}w_4+w_5+\frac{2\,926\,960\,064}{7\,137}, \\ & \frac{177\,149\,431}{288}w_1+\frac{15\,397}{12}w_2+\frac{1\,378\,919}{6}w_3+\frac{23\,011}{24}w_4+w_5+\frac{823\,145}{2}, \\ & \frac{1\,136\,118\,017}{1\,848}w_1+\frac{1\,577\,633}{1\,232}w_2+\frac{5\,070\,793}{22}w_3+\frac{126\,745}{132}w_4+w_5+\frac{5\,739\,285}{14}, \\ & \frac{60\,859\,307}{99}w_1+\frac{1\,435\,585}{1\,122}w_2+\frac{129\,501\,640}{561}w_3+\frac{1\,617\,226}{1\,683}w_4+w_5+\frac{229\,601\,159}{561} \end{aligned}\right\} \tag{14}$$