

Hessian-Aware Pruning and Optimal Neural Implant

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Abstract

Pruning is an effective method to reduce the memory footprint and FLOPs associated with neural network models. However, existing structured-pruning methods often result in significant accuracy degradation for moderate pruning levels. To address this problem, we introduce a new Hessian Aware Pruning (HAP) method coupled with a Neural Implant approach that uses second-order sensitivity as a metric for structured pruning. The basic idea is to prune insensitive components and to use a Neural Implant for moderately sensitive components, instead of completely pruning them. For the latter approach, the moderately sensitive components are replaced with a low rank implant that is smaller and less computationally expensive than the original component. We use the relative Hessian trace to measure sensitivity, as opposed to the magnitude based sensitivity metric commonly used in the literature. We test HAP for both computer vision tasks and natural language tasks, and we achieve new state-of-the-art results. Specifically, HAP achieves less than 0.1%/0.5% degradation on PreResNet29/ResNet50 (CIFAR-10/ImageNet) with more than 70%/50% of parameters pruned. Meanwhile, HAP also achieves significantly better performance (up to 0.8% with 60% of parameters pruned) as compared to gradient based method for head pruning on transformer-based models. The framework has been open sourced and available online: <https://github.com/yaozhewei/HAP>.

1. Introduction

There has been a significant increase in the computational resources required for Neural Network (NN) training and inference. This is in part due to larger input sizes (e.g., higher image resolution) as well as larger NN models requiring more computation with a significantly larger memory footprint. The slowing down of Moore’s law, along with

challenges associated with increasing memory bandwidth, has made it difficult to deploy these models in practice. Often, the inference time and associated power consumption is orders of magnitude higher than acceptable ranges. This has become a challenge for many applications, e.g., health care and personalized medicine, which have restrictions on uploading data to cloud servers, and which have to rely on local servers with limited resources. Other applications include inference on edge devices such as mobile processors, security cameras, and intelligent traffic control systems, all of which require real-time inference. Importantly, these problems are not limited to edge devices, and state-of-the-art models for applications such as speech recognition, natural language processing, and recommendation systems often cannot be efficiently performed even on high-end servers.

A promising approach to address this is pruning. [3, 7, 14–17, 35, 36, 38, 39, 45, 47, 48, 52, 55, 61, 64, 65, 69, 75, 78, 79], However, an important challenge is determining which parameters are insensitive to the pruning process. A brute-force method is not feasible since one has to test each parameter in the network separately and measure its sensitivity. The seminal work of [36] proposed Optimal Brain Damage (OBD), a second-order based method to determine insensitive parameters. However, this approach requires pruning the parameters one at a time, which is time-consuming. To address this problem, we propose a simple, yet effective, modification of OBD by using the Hessian trace to prune a group of parameters along with a low rank Neural Implant. In more detail, our contributions are as follows:

- We propose HAP, a Hessian Aware Pruning method that uses a fast second-order metric to find insensitive parameters in a NN model. In particular, we use the average Hessian trace to weight the magnitude of the parameters in the NN. Parameters with large second-order sensitivity remain unpruned, and those with relatively small sensitivity are pruned. In contrast to OBD [36], HAP finds groups of insensitive parameters, which is faster than pruning a single parameter at a time. Details of the HAP method are discussed in Section 3.
- We propose a novel Neural Implant (denoted by

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HAP+IMPLANT) technique to alleviate accuracy degradation. In this approach, we replace moderately sensitive model components with a low rank implant. The model along with the implant is then fine-tuned. We find that this approach helps boost the accuracy. For details, see Section 3.2.

- We perform detailed empirical testing and show that HAP achieves 94.3% accuracy ($< 0.1\%$ degradation) on PreResNet29 (CIFAR-10), with only 31% parameters left (Fig. 2). In comparison to EigenDamage, a recent second-order pruning method, we achieve up to 1.2% higher accuracy with fewer parameters and FLOPs (Fig. 2). Moreover, for ResNet50, HAP achieves 75.1% top-1 accuracy (0.5% degradation) on ImageNet, with only half of the parameters left (Tab. 2). In comparison to prior state-of-the-art HRank [40], HAP achieves up to 2% higher accuracy with fewer parameters and FLOPs (Tab. 2). For head pruning of RoBERTa on MRPC/QNLI, HAP achieves up to 0.82%/0.89% higher accuracy than the gradient based method [53] (Tab. 4).
- We perform detailed ablation experiments to illustrate the efficacy of the second-order sensitivity metric. In particular, we compare the second-order sensitivity with a random method, and a reverse-order in which the opposite order sensitivity order given by HAP is used. In all cases, HAP achieves higher accuracy (Tab. 5).

2. Related work

Several different approaches have been proposed to make NN models more efficient by making them more compact, faster, and more energy efficient. These efforts could be generally categorized as follows: (i) efficient NN design [25, 26, 30, 50, 59, 76]; (ii) hardware-aware NN design [5, 8, 13, 27, 43, 49, 60, 66, 70]; (iii) quantization [9, 10, 12, 29, 31, 32, 67]; (iv) distillation [24, 54, 57, 73]; and (v) pruning.

Here we briefly discuss the related work on pruning, which can be broadly categorized into: unstructured pruning [7, 37, 56, 68]; and structured pruning [22, 28, 41, 48, 74, 77]. Unstructured pruning prunes out neurons without any structure. However, this leads to sparse matrix operations which are hard to accelerate and are typically memory-bounded [4, 11]. This can be addressed with structured pruning, where an entire matrix operation (e.g., an output channel) is removed. However, the challenge here is that high degrees of structured pruning often leads to significant accuracy degradation.

In both approaches, the key question is to find which parameters to prune. A simple and popular approach is magnitude-based pruning. In this approach, the magnitude of parameters is used as the pruning metric. The assumption here is that small parameters are not important and can be removed. A variant of this approach was used in [46], where the scaling factor of the batch normalization layer is used as

the sensitivity metric. In particular, channels with smaller scaling factors (or output values) are considered less important and got pruned. Another variation is proposed by [39], where channel-wise summation over weights is used as the metric. Other methods have been proposed as alternative sensitivity metrics. For instance, [40] uses channel rank as sensitivity metric; [23] uses a LASSO regression based channel selection criteria; and [21] uses the geometric median of the convolutional filters. An important problem with magnitude-based pruning methods is that parameters with small magnitudes can actually be quite sensitive. It is easy to see this through a second-order Taylor series expansion, where the perturbation is dependent on not just the weight magnitude but also the Hessian [36]. In particular, small parameters with large Hessian could in fact be very sensitive, as opposed to large parameters with small Hessian (here, we are using small/large Hessian loosely; the exact metric to measure is given by the second-order perturbation in Eq. 3). For this reason, OBD [36] proposes to use the Hessian diagonal as the sensitivity metric. The follow up work of Optimal Brain Surgeon (OBS) [18, 19] used a similar method, but considered off-diagonal Hessian components, and showed a correlation with inverse Hessian. One important challenge with these methods is that pruning has to be performed one parameter at a time. The recent work of [7] extends this to layer-wise pruning in order to reduce the cost of computing Hessian information for one parameter at a time. However, this method can result in unstructured pruning. Another second-order pruning method is EigenDamage [64], where the Gauss-Newton operator is used instead of Hessian. In particular, the authors use Kronecker products to approximate the GN operator. Our findings below show that using the average Hessian trace method significantly outperforms EigenDamage. We also find that it is very helpful to replace moderately sensitive layers with a low rank Neural Implant, instead of completely pruning them, as discussed next.

3. Methodology

Here, we focus on supervised learning tasks, where the nominal goal is to minimize the empirical risk by solving the following optimization problem:

$$L(w) = \frac{1}{N} \sum_{i=1}^N l(x_i, y_i, w), \quad (1)$$

where $w \in \mathbb{R}^n$ is the trainable model parameters, $l(x_i, y_i, w)$ is the loss for the input datum x_i , where y_i is the corresponding label, and N is the training set cardinality. For pruning, we assume that the model is already trained and converged to a local minima which satisfies the first and second-order optimality conditions (that is, the gradient $\nabla_w L(w) = 0$, and the Hessian is Positive Semi-Definite (PSD), $\nabla_w^2 L(w) \succcurlyeq 0$). The problem statement is to prune (remove) as many parameters as possible to reduce the model size and FLOPs to a target threshold with minimal accuracy degradation.

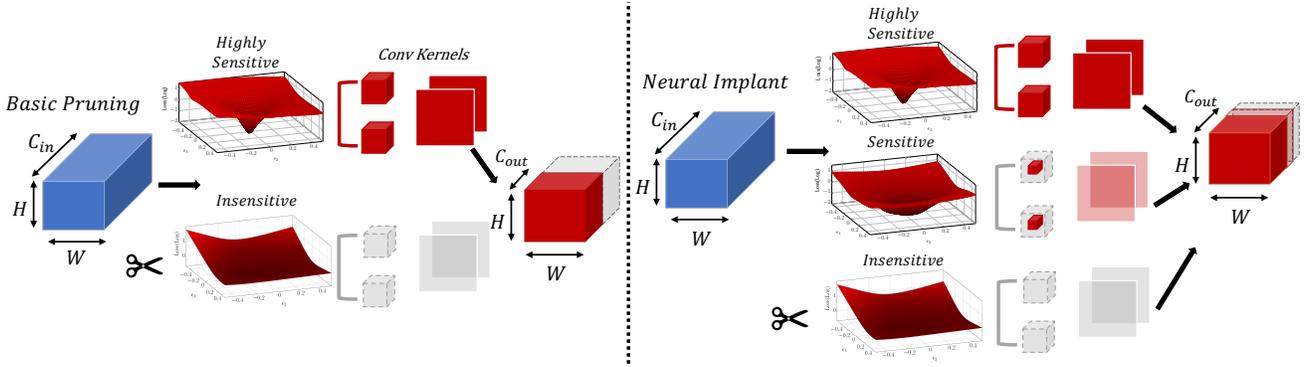


Figure 1: (Left) The HAP method is a structured pruning method that prunes channels based on their second-order sensitivity, which measured flatness/sharpness of loss landscape. Channels are sorted based on this metric, and only insensitive channels are pruned. (Right) Similar to other structured-pruning methods, HAP at large pruning ratios results in accuracy degradation. This because one has to inevitably prune moderately sensitive channels at high pruning ratios, which may contain individual neurons that are very sensitive. The removal of the entire channel, along with the sensitive neurons results in accuracy degradation. To address this, we propose HAP+IMPLANT method, where such channels are replaced with a light-weight, low-rank Neural Implant.

We first start with a general description of the problem and then derive our method. Let $\Delta w \in \mathbb{R}^n$ denote the pruning perturbation such that the corresponding weights become zero (that is $w + \Delta w = 0$). We denote the corresponding change of loss as ΔL :

$$\Delta L = L(w + \Delta w) - L(w) = g^T \Delta w + \frac{1}{2} \Delta w^T H \Delta w + O(\|\Delta w\|^3). \quad (2)$$

where the second equation comes from Taylor series expansion. Here $g \in \mathbb{R}^n$ denotes the gradient of loss function L w.r.t. weights w , and $H \in \mathbb{R}^{n \times n}$ is the corresponding Hessian operator (i.e. second-order derivative). For a pre-trained neural network that has already converged to a local minimum, we have $g = 0$, and the Hessian is a PSD matrix. As in prior work [19], we assume higher-order terms, e.g., $O(\|\Delta w\|^3)$, in Eq. 2 can be ignored.

The pruning problem is to find the set of weights that result in minimum perturbation to the loss (ΔL). This leads to the following constrained optimization problem:

$$\begin{aligned} \min_{\Delta w} \frac{1}{2} \Delta w^T H \Delta w &= \frac{1}{2} \begin{pmatrix} \Delta w_p \\ \Delta w_l \end{pmatrix}^T \begin{pmatrix} H_{p,p} & H_{p,l} \\ H_{l,p} & H_{l,l} \end{pmatrix} \begin{pmatrix} \Delta w_p \\ \Delta w_l \end{pmatrix}, \\ \text{s.t. } \Delta w_p + w_p &= 0. \end{aligned} \quad (3)$$

Here, we denote the channels that are pruned with p as the subscript (e.g. $w_p \in \mathbb{R}^p$), and denote the remaining parameters with l as the subscript (e.g. $w_l \in \mathbb{R}^{n-p}$). Similarly, we use Δw_p and Δw_l to denote the corresponding perturbations. Note that $\Delta w_p = -w_p$ since p -channels are pruned. Moreover, $H_{l,p}$ denotes the cross Hessian w.r.t. l -channels and p -channels (and similarly $H_{p,p}$ and $H_{l,l}$ are Hessian w.r.t. pruned and unpruned parameters). Using Lagrangian method, we can finally get (see Appendix A for more de-

tails):

$$\frac{1}{2} \Delta w^T H \Delta w = \frac{1}{2} w_p^T (H_{p,p} - H_{p,l} H_{l,l}^{-1} H_{l,p}) w_p. \quad (4)$$

Eq. 4 gives us the perturbation to the loss when a set of parameters w_p is removed. It should be noted that OBS [19] and L-OBS [7], where OBS is applied for each layer under the assumption of cross-layer independence, is a degenerate case of Eq. 4 for the special case of $w_p \in \mathbb{R}^1$. Next we discuss how this general formulation can be simplified.

3.1. Hessian-aware Pruning

There are three major disadvantages with OBS. First, computing Eq. 4 requires computing (implicitly) information from the inverse Hessian, $H_{l,l}^{-1}$. This can be costly, both in terms of computations and memory (even when using matrix-free randomized methods). The work of L-OBS [7] attempted to address this challenge by ignoring cross-layer dependencies, but it still requires computing block-diagonal inverse Hessian information, which can be costly. Second, in both OBS and L-OBS, one has to measure this perturbation for all the parameters separately, and then prune those parameters that result in the smallest perturbation. This can have a high computational cost, especially for deep models with many parameters. Third, this pruning method results in unstructured pruning, which is difficult to accelerate with current hardware architectures.

In the OBD [36] method the first problem does not exist as the the Hessian is approximated as a diagonal operator, without the need to compute inverse Hessian:

$$\frac{1}{2} \Delta w^T H \Delta w \approx \frac{1}{2} w_p^T \text{Diag}(H_{p,p}) w_p. \quad (5)$$

Here $Diag(H_{p,p})$ denotes the diagonal elements of $H_{p,p}$. However, the second and third of these disadvantages still remain with OBD.

To address the second and third of these disadvantages, we propose to group the parameters and to compute the corresponding perturbation when that group is pruned, rather than computing the perturbation for every single parameter separately. Note that this can also address the third disadvantage, since pruning a group of parameters (for example parameters in a convolution channel) results in structured pruning. This can be achieved by considering the Hessian as a block diagonal operator, and then approximating each block with a diagonal operator, with Hessian trace as the diagonal entries. In particular, we use the following approximation:

$$\begin{aligned} \frac{1}{2} \Delta w^T H \Delta w &= \frac{1}{2} w_p^T [H^{-1}]_{p,p}^{-1} w_p \\ &\approx \frac{1}{2} w_p^T \frac{Trace(H_{p,p})}{p} w_p = \frac{Trace(H_{p,p})}{2p} \|w_p\|_2^2, \end{aligned} \quad (6)$$

where $Trace(H_{p,p})$ denotes the trace of the block diagonal Hessian (the corresponding Hessian block for pruned parameters $H_{p,p}$). The Hessian can be computed very efficiently with randomized numerical linear algebra methods, in particular Hutchinson’s method [1, 2, 71, 72]. Importantly, this approach requires computing only the application of the Hessian to a random input vector. This has the same cost as back-propagating the gradient [71, 72]. (Empirically, in our experiments corresponding to ResNet50 on ImageNet, the longest time for computing this trace was three minutes.) A similar approach was proposed by [9] in the context of quantization.

In more detail, HAP performs structured pruning by grouping the parameters and approximating the corresponding Hessian as a diagonal operator, with the average Hessian trace of that group as its entries. For a convolutional network, this group can be an output channel. We found that this simple modification results in a fast and efficient pruning method that when combined with the Neural Implant approach exceeds state-of-the-art. This is discussed next.

3.2. Hessian-aware Neural Implant

In HAP, we sort the channels from most sensitive to least sensitive (based on Eq. 6). For a target model size or FLOPs budget, one has then to prune by starting from insensitive channels. This approach works well, as long as all these channels are extremely insensitive. However, in practice, some of the sorted channels will exhibit some level of sensitivity. Entirely pruning these channels, and leaving the rest of the sensitive ones unpruned, can result in significant accuracy degradation. This is one of the major problems with structured pruning methods, as very few groups of parameters are completely insensitive. When those are pruned, the remaining groups/set of parameters always include some

subset of highly sensitive neurons that if pruned, would result in high accuracy loss.

Here, we propose an alternative strategy to replace moderately sensitive parameter groups with a low rank *Neural Implant*. The basic idea is to prune insensitive layers completely, but detect the moderately sensitive layers, and instead of completely removing all of its parameters (which can contain some sensitive ones as discuss above), replace them with a low rank decomposition. As an example, a spatial convolution could be replaced with a new point-wise convolution that has smaller parameters and flops. One could also consider other types of low rank decomposition (e.g. CP/Tucker decomposition, depth-wise/separable convolution, etc). However, for simplicity we only use a pointwise convolution implant in this paper.

After the implant, the model is fine-tuned to recover accuracy. We denote this approach as HAP+IMPLANT, which is schematically illustrated in Fig. 1. In summary, we use the Hessian metric in Eq. 6, and then we apply a Neural Implant to the most sensitive channels to be pruned.

We have to emphasize that many prior works have investigated low-rank matrix approximation [51]. However, existing methods for NN pruning replace all or part of the model, irrespective of their sensitivity, whereas in our approach we perform a targeted low-rank approximation, and only replace the sensitive parts of the model, quantified through the Hessian in Eq. 6. We empirically found that this approach is quite effective, especially for moderate pruning ratios, as discussed next.

4. Results

4.1. Experimental Settings

Computer Vision. For evaluating the performance of HAP, we conduct experiments for image classification on CIFAR-10 (ResNet56/WideResNet32/PreResNet29/VGG16) and ImageNet (ResNet50). Our main target comparison for HAP (without Implant) is EigenDamage, a recent second-order pruning method. For fair comparison, we use the same pretrained model used by EigenDamage when available (WideResNet32 on Cifar-10), and otherwise train the model from scratch (ResNet56, VGG16, and PreResNet29 on Cifar-10). For all cases, we ensure comparable baseline accuracy, and when not possible, we report the baseline used by other methods. For comparison we consider a wide range of pruning ratios, and consider validation accuracy, FLOPs, and parameter size as the metrics. The goal is to achieve higher accuracy with lower FLOPs/parameter size.

Natural Language Understanding. We use RoBERTa-base [44], which consists of 12 attention heads for each of 12 Transformer encoder layers, as the baseline model. It has been explored in [53] that not all heads in Transformer

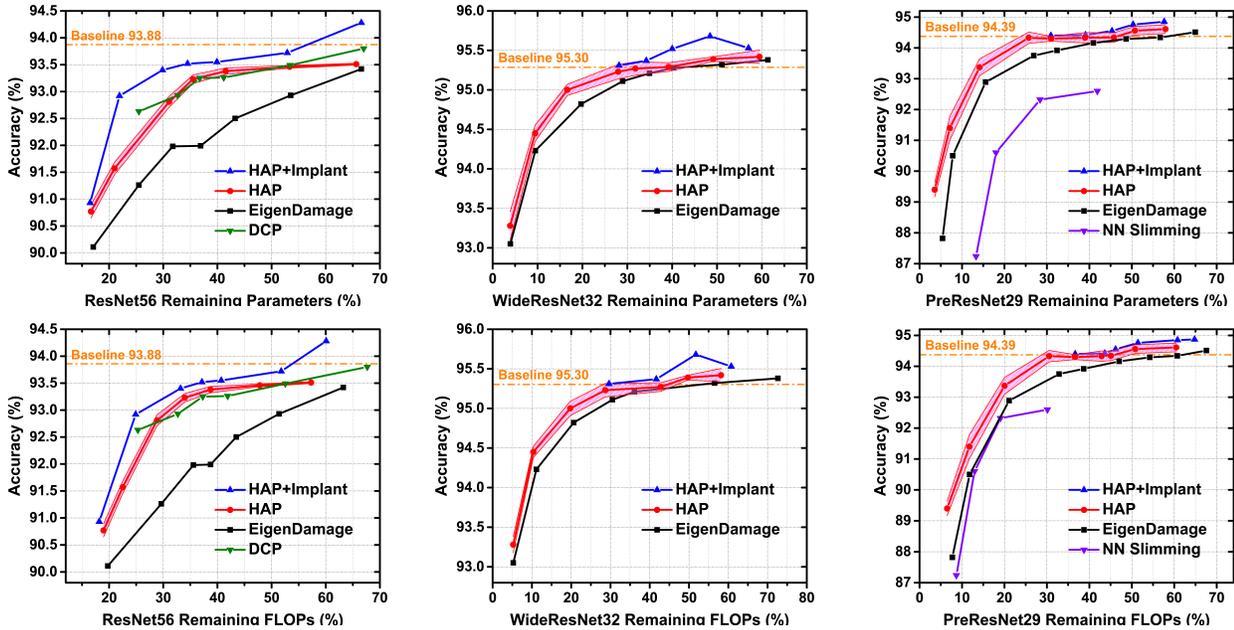


Figure 2: Comparison of accuracy with different pruning ratios among HAP+IMPLANT, HAP, NN Slimming, EigenDamage, and DCP, on the CIFAR-10 dataset, for ResNet56, WideResNet32, and PreResNet29. (Top) Remaining parameters in the network after pruning is used for x-axis. (Bottom) Remaining FLOPs in the network after pruning is used for x-axis. HAP consistently outperforms EigenDamage and NN Slimming, and HAP+IMPLANT boosts performance for moderate pruning ratios and surpasses DCP.

architectures are equally important and thus a great portion of them can be removed without degrading the accuracy.

Table 1: Comparison between HAP and other pruning methods on CIFAR-10. Here, VGG16 denotes the baseline used in HRank [40], and VGG16-HAP denotes the baseline used by HAP method. As one can see, HAP consistently outperforms other pruning methods, even though its pruned models have fewer parameters (Param.) and FLOPs.

| Method | Acc.(%) | Param.(%) | FLOPs(%) |
|--------------|--------------|-------------|-------------|
| VGG16 | 93.96 | 100.0 | 100.0 |
| VGG16-HAP | 93.88 | 100.0 | 100.0 |
| L1[39] | 93.40 | 36.0 | 65.7 |
| SSS[28] | 93.02 | 26.2 | 58.4 |
| VarP[77] | 93.18 | 26.7 | 60.9 |
| HRank[40] | 93.43 | 17.1 | 46.5 |
| GAL-0.05[42] | 92.03 | 22.4 | 60.4 |
| HRank[40] | 92.34 | 17.9 | 34.7 |
| GAL-0.1[42] | 90.73 | 17.8 | 54.8 |
| HAP | 93.66 | 10.1 | 29.7 |
| HRank[40] | 91.23 | 8.0 | 23.5 |
| HAP | 93.37 | 5.1 | 20.3 |
| HAP | 91.22 | 1.6 | 7.5 |

4.2. HAP Results on CIFAR-10

We first start with evaluating HAP without Neural Implant, and then discuss the specific improvement of using Neural Implant. The results on CIFAR-10 for different pruning ratios and various models are presented in Fig. 2. In particular, we report both the validation accuracy versus remaining parameters after pruning, as well as validation accuracy versus the FLOPs. For comparison, we also plot the performance of NN slimming [46], EigenDamage [64] and DCP [79] for different pruning ratios. For all the points that we compare, HAP achieves higher accuracy than EigenDamage, even for cases with fewer parameters/FLOPs. We generally observe that the difference between HAP and EigenDamage is more noticeable for higher pruning ratios (i.e., fewer remaining parameter). This is expected, since small amounts of pruning does not lead to significant accuracy degradation, while higher pruning ratios are more challenging. In particular, when the parameter remaining percentage is around 35% (i.e., 65% of the parameters are pruned), HAP achieves 93.2% accuracy, which is 1.24% higher than EigenDamage, with fewer FLOPs (34.0% versus 38.7% for EigenDamage). We observe a similar trend on WideResNet32, where HAP consistently outperforms EigenDamage.

We also plot the performance of DCP, which is not a second-order method, but is known to achieve good pruning

accuracy. HAP achieves higher accuracy as compared to NN slimming, and comparable accuracy to DCP. As for the latter, the benefit of HAP is that we do not need to perform any greedy channel selection and the entire Hessian calculation and channel selection is performed in one pass.¹

For PreResNet29, we also compare with NN Slimming [46], to compare with prior reported results on this model. We observe that HAP achieves up to 6% higher accuracy as compared to NN Slimming method, and slightly higher accuracy as compared to EigenDamage. It is interesting to note that HAP can keep the accuracy the same as baseline, up to pruning 70% of the parameters (corresponding to 30% remaining parameters in Fig. 2).

We also present results on VGG16 and compare with other works in the literature, including GAL [42], HRank [40], and VarP [77], as reported in Table 1. Here, we consistently achieve higher accuracy. In particular, HAP with 29.7% FLOPs and 10.1% parameters achieves the highest accuracy (despite using a pretrained model with lower baseline accuracy). Similarly, HAP with 20.3% FLOPs and 5.1% parameters achieves 93.37% accuracy, with less than 2× FLOPs and 3× fewer parameters as compared to HRank in the same block. For extreme pruning, HAP achieves 91.22% accuracy with only 1.6% of the parameters remaining. To the best of our knowledge, this level of aggressive pruning, while maintaining such high accuracy, has not been reported in the literature.

It is interesting to visualize how HAP performs channel selection using the second-order sensitivity discussed in Sec. 3.1, and the result can be found in Appendix E

4.3. Neural Implant Results on CIFAR-10

Despite HAP’s competitive results as compared to prior pruning methods, it still has lower accuracy as compared to baseline. This is known problem and shortcoming of structured pruning methods. We propose to use a low rank Neural Implant to address this problem, and find it particularly helpful for moderate levels of structured pruning. In particular, for the CNNs tested in this paper we replace sensitive 3×3 spatial convolutions with a pointwise convolution. This replacement still reduces the number of parameters for the 3×3 convolution by a factor of $9 \times$.

We repeated the previous experiments with this approach, and report the results in Fig. 2 (blue line). We observe that HAP+IMPLANT consistently achieves better performance than HAP for both the same parameter size (first row) and the same FLOPs (second row), which also surpasses the performance of DCP [79] that has a competitive result with HAP.

¹We also tried to test DCP on other models but the code base is old and we were not able to use it for WideResNet32 or PreResNet29. As such we considered other pruning methods, besides EigenDamage, for comparison with those models.

Table 2: Comparison of FLOPs and accuracy on CIFAR-10 using ResNet56 for different pruning methods. We report the baseline accuracy used in each work, as well as the corresponding final accuracy after pruning. For ease of comparison, we also report the accuracy drop (Acc. ↓) w.r.t. each baseline. As one can see, HAP and HAP+IMPLANT consistently outperform other work reported in the literature.

| Method | Base-acc. | Final-acc. | Acc. ↓ | FLOPs (%) |
|--------------------|--------------|--------------|-------------|-------------|
| CP[23] | 92.80 | 91.80 | 1.00 | 50.0 |
| AMC[22] | 92.80 | 91.90 | 0.90 | 50.0 |
| FPGM[21] | 93.59 | 93.26 | 0.33 | 47.4 |
| LFPC[20] | 93.59 | 93.24 | 0.35 | 47.1 |
| HAP+IMPLANT | 93.88 | 93.55 | 0.33 | 40.7 |
| GAL-0.8[42] | 93.26 | 90.36 | 2.90 | 39.8 |
| HRank[40] | 93.26 | 90.72 | 2.54 | 25.9 |
| HAP | 93.88 | 91.57 | 2.31 | 21.0 |
| HAP+IMPLANT | 93.88 | 92.92 | 0.96 | 23.9 |

For some cases, the performance of the pruned network slightly exceeds the baseline accuracy. In particular, for ResNet56, we observe up to 1.5% higher accuracy as compared to HAP, and up to 2% higher accuracy as compared to EigenDamage. We observe a similar trend for both WideResNet32/PreResNet29, where HAP+IMPLANT consistently performs better than both HAP and EigenDamage.

It should be noted that the gains from HAP+IMPLANT diminish for higher pruning ratios (around 20% remaining parameters for ResNet56, and around 30% remaining for WideResNet32/PreResNet29). This is expected, since there is a trade-off associated with adding the Neural Implant. While the implant helps reduce the information loss from completely removing sensitive channels, it does so by adding additional parameters. As such, we actually have to enforce a larger pruning ratio to meet a target model size. As the channels are sorted based on their sensitivity (from Eq. 6), this means that we have to prune the next set of more sensitive channels to satisfy the target. However, if such channels have much higher sensitivity, then that can actually degrade the performance. This is what happens for extreme pruning cases, since most of the remaining parameters will be highly sensitive; and, as such, the gains achieved by the Neural Implant will not be enough.

In addition to parameter percentage, we also compare HAP+IMPLANT results based on remaining FLOPs with other methods reported in the literature. This is shown in Tab. 2. As one can see, with a high remaining FLOPs percentage, HAP+IMPLANT can reach 93.55% accuracy with only 0.33% degradation as compared with the corresponding pretrained baseline model. It should be noted that state-of-the-art methods such as FPGM [21] and LFPC [20] requires

Table 3: Comparison between HAP and HAP+IMPLANT, and other state-of-the-art pruning methods on ImageNet. Here, ResNet50 is the baseline used in HRank [40]’s table, while ResNet50-HAP is the baseline used by HAP.

| Method | Top-1 | Param.(%) | FLOPs(%) |
|-------------------|--------------|--------------|--------------|
| ResNet50-MetaP | 76.16 | 100.0 | 100.0 |
| ResNet50 | 76.15 | 100.0 | 100.0 |
| ResNet50-HAP | 75.62 | 100.0 | 100.0 |
| SSS-32[28] | 74.18 | 72.94 | 68.95 |
| CP[23] | 72.30 | - | 66.75 |
| GAL-0.5[42] | 71.95 | 83.14 | 56.97 |
| MetaP[45] | 72.27 | 61.35 | 56.36 |
| HRank[40] | 74.98 | 63.33 | 56.23 |
| HAP | 75.12 | 55.41 | 66.18 |
| HAP+IMPLANT | 75.36 | 53.74 | 55.49 |
| GDP-0.6[41] | 71.19 | - | 45.97 |
| GDP-0.5[41] | 69.58 | - | 38.39 |
| SSS-26[28] | 71.82 | 61.18 | 56.97 |
| GAL-1[42] | 69.88 | 57.53 | 38.63 |
| GAL-0.5-joint[42] | 71.82 | 75.73 | 44.99 |
| HRank[40] | 71.98 | 54.00 | 37.90 |
| HAP | 74.00 | 34.74 | 40.44 |
| ThiNet-50[48] | 68.42 | 33.96 | 26.89 |
| GAL-1-joint[42] | 69.31 | 40.04 | 27.14 |
| HRank[40] | 69.10 | 32.43 | 23.96 |
| MetaP[45] | 70.07 | 41.44 | 25.69 |
| HAP | 71.18 | 20.47 | 32.85 |

6.4% more FLOPs to reach comparable performance. Moreover, when the target percentage of remaining FLOPs is small, HAP+IMPLANT only incurs 0.96% accuracy degradation as compared with 2.31% for HAP and 2.54% for HRank (with comparable FLOPs and baseline accuracy).

4.4. HAP Results on ImageNet

We also test HAP on ImageNet using ResNet50, and report the results in Tab. 3. We compare with several previous structured pruning methods including SSS [28], CP [23], ThiNet [48], and HRank [40]. It should be noted that the accuracy of our pretrained baseline is slightly lower than HRank, yet our HAP method still achieves higher accuracy. For instance, in all cases, HAP achieves higher accuracy with smaller number of parameters as compared to all prior work reported on ResNet50. The highest difference corresponds to 34.74% remaining parameters (i.e., pruning 65.26% of parameters), where HAP has 2% higher Top-1 accuracy with 19.26% fewer parameters as compared to HRank (although for fairness our FLOPs are slightly larger). We

observe a consistent trend even for high pruning ratios. For example, with 20.47% remaining parameters, HAP still has more than 2% higher accuracy as compared to HRank. We should also note that despite using second-order information, HAP is quite efficient, and the end-to-end Hessian calculations were completed three minutes on a single RTX-6000 GPU.

Table 4: Comparison of HAP and the gradient-based [53] heads pruning methods for RoBERTa-base, evaluated on MRPC and QNLI. For evaluation metric, we report accuracy for QNLI and the average of accuracy and F1 score for MRPC.

| (a) MRPC | | | | | |
|---------------------|-------|--------------|--------------|--------------|--------------|
| Parameter (%) | 100 | 80 | 60 | 50 | 40 |
| Gradient-based [53] | 92.06 | 92.19 | 89.52 | 89.36 | 89.07 |
| HAP | 92.06 | 91.97 | 90.74 | 90.43 | 89.89 |
| Diff | 0.00 | -0.22 | +1.22 | +1.07 | +0.82 |
| (b) QNLI | | | | | |
| Parameter (%) | 100 | 80 | 60 | 50 | 40 |
| Gradient-based [53] | 93.12 | 92.4 | 91.78 | 91.71 | 91.38 |
| HAP | 93.12 | 93.30 | 92.93 | 92.59 | 92.27 |
| Diff | 0.00 | +0.90 | +1.15 | +0.88 | +0.89 |

4.5. HAP Results on RoBERTa

Additionally, we show in this section that our HAP method can be applied to Transformer [62] architectures to prune unnecessary attention heads from multi-head attention layers. The results are reported as Tab. 4, where we test over different heads prune ratio. As shown in the table, HAP outperforms the gradient-based method with 40~60% prune ratio for MRPC and with all prune ratio for QNLI by a noticeable margin of ~1 point. The results indicate that Hessian could be more informative than gradient in determining sensitivities of attention heads.

4.6. Ablation Study

We conducted different ablation experiments to study the effectiveness of the second-order based metric in HAP. For all the experiments, we use ResNet56 on CIFAR-10.

One of the main components of HAP is the Hessian trace metric used to sort different channels to be pruned. In particular, this ordering sorts the channels from the least sensitive to most sensitive, computed based on Eq. 6. In the first ablation study, we use the reverse order of what HAP recommends, and denote this method as R-HAP. The results are shown in Tab. 5. It can be clearly observed that for all cases R-HAP achieves lower accuracy as compared to HAP (more than 3% for the case with 35.50% remaining parameters).

Table 5: Ablation study on the sensitivity metric. R-HAP denotes pruning by reversely using sensitivity in HAP. Random is conducted by randomly allocating channel-wise sensitivity.

| Method | Acc. | Param.(%) | FLOPs(%) | Channel |
|------------|--------------|--------------|--------------|-----------|
| R-HAP | 89.77 | 47.48 | 46.98 | 65 |
| Random | 93.12 | 45.60 | 46.68 | 60 |
| Magnitude | 93.29 | 61.82 | 39.29 | 55 |
| HAP | 93.38 | 41.53 | 38.74 | 50 |
| R-HAP | 89.97 | 42.85 | 39.99 | 60 |
| Random | 92.21 | 36.01 | 33.99 | 50 |
| Magnitude | 92.99 | 56.15 | 34.97 | 50 |
| HAP | 93.23 | 35.50 | 33.99 | 45 |
| R-HAP | 88.83 | 27.15 | 30.61 | 50 |
| Random | 90.95 | 29.64 | 31.32 | 40 |
| Magnitude | 92.45 | 47.28 | 28.97 | 42.2 |
| HAP | 92.81 | 31.08 | 28.85 | 40 |
| R-HAP | 88.18 | 23.34 | 28.04 | 45 |
| Random | 90.18 | 25.33 | 25.69 | 30 |
| Magnitude | 91.65 | 38.25 | 22.93 | 35 |
| HAP | 92.06 | 22.05 | 22.86 | 31 |

Table 6: Hessian Aware Low Rank (Neural Implant) vs Low Rank. Ablation study on Hessian aware Neural Implant. The R-HAP+IMPLANT uses low rank replacement without using Hessian information. Specially, the Neural Implant is applied on channels that are most sensitive.

| ResNet56/Cifar10 | Acc. | Param.(%) | FLOPs(%) |
|--------------------|--------------|--------------|--------------|
| Low Rank | 90.79 | 39.56 | 40.33 |
| HAP+IMPLANT | 93.52 | 34.49 | 37.15 |
| Low Rank | 90.34 | 34.39 | 37.24 |
| HAP+IMPLANT | 93.40 | 29.87 | 33.20 |
| Low Rank | 89.83 | 23.05 | 25.16 |
| HAP+IMPLANT | 92.92 | 21.92 | 24.90 |

In the second ablation experiment, we use a random order for pruning the layers, irrespective of their second-order sensitivity, and denote this as Random in Tab. 5. Similar to the previous case, the random ordering achieves consistently lower accuracy as compared to HAP. In addition, its results exhibit a larger variance.

Another important ablation study is to compare the performance of the Hessian-based pruning with the commonly used magnitude based methods that use variants of $\|w\|_2^2/p$ (denoted as Magnitude in Tab. 5). To make a fair comparison, we set the FLOPs of the model after pruning to be the same for HAP and the magnitude based pruning (and slightly higher for the latter to be fair). The results are re-

ported in Tab. 5. As the results show, HAP achieves the same accuracy as magnitude based pruning but with much fewer parameters (i.e., higher pruning ratio). In particular, for pruning with 22.53% of FLOPs (last row of Tab. 5), HAP achieves 92.06% which is almost the same as magnitude based pruning (91.65%). However, HAP achieves this accuracy with only 21.01% of the parameters remaining, as compared to 38.25%, which is quite a significant difference. This is expected, as HAP’s performance was higher than the different magnitude based results reported in the literature, for both the CIFAR-10 and ImageNet tests of the previous subsections (Sec. 4.2 and 4.4, respectively).

To study the role played by Hessian analysis in Neural Implant, we’ve run another set of experiment to prove that Hessian-aware Neural Implant is a combination of sensitivity analysis and Low Rank approximation. The way of using Hessian to guide where Low Rank approximation happens is way more effective than directly applying it to the original model. The key idea is to only apply low rank for sensitive layers, as measured by Hessian trace. We compare Neural Implant with Low Rank in Tab. 6, which clearly shows that we can get up to 2% higher accuracy with lower Params/FLOPs.

5. Conclusion

Existing structured-pruning methods often result in significant accuracy degradation for moderate pruning levels. To address this, we propose HAP, a new second-order structured-pruning method that uses the Hessian trace as the sensitivity metric for pruning a NN model. We also proposed a new Neural Implant approach that uses HAP’s sensitivity metric to perform targeted replacement of sensitive neuron’s with a light-weight low rank implant. The main intuition is to prune insensitive components and to use the Neural Implant for moderately sensitive components., instead of completely pruning them. We performed extensive empirical tests using multiple NN models. We compared with several prior works, including both the second-order based structured pruning method of EigenDamage, as well as several magnitude-based pruning methods. HAP consistently achieved higher accuracy with fewer parameters. Specifically, HAP achieves 94.3% accuracy ($< 0.1\%$ degradation) on PreResNet29 (CIFAR-10), with more than 70% of parameters pruned. In comparison to EigenDamage, we achieve up to 1.2% higher accuracy with fewer parameters and FLOPs. Moreover, for ResNet50 HAP achieves 75.1% top-1 accuracy (0.5% degradation) on ImageNet, after pruning almost half of the parameters. In comparison to the prior state-of-the-art of HRank, we achieve up to 2% higher accuracy with fewer parameters and FLOPs. For head-pruning of RoBERTa, HAP can achieve more than 0.8% better performance as compared to previous gradient based method with 60% pruned heads.

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