

Supplementary material for the paper: A Riemannian Framework for Analysis of 3D Human Body

1. SMPL Template

The Figure 1 shows the used SMPL [3] template \mathcal{T} .

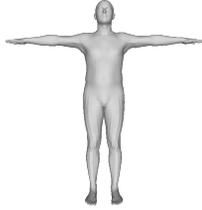


Figure 1: SMPL template

2. Examples from FAUST dataset

The Figure 2 shows some examples of human shapes from FAUST dataset [1]. One can see that there is a significant variability in shape and pose.

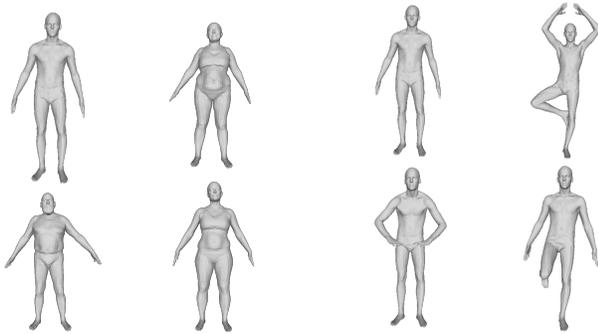


Figure 2: Human shapes from the FAUST dataset.

3. Shape Space as Section of a Fiber Bundle

In this paper, each human body surface is aligned to a given human template (SMPL template). As illustrated in Figure 3, the geometric features of the template are aligned

with geometric feature of the human surface (for instance, the finger tips of the template correspond to the finger tips of the other humans bodies).

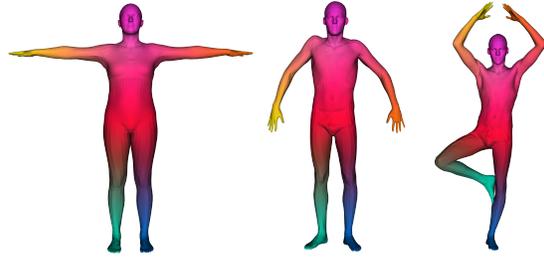


Figure 3: Alignment with the template: The 3 meshes in different poses are displayed with different color on extremities. This validate the choice to work on this particular section of the fiber bundle $\Pi : \mathcal{H} \rightarrow \mathcal{H}/G$

Mathematically the choice of a preferred alignment with the template is called a section \mathcal{S}_0 of the fiber bundle $\Pi : \mathcal{H} \rightarrow \mathcal{H}/G$. A section of Π is a (smooth) map assigning to each equivalence class $[f_0] \in \mathcal{H}/G$ a representative $f_0 \in \mathcal{H}$ in this class, i.e. such that $\Pi(f_0) = [f_0]$. This notion is illustrated in Figure 4. The section we are using, i.e. the correspondence with the template, is smooth, thanks to the geometric alignment as explained above. An illustration of the section is displayed in Figure 4.

In this paper, we pull-back the Riemannian metrics that are defined on shape space (see in section 4) on the preferred section \mathcal{S}_0 given by the correspondence with the template.

4. Numerical Computation of the First Fundamental Form g

The computation of the first fundamental g is given by:

$$g = \left(\begin{array}{cc} \left\langle \frac{\partial f}{\partial u}, \frac{\partial f}{\partial u} \right\rangle & \left\langle \frac{\partial f}{\partial u}, \frac{\partial f}{\partial v} \right\rangle \\ \left\langle \frac{\partial f}{\partial u}, \frac{\partial f}{\partial v} \right\rangle & \left\langle \frac{\partial f}{\partial v}, \frac{\partial f}{\partial v} \right\rangle \end{array} \right)$$

It is always computed relatively to a given parameterization. Here the parameterization is given relatively to the template.

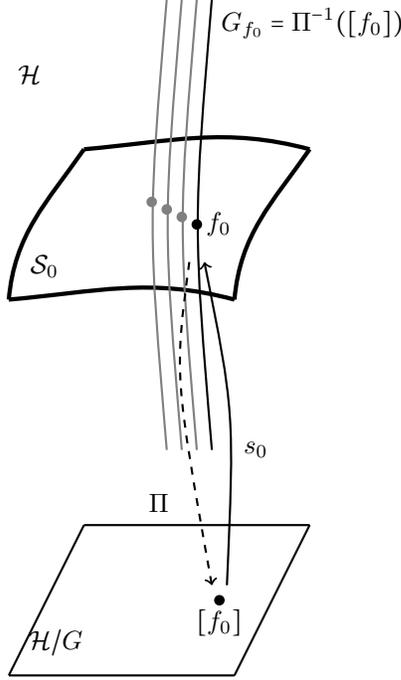


Figure 4: Section of the fiber bundle $\Pi : \mathcal{H} \rightarrow \mathcal{H}/G$: the one-to-one correspondance with the template mesh allows us to work on the corresponding section \mathcal{S}_0 as a shape space. The correspondance initially gives the section for $\text{Diff}^+(\mathcal{T})$, but with Procrustes analysis, the section for $\text{SO}(3)$ comes straightforwardly.

The first step is to describe a canonical frame for each template triangle t_i^T : Let p_1, p_2, p_3 be the 3 corners of the triangle. We set the origin of the frame to 0. We design the local coordinates by (u, v) in the plane denoted by the triangle, that we define with : the coordinates of the triangle corners being $(0, 0), (u_2, 0), (u_3, v_3)$.

Now we need to find the f that maps the triangle t_i in the template to the triangle t_i^m in the destination mesh. Let q_1, q_2, q_3 be the 3 corners of the triangle.

In a given (u, v) of t_i , the parameterization is of the form:

$$f(u, v) = \lambda_1 q_1 + \lambda_2 q_2 + \lambda_3 q_3$$

In order to compute the derivatives, we need to derive f . The derivative is given by:

$$\frac{\partial f(u, v)}{\partial u} = \frac{\partial \lambda_1}{\partial u} q_1 + \frac{\partial \lambda_2}{\partial u} q_2 + \frac{\partial \lambda_3}{\partial u} q_3$$

$$\frac{\partial f(u, v)}{\partial v} = \frac{\partial \lambda_1}{\partial v} q_1 + \frac{\partial \lambda_2}{\partial v} q_2 + \frac{\partial \lambda_3}{\partial v} q_3$$

Since the triangle are in correspondance, only the λ_i – the barycentric coordinates – depend on (u, v) . Given (u, v) ,

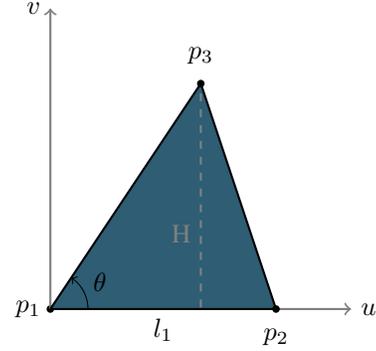


Figure 5: Ordered triangle with the corresponding local reference frame. The quantities l_1, H, θ are useful in the computation of the derivatives.

those λ_i are defined as the solution of the following equation (see [2] for similar calculations):

$$\begin{pmatrix} 0 & u_2 & u_3 \\ 0 & 0 & v_3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

The solution is straightforward and given by:

$$\frac{\partial f(u, v)}{\partial u} = \frac{1}{u_2} (q_2 - q_1)$$

$$\frac{\partial f(u, v)}{\partial v} = \frac{u_3}{v_3 u_2} (q_1 - q_2) + \frac{1}{v_3} (q_3 - q_1)$$

The values of u_2 is simply the length of the first edge of the triangle (l_1). u_3 and v_3 are the projection of first and second edge on the u -axis and v -axis. So v_3 is the height H of the triangle (relative to first edge as the basis), and $\frac{u_3}{v_3}$ is $\tan(\theta)$, where θ is the angle between first edge and second edge:

$$\frac{\partial f(u, v)}{\partial u} = \frac{1}{l_1} (q_2 - q_1)$$

$$\frac{\partial f(u, v)}{\partial v} = \frac{\tan \theta}{l_1} (q_1 - q_2) + \frac{1}{H} (q_3 - q_1)$$

References

- [1] Federica Bogo, Javier Romero, Matthew Loper, and Michael J. Black. FAUST: Dataset and evaluation for 3D mesh registration. In *Proceedings IEEE Conf. on Computer Vision and Pattern Recognition (CVPR)*, Piscataway, NJ, USA, June 2014. IEEE. 1
- [2] Bruno Lévy. *Géométrie Numérique*. Habilitation à diriger des recherches, Institut National Polytechnique de Lorraine - INPL, Feb. 2008. 2
- [3] Matthew Loper, Naureen Mahmood, Javier Romero, Gerard Pons-Moll, and Michael J. Black. SMPL: A skinned multi-person linear model. *ACM Trans. Graphics (Proc. SIG-GRAPH Asia)*, 34(6):248:1–248:16, Oct. 2015. 1