Supplimentary Material: No Reference Opinion Unaware Quality Assessment of Authentically Distorted Images

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A. Steps to obtain the variational approximation loss

Our proposed optimization problem for variational approximation is

$$\min_{\theta} \left[\mathrm{KL}(p(Y_k, Z_k) \| q_\theta(Y_k, Z_k)) - \mathrm{KL}(p(Y_k)p(Z_k) \| q_\theta(Y_k, Z_k)) \right]. \tag{1}$$

Following Theorem 3.2 in [1], defining $q_\theta(Y_k, Z_k) = q_\theta(Y_k|Z_k)p(Z_k)$, we can rewrite the above term as

$$\min_{\theta} \left\{ \mathbb{E}_{p(Y_k,Z_k)} \left[\log \left(\frac{p(Y_k|Z_k)p(Z_k)}{q_\theta(Y_k|Z_k)p(Z_k)} \right) \right] - \mathbb{E}_{p(Y_k)p(Z_k)} \left[\log \left(\frac{p(Y_k)p(Z_k)}{q_\theta(Y_k|Z_k)p(Z_k)} \right) \right] \right\}. \tag{2}$$

Removing the terms that do not depend on the parameter θ, the optimization problem reduces to

$$\max_{\theta} \left[\mathbb{E}_{p(Y_k,Z_k)} \left[\log q_\theta(Y_k|Z_k) \right] - \mathbb{E}_{p(Y_k)p(Z_k)} \left[\log q_\theta(Y_k|Z_k) \right] \right]. \tag{3}$$

Similar to [1], we can use samples to obtain the following unbiased estimate of the objective function to be maximized as

$$\mathcal{L}_{\theta_k} = \frac{1}{N} \sum_{n=1}^{N} \log q_\theta(y_k^{(n)}|z_k^{(n)}) - \frac{1}{N^2} \sum_{n=1}^{N} \sum_{m=1}^{N} \log q_\theta(y_k^{(m)}|z_k^{(n)}). \tag{4}$$

$$\mathcal{L}_{\theta_k} = \frac{1}{N} \sum_{n=1}^{N} \left[\log q_\theta(y_k^{(n)}|z_k^{(n)}) - \frac{1}{N} \sum_{m=1}^{N} \log q_\theta(y_k^{(m)}|z_k^{(n)}) \right]. \tag{5}$$

References

[1] Pengyu Cheng, Weituo Hao, Shuyang Dai, Jiachang Liu, Zhe Gan, and Lawrence Carin. Club: A contrastive log-ratio upper bound of mutual information. In *International conference on machine learning*, pages 1779–1788. PMLR, 2020.