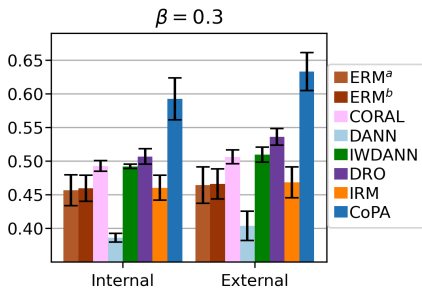


A. Complete Results

(a) 2-dim, ERM^a : input=X, ERM^b : input=X,Z



(b) CMNIST, ERM^c : greyscale input

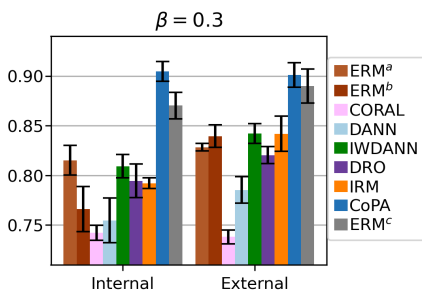
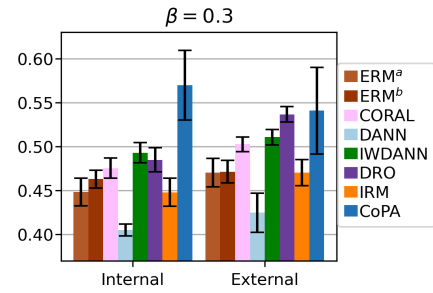


Figure 13. F1-score at test site, multiple training sites. $Y \leftarrow S \rightarrow Z$

(a) 2-dim, ERM^a : input=X, ERM^b : input=X,Z



(b) CMNIST, ERM^c : greyscale input

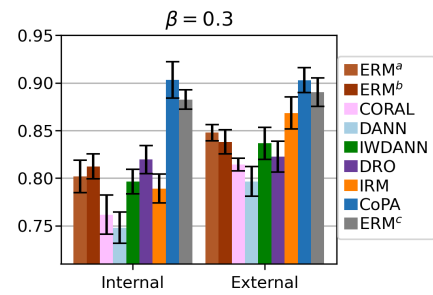
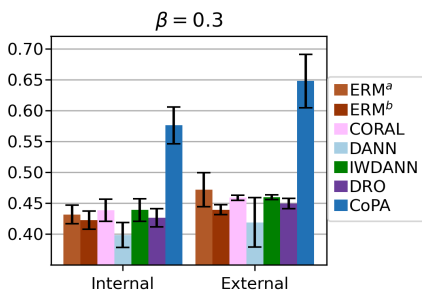


Figure 15. F1-score at test site, multiple training sites. $Y \leftarrow Z$

(a) 2-dim, ERM^a : input=X, ERM^b : input=X,Z



(b) CMNIST, ERM^c : greyscale input

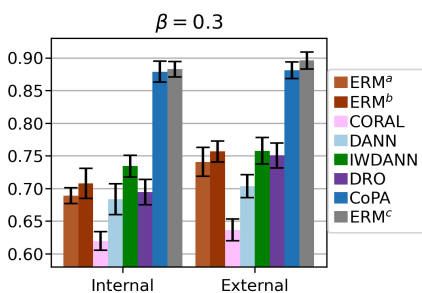
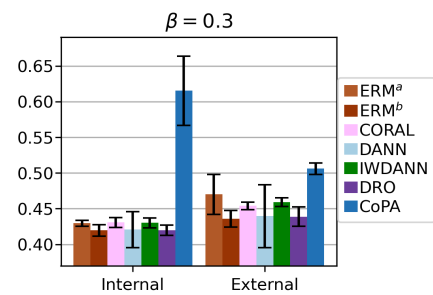


Figure 14. F1-score at test site, a single training site. $Y \leftarrow S \rightarrow Z$

(a) 2-dim, ERM^a : input=X, ERM^b : input=X,Z



(b) CMNIST, ERM^c : greyscale input

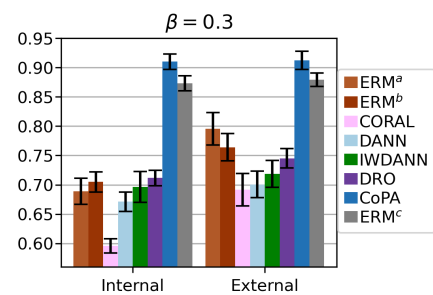
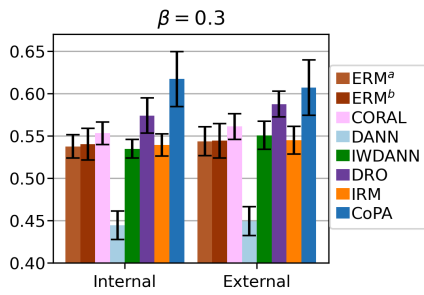


Figure 16. F1-score at test site, a single training site. $Y \leftarrow Z$

(a) 2-dim, ERM^a : input=X, ERM^b : input=X,Z



(b) CMNIST, ERM^c : greyscale input

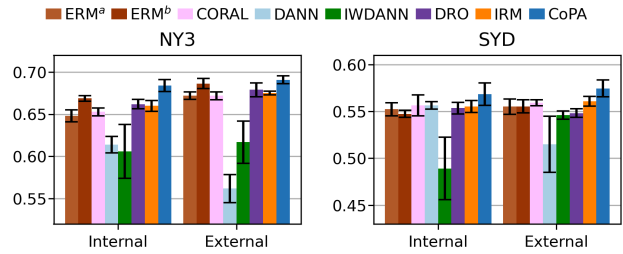
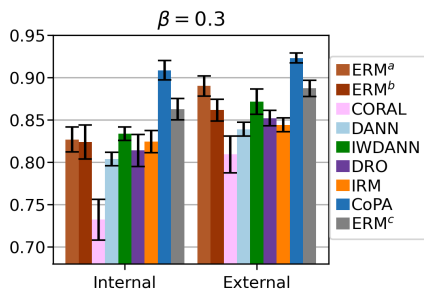
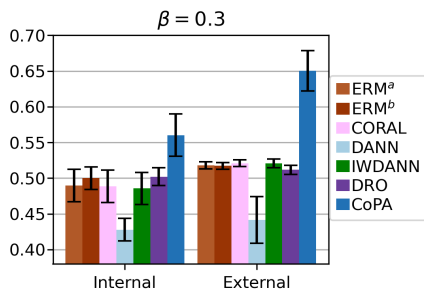


Figure 19. F1-score at ISIC test sites, multiple training sites.

Figure 17. F1-score at test site, multiple training sites. $Y \rightarrow Z$

(a) 2-dim, ERM^a : input=X, ERM^b : input=X,Z



(b) CMNIST, ERM^c : greyscale input

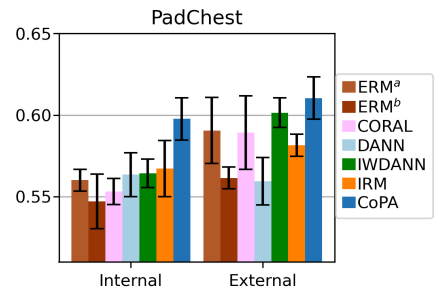
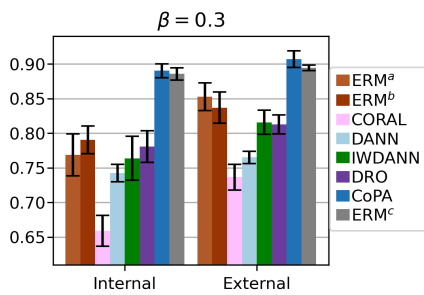


Figure 20. F1-score at CXR test site, a single training site

Figure 18. F1-score at test site, a single training site. $Y \rightarrow Z$

B. Complete Ablations

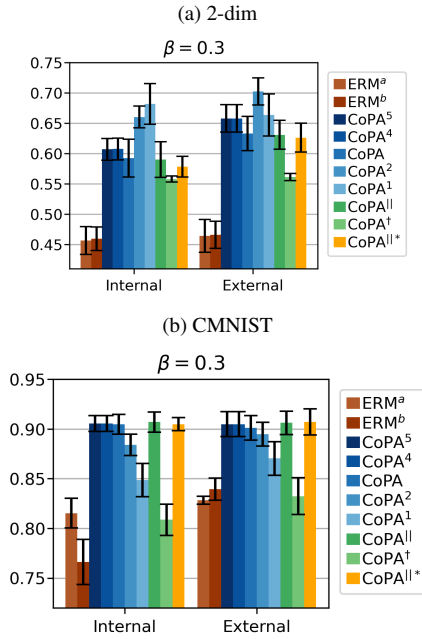


Figure 21. Ablation on synthetic data. Test F1-score. $Y \leftarrow S \rightarrow Z$

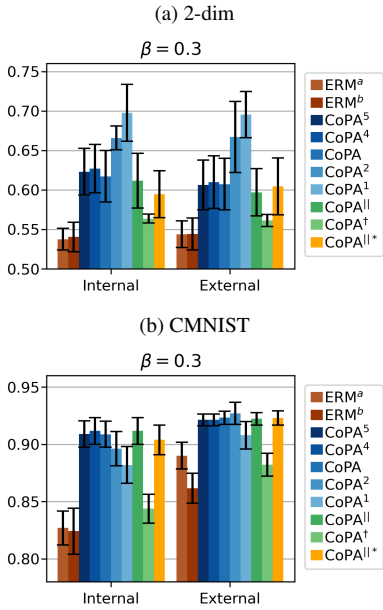


Figure 22. Ablation on synthetic data. Test F1-score. $Y \rightarrow Z$

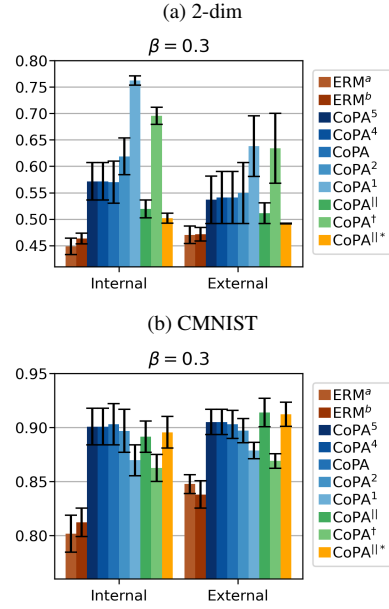


Figure 23. Ablation on synthetic data. Test F1-score. $Y \leftarrow Z$

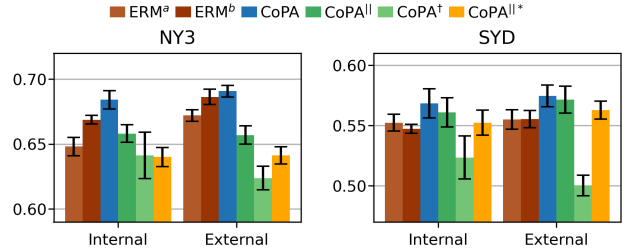


Figure 24. Ablation on ISIC data. Test F1-score.

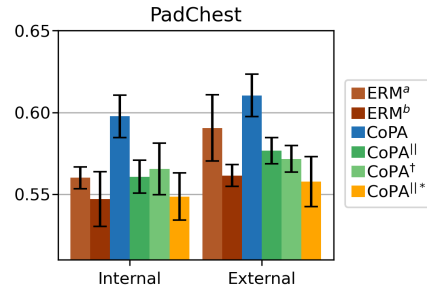


Figure 25. Ablation on CXR data. Test F1-score.

C. Examples of CMNIST, ISIC, and CXR data

C.1. CMNIST

Figure 26 shows some examples of the CMNIST data from two sites: $\beta=0.7$ and $\beta=0.3$. The correlation between red color and $Y=1$ is strong when $\beta=0.7$ but is very weak when $\beta=0.3$. Besides, there are far fewer images with $Y=1$

label in $\beta=0.3$ site than in $\beta=0.7$ site, indicating a change in the $P(Y|E)$ distribution.

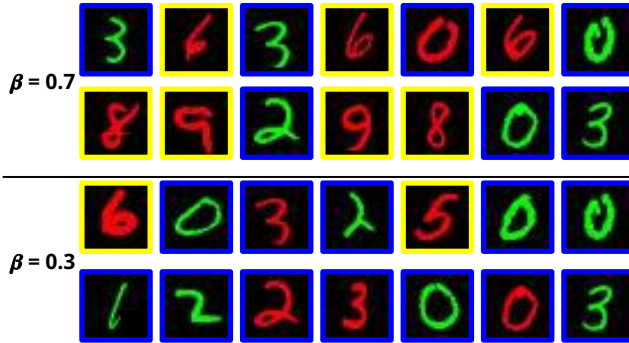


Figure 26. CMNIST data from two sites: $\beta=0.7$ and $\beta=0.3$. Yellow border: $Y=1$, blue border: $Y=0$.

C.2. ISIC Data

The skin cancer dataset is from the International Skin Imaging Collaboration (ISIC) archive. Data from the archive are collected by Memorial Sloan Kettering Cancer Center, Medical University of Vienna, Hospital Clinic de Barcelona, Melanoma Institute Australia, the University of Queensland, and Boston University at different points in time. There are about 70k data samples in total. Each data sample consists of an input image X , a binary target label Y (having melanoma or not) and three confounding variables Z (*Age*, *Anatomical Site*, *Sex*). Table 4 shows the list of values of *Anatomical Site*.

| <i>Anatomical Site</i> | Frequency (%) |
|------------------------|---------------|
| anterior torso | 17.94 |
| head/neck | 9.83 |
| lateral torso | 1.24 |
| lower extremity | 19.91 |
| oral/genital | 0.27 |
| palms/soles | 1.13 |
| posterior torso | 16.67 |
| upper extremity | 12.08 |

Table 4. *Anatomical Site* as a confounding variable (Z)

Figure 27 shows the ISIC data from two sites: *BCN1* and *BCN2*. There are also far fewer images with $Y=1$ label in *BCN2* than in *BCN1*, indicating a change in the $P(Y|E)$ distribution.

C.3. Chest X-Ray (CXR) Data

Figure 28 shows data from two datasets: CheXpert and PadChest.

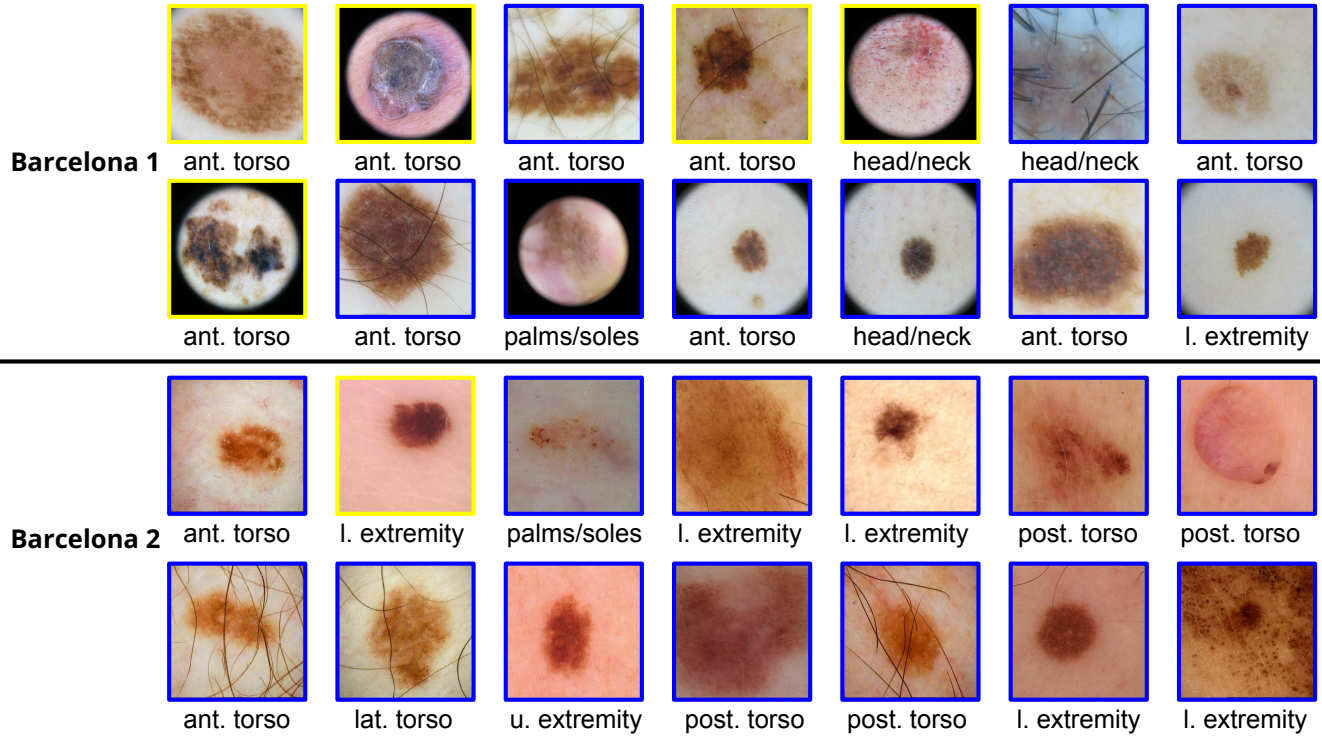


Figure 27. ISIC data from two sites: *Barcelona 1 (BCN1)* and *Barcelona 2 (BCN2)*. Yellow border: $Y=1$, blue border: $Y=0$. The captions under the images indicate the *Anatomical Site* where the images were taken.

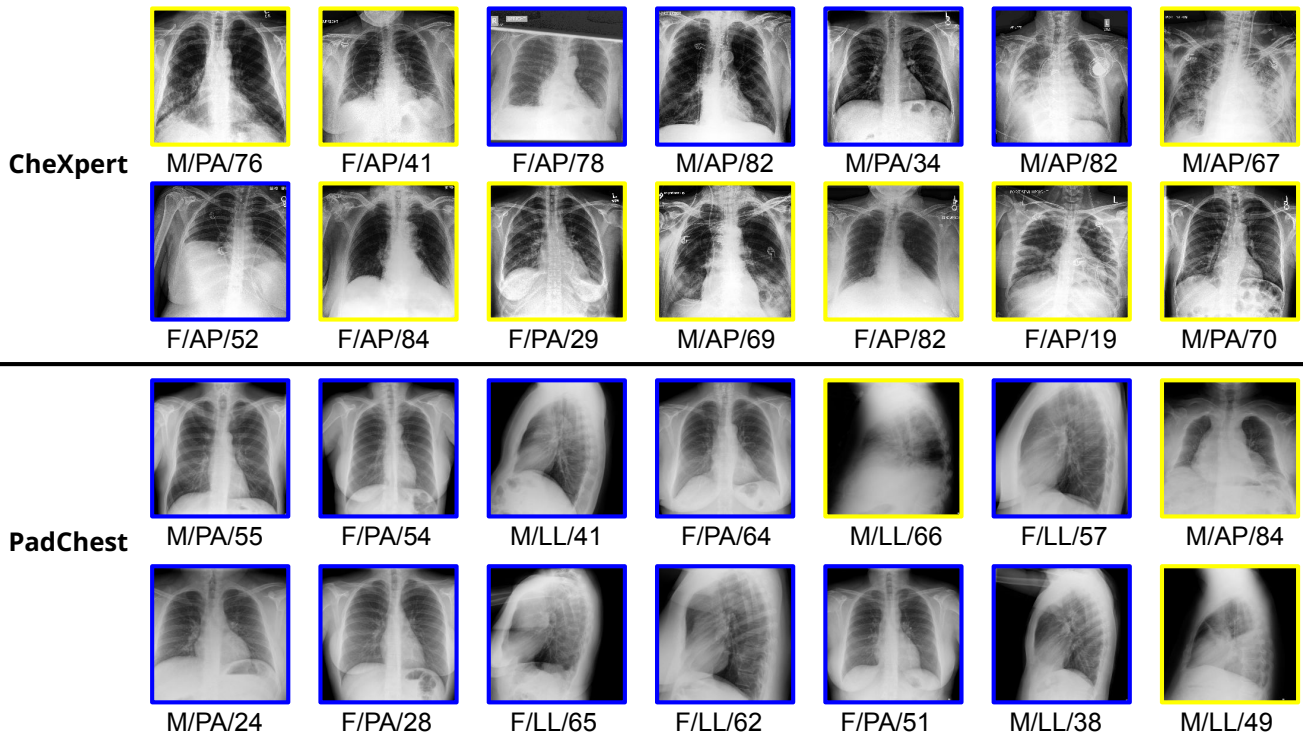


Figure 28. Data from two chest X-Ray datasets: CheXpert and PadChest. Yellow border: $Y=1$, blue border: $Y=0$. The captions under the images indicate the *Sex/Projection/Age* values.