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Blind Image Deblurring with FFT-ReLU Sparsity Prior: Supplementary Material

Anonymous WACV Algorithms Track submission

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1. Closed-form Solution for I

In our algorithm, the blind deconvolution section involves estimating the nonlinear RFT(I). This is done by using a linear operator F , such that $FI = \text{RFT}(I)$. The computation of F is done using gradient descent with Adam optimizer.

Given F , we can solve for I from:

$$\min_I \|\mathbf{T}_k I - \mathbf{B}\|_2^2 + \gamma \|\nabla I - \mathbf{g}\|_2^2 + \beta \|FI - \mathbf{h}\|_2^2 \quad (1)$$

where \mathbf{T}_k is a Toeplitz matrix of k , which is multiplied with vectors using FFT. \mathbf{B} , \mathbf{g} and \mathbf{h} are denoted in their vector forms, respectively.

The solution for Equation (1) is computed as follows:

$$\begin{aligned} L &= \min_I \|\mathbf{T}_k I - \mathbf{B}\|_2^2 + \gamma \|\nabla I - \mathbf{g}\|_2^2 + \beta \|FI - \mathbf{h}\|_2^2 \\ &= (\mathbf{T}_k I - \mathbf{B})^T (\mathbf{T}_k I - \mathbf{B}) + \gamma (\nabla I - \mathbf{g})^T (\nabla I - \mathbf{g}) + \\ &\quad \beta (FI - \mathbf{h})^T (FI - \mathbf{h}) \\ &= \mathbf{I}^T \mathbf{T}_k^T \mathbf{T}_k \mathbf{I} - \mathbf{I}^T \mathbf{T}_k^T \mathbf{B} - \mathbf{B}^T \mathbf{T}_k \mathbf{I} + \mathbf{B}^T \mathbf{B} + \gamma \mathbf{I}^T \nabla^T \nabla I - \\ &\quad \gamma \mathbf{I}^T \nabla^T \mathbf{g} - \gamma \mathbf{g}^T \nabla I + \gamma \mathbf{g}^T \mathbf{g} + \beta \mathbf{I}^T \mathbf{F}^T \mathbf{F} \mathbf{I} - \beta \mathbf{I}^T \mathbf{F}^T \mathbf{h} - \\ &\quad \beta \mathbf{h}^T \mathbf{F} \mathbf{I} + \beta \mathbf{h}^T \mathbf{h} \end{aligned} \quad (2)$$

Here, $\nabla = (\nabla_x, \nabla_y)$ represents the matrix for computing axiswise gradients. Differentiating Equation (2) with respect to I , we get:

$$\begin{aligned} \frac{dL}{dI} &= 2\mathbf{T}_k^T \mathbf{T}_k \mathbf{I} - \mathbf{T}_k^T \mathbf{B} - \mathbf{T}_k^T \mathbf{B} + 0 + 2\gamma \nabla^T \nabla I - \gamma \nabla^T \mathbf{g} - \\ &\quad \gamma \nabla^T \mathbf{g} + 0 + 2\beta \mathbf{F}^T \mathbf{F} \mathbf{I} - \beta \mathbf{F}^T \mathbf{h} - \beta \mathbf{F}^T \mathbf{h} \\ &= 2\mathbf{T}_k^T \mathbf{T}_k \mathbf{I} - 2\mathbf{T}_k^T \mathbf{B} + 2\gamma \nabla^T \nabla I - 2\gamma \nabla^T \mathbf{g} + 2\beta \mathbf{F}^T \mathbf{F} \mathbf{I} - \\ &\quad - 2\beta \mathbf{F}^T \mathbf{h} \end{aligned} \quad (3)$$

In the case of convergence, $\frac{dL}{dI} = 0$. Therefore,

$$\begin{aligned} \mathbf{I} &= \frac{\mathbf{T}_k^T \mathbf{B} + \gamma \nabla^T \mathbf{g} + \beta \mathbf{F}^T \mathbf{h}}{\mathbf{T}_k^T \mathbf{T}_k + \gamma \nabla^T \nabla + \beta \mathbf{F}^T \mathbf{F}} \\ &= \mathcal{F}^{-1} \left(\frac{\overline{\mathcal{F}(\mathbf{K})} \mathcal{F}(\mathbf{B}) + \gamma \overline{\mathcal{F}(\nabla)} \mathcal{F}(\mathbf{g}) + \beta \overline{\mathcal{F}(\mathbf{F})} \mathcal{F}(\mathbf{h})}{\overline{\mathcal{F}(\mathbf{K})} \mathcal{F}(\mathbf{K}) + \gamma \overline{\mathcal{F}(\nabla)} \mathcal{F}(\nabla) + \beta \overline{\mathcal{F}(\mathbf{F})} \mathcal{F}(\mathbf{F})} \right) \end{aligned} \quad (4)$$

This is the closed-form solution presented in our paper.

2. Parameter Sensitivity

As mentioned in the text, for the variable λ , we experimented with some other values to find the optimum, with respect to kernel similarity (defined as SSIM between the estimated kernel and ground-truth kernel).

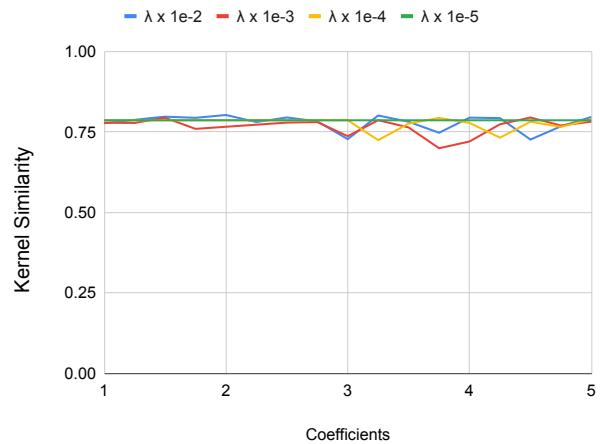
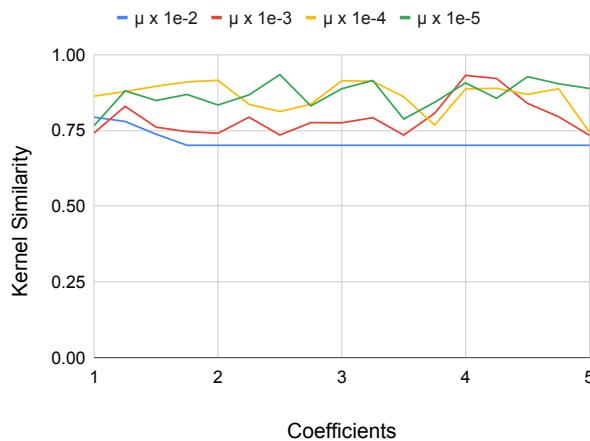


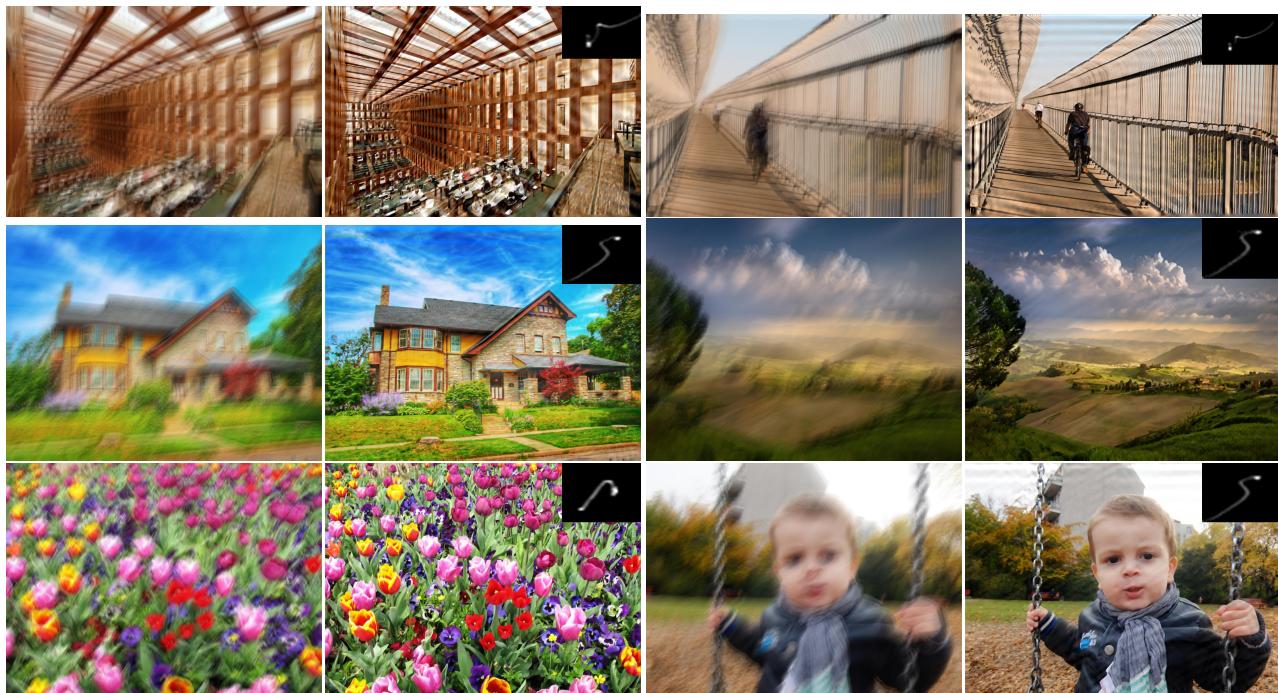
Figure 1. Finding Optimum Value of λ

In Fig. 1, we can see that for ranges in 10^{-3} , the kernel similarity is lower. In ranges 10^{-4} and 10^{-5} , the levels of kernel similarity are similar. However, in our experiments, using values around 10^{-5} increased the runtime of our algorithm. Therefore, we chose 3×10^{-4} since it provided

108 the best results. For μ , we conducted similar experiments
 109 and found the optimum value.
 110



125 Figure 2. Finding Optimum Value of μ
 126



153 Figure 3. Results of our blind image deblurring algorithm, presented with the corresponding estimated blur kernel. For each pair, the input
 154 image is on the left, and the output of our algorithm is on the right.
 155

162 We can see from the experiment results in Fig. 2 that the
 163 highest levels of kernel similarity are achieved in the ranges
 164 of 10^{-3} , and we set the value of μ to 0.004.
 165

3. More Experimental Results

166 In this section, we show more results of our algorithm on
 167 input blurry images. We present them as input and output
 168 image pairs, and the estimated blur kernel from our algo-
 169 rithm is embedded into the top right corner of the output
 170 image. The images are obtained from datasets provide by
 171 Lai *et al.* [1] and Levin *et al.* [2].
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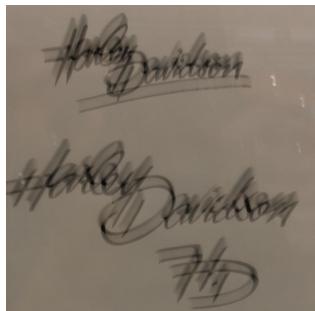
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Figure 4. More results of our blind image deblurring algorithm, presented with the corresponding estimated blur kernel. For each pair, the input image is on the left, and the output of our algorithm is on the right for each pair.

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References

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| <p><i>Pattern Recognition</i>, 2016. 2</p> <p>[2] Anat Levin, Yair Weiss, Fredo Durand, and William T. Freeman. Understanding and evaluating blind deconvolution algorithms. In <i>2009 IEEE Conference on Computer Vision and Pattern Recognition</i>, pages 1964–1971, 2009. 2</p> | |