Exemplar based Underwater Image Enhancement augmented by Wavelet Corrected Transforms

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Abstract

In this paper we propose a novel deep learning framework to enhance underwater images by augmenting our network with wavelet corrected transformations. Wavelet transforms have recently made way into deep learning frameworks and their ability to reconstruct arbitrary signals accurately makes them favorable for many applications. Underwater images are subjected to unique distortions, this is mainly attributed to the fact that red wavelength light gets absorbed dominantly giving a greenish, blue hue. This wavelength dependent selective absorption of light and also scattering by the suspended particles introduce non-linear distortions that affect the quality of the images. We propose an encoder-decoder module with wavelet pooling and unpooling as one of the network components to perform progressive whitening and coloring transforms to enhance underwater images via realistic style transfer. We give a sound theoretical proof as to why wavelet transforms are better for signal reconstruction. We demonstrate our proposed framework on popular underwater images dataset and evaluate it using metrics like SSIM, PSNR and UCIQE and show that we achieve state-of-the-art results compared to those mentioned in the literature.

1. Introduction

In this paper we propose a novel deep learning framework that encompasses wavelet transforms as one of the network architecture modules to enhance underwater images. Underwater imaging is an important task in many ocean research and engineering tasks. Autonomous Underwater Vehicles (AUVs) and Remotely Operated Vehicles (ROVs) which rely on visual sensing as one of their sensor modal are ubiquitous for embarking journey into the depths of water, however visual sensing faces variety of challenges owing to the wavelength selective absorption of water that makes it appear greenish, blue hue. The problem is further worsened as one traverses the depth of water due to suspended particles, time of the day, amount of light present which introduce unique non-linear distortions degrading the image quality. AUVs which use vision-based algorithms should take into account the change in color, illumination and the distortions to effectively make intelligent decisions. This prompts for frameworks that will help in restoring or enhancing underwater images to increase the visual quality. In order to restore or enhance underwater images, we should first study the basic physics laws that govern the formation of image underwater. The water medium presents unique challenges that are usually not encountered while capturing images in the air medium. Light gets attenuated in an exponential fashion when it enters the water medium resulting in highly degraded images. The forward and backward scattering of light results in a veil that obscures the actual objects and scene under consideration. As authors in [18] summarize, the images of interest are usually plagued by one or more of these problems : blurring, bright artifacts, low contrast, noise etc. To deal with these problems, underwater image processing can be broadly categorized as either an image restoration technique that involves modeling the image degradation and treating it as an inverse problem where various parameters like attenuation and diffusion constants or depth estimation of the scene/object have to be considered to restore the underwater image. The other category involves enhancing the image qualitatively without relying on any physical model. In this work we focus on enhancing the underwater image without relying on any physical model. We propose a novel encoder-decoder architecture which has wavelet corrected transforms as one of the network module, using this we treat the problem of image enhancement as a photo-realistic style transfer problem and show that we achieve state-of-the-art results. Towards this we make the following contributions :

1. We treat underwater image enhancement problem as a photo-realistic style transfer problem and propose a

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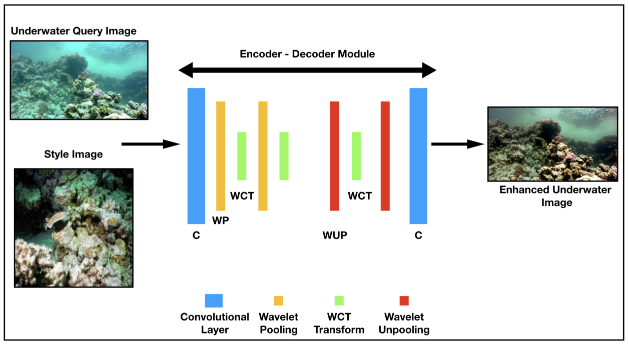

Figure 1. The query underwater image along with the style image is fed to the encoder-decoder module which has wavelet pooling and unpooling layers augmented by whitening and color transforms to perform stylization. The resultant image is enhanced and has low noise and improved global contrast.

novel deep learning framework that leverages wavelet transforms as its network module to help restore underwater images.

2. Our framework is able to restore images with high fidelity leading to an improved global contrast, reduced noise level and also retaining intricate and sharp details of the original images.

3. We demonstrate our framework on popular underwater images dataset and use popular evaluation metrics like SSIM, PSNR and UCIQE to show how effective our proposed framework is.

In what follows, section 2 will highlight some of the contemporary works on the problem, in section 3 we will discuss why wavelet assisted transforms neural network architectures are better suited for inverse problems such as image enhancement and restoration. In section 4 we discuss in detail our proposed methodology followed by results and conclusion. Figure 1 gives an overall schematic of proposed methodology.

2. Related Work

In this section we discuss few contemporary, related works that have been carried out in this area. Underwater image enhancement and restoration techniques are important for many applications like aquatic robot inspection, marine life and environment surveillance and also in oceanography. Unmanned underwater vehicles have replaced traditional human surveillance methods, the evolving sensor technology and also reliance on vision algorithms for making intelligent decisions prompt for better frameworks that could circumvent various image degradation problems. Authors in [6] propose to use a Cycle-GAN approach to first generate synthetic underwater images and then use the generated pairs as training data. They learn a forward and inverse mapping between distorted and undistorted images by performing style transfer using Cycle-GAN. The problem with using GAN is the introduction of unnatural artifacts in the images making them appear less realistic. Authors in [2] propose a deep learning framework based on convolutional neural networks to enhance the underwater images that attempts to jointly optimize MSE and SSIM losses along with using underwater image formation model to generate synthetic images which are further used by their network for training. In paper [12] the authors propose to use generative adversarial networks to generate realistic underwater images by using in-air images and their corresponding depth pairings which form the training data for their two-stage network that performs color correction. Many works [4, 11] also propose to use dehazing algorithms to restore underwater images, but the problem is dehazing and restoring underwater images are two different physical phenomena and the algorithm may or may not perform adequately. Some works [8, 1, 17] leverage fusion techniques and other optical properties of light, color correction mechanisms and dark channel priors etc to restore underwater images. In this work we propose a simple yet effective framework to enhance underwater images. We treat underwater image enhancement problem as a realistic style transfer problem using an encoder-decoder architecture augmented by wavelet pooling and unpooling which performs progressive stylization of images using whitening and coloring transforms 4.3.

3. High Fidelity Signal Reconstruction

In this section we briefly talk about the motivation behind using wavelet pooling and unpooling as network components as opposed to max-pooling operations. Deep Learning frameworks have found immense applications in many low-level computer vision tasks like segmentation, super-resolution etc. Recently many works[23, 22] have proposed to augment regular deep learning frameworks with classical signal processing techniques like wavelets, non-local processing etc. The use of wavelet transforms has found extensive use in recent works like [3] for super resolution, for image denoising [9], feature dimension reduction [20] etc. The idea of using Haar wavelets to recover the spatial signal without noise amplification for performing photo-realistic style transfer [24] and the work in [22] shows how classical signal processing approaches like wavelets in conjunction with deep learning frameworks are suitable for inverse problems has inspired this work of underwater image enhancement and restoration. We leverage the work done in [24] to transform underwater image enhancement problem into an exemplar based style transfer problem to restore and enhance degraded and distorted images.The work in [24] is inspired by the interpretation of deep learning frameworks in terms of frames and wavelets [23, 22]. A frame ϕ_j is a subset of Hilbert space $\mathcal{H}$ such that it allows for a function $f \in \mathcal{H}$ to be numerically reconstructed from its frame coefficients in L^2. Mathematically, for ϕ_j to be called a frame, $\phi_j \subset$ of $\mathcal{H}$, with $A, B > 0$, satisfying,

$$A||f||^2 \leq \sum(| < \phi_j, f > |)^2 \leq B||f||^2, \forall f \in \mathcal{H} \quad (1)$$

here A, B are the upper and lower bounds respectively and if $A = B$ then its called a *tight frame*. The values $(| < \phi_j, f > |)_{j=1}^{K}$ are called the frame co-efficients. These co-efficients are responsible for helping in reconstructing the original frame by using *Dual Frame* ($\tilde{\phi_j}$). The expression for the frame and its dual is given by,

$$Frame(\phi_j) = F^* F(\tilde{\phi_j}) \quad (2)$$

$$Dual(\tilde{\phi_j}) = (F^* F)^{-1} \phi_j \quad (3)$$

With this motivation, the wavelet transforms are used as one of the components in the encoder-decoder architecture to enhance its performance. More specifically, Haar wavelet transforms replace the max-pooling operations. The Haar wavelet pooling operation has four kernels consisting of high frequency and low frequency filters represented as $\{LL^T, LH^T, HL^T, HH^T\}$, which are represented as LL, LH, HL and HH respectively for brevity. The low frequency kernel (LL) captures the smooth surface and texture information while the high frequency kernels (LH,HL,HH) capture edge-like information. The mirror operation of wavelet pooling which is unpooling is capable of reconstructing the original signal with minimal noise amplification by performing component wise transposed convolution and summation.This alluring property helps in retaining the structural information of the image while also performing faithful style transfer. We use this idea to perform style transfer on under-water images and in-turn restore them.

4. Proposed Methodology

In this section we provide detailed overview of the network architecture and how we use wavelet guided network components to restore underwater images.

4.1. Model Architecture

The model architecture consists of an auto-encoder network which is used for general image reconstruction process. We leverage the network architecture proposed by Yoo et. al [24] which mainly consists of the VGG19 network [19] trained on ImageNet to act as an encoder. The network is modified such that *conv1-1* to *conv4-1* layers are treated as encoders followed by replacement of regular max-pooling layers with wavelet pooling layers. The decoder is the exact mirror image of encoder with wavelet unpooling layers in between. This novel modification is because using simple upsampling or max-pooling masks as proposed in works [13, 14] to invert the features to RGB space leads to loss of spatial information in the feature maps which further leads to poor quality image reconstruction. The information captured by the low frequency filters of the wavelet pooling is passed onto the next encoder module while the high frequency filters are skip connected to the decoder module directly.

4.2. Wavelet Pooling and Unpooling

Convolutional Neural Networks (CNN) have become defacto standard for various image and video related tasks. The state-of-the-art methods are capable of classifying images, objects, videos etc with significant accuracies. This performance has motivated researchers to come up with better foundational concepts and tweaks to further enhance the performance of network models [20]. The major components of the CNN are the convolutional layers and the pooling layers. The convolutional layer acts as a feature extractor while the pooling layer aggregates all the extracted features and tries to reduce the dimensions of the features. The most common type of pooling operations are - Max-Pooling and Average-Pooling. The problem with these methods is that they employ a neighborhood approach to subsample the features as outlined by authors in [20]. Pooling is basically involves condensing the features extracted by the convolutional layers, by summarizing them into a single neuronal value. To circumvent these problems, the concept of wavelet pooling has become popular [9, 24, 23, 22, 20], in this work we also employ wavelet pooling and unpooling operations based on a recent mathematical development called frames and wavelets. As outlined in section 3, the max-pooling layers of the encoder-decoder module is replaced by the wavelet pooling and unpooling layers which are based on the Haar wavelet transform. The Haar wavelet is just one of many transforms [7] which is simple and satisfies the *tight frame* condition outlined in section 3 along with providing a way to decompose the signal into high and low frequency sub-bands which captures complementary information.

4.3. Whitening and Color Transforms

Convolutional Auto-Encoders have been explicitly used as unsupervised feature extractors for image data. Whitening transforms have been used as a preprocessing step for images to reduce the redundant information before feeding them to neural networks. The photo-realistic style transfer method attempts to use whitening and color transforms described in [13]. The idea is to use VGG-19 to extract feature maps of both the content image I_c and the style image I_s and then find feature correlation in the VGG domain. By using singular value decomposition (SVD) the features extracted from content image are projected onto the eigenspace of style image. More precisely, the whitening transform strips off the style of the image while keeping the overall structure of the image intact and the coloring transform does the inverse process of whitening transform. These transferred features are fed to the decoder to obtain the final stylized image. Authors in [15] give a detailed account of the effect of using whitening transforms on autoencoders. In practice, the whitening transform is usually carried out by a technique called ZCA - Zero Component Analysis [16], we demonstrate the effect of whitening transform and trace the steps to whiten the data on the CIFAR10 dataset [10]. We first rescale the images to have values between [0,1], by dividing them by maximum pixel value of 255. Now, we perform a per-pixel mean subtraction for all images rendering each pixel value of all images centered around 0. This means when feature extraction is performed, the image features are treated as separate. Now the co-variance matrix of this zero-centered data is calculated to obtain the eigenvalues necessary for performing singular value decomposition (SVD) to rotate the data, the equation to perform this transformation is given by,

$$\mathbf{X}_{zca} = \mathbf{U}.diag(\frac{1}{\sqrt{diag(\mathbf{S}) + \epsilon}}).\mathbf{U}^T.\mathbf{X} \tag{4}$$

where, **U** consists of left singular vectors, **S** consists of singular values of the covariance of the normalized dataset, **X** is the normalized dataset and ϵ is the hyperparameter responsible for controlling the whitening effect. The results of the whitening transform are presented in figure 2.

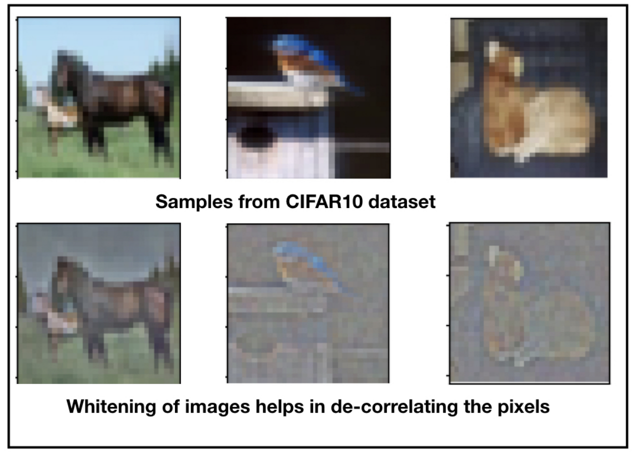

Figure 2. The whitening transform is a pre-processing step that is used to make the image pixels de-correlated so that the performance of a deep learning framework is improved. In our case, the whitening transform is used to strip off the style of the image while preserving the overall structural integrity of the image. We have demonstrated on few samples of CIFAR10 dataset.

5. Experiments

In this section we discuss about the various experiments carried out on paired and unpaired datasets. Here paired dataset means, we have the content image which is underwater and degraded and the corresponding style image which is clear and undistorted. This dataset was obtained from [6]. The other dataset is unpaired, in the sense we have no particular style image, but we experiment with arbitrary style images. We report the PSNR and SSIM values of enhanced images as part of the quantitative evaluation.

5.1. Datasets and Implementation Details

Deep learning frameworks for underwater image processing is largely limited by lack of dataset. To circumvent this problem, many works [6, 2] generate synthetic image pairs to train the network. We use the image pair generated by authors in [6] and another state-of-the-art dataset that had degraded underwater images obscured by turbidity [5]. We also demonstrate our proposed approach on various stock underwater images with arbitrary styles and also report results by performing video stylization. As mentioned in section 4 we transform a VGG19 network into an encoder module pre-trained on ImageNet by imposing a L2 and feature Gram matching loss with the encoder[24]. All the implementation was done on a local machine - MacBook Pro, 8GB RAM, Integrated graphics, The stylization process took approximately 22-26 seconds to generate a 512 resolution sized output image.

5.2. Evaluation Metrics

The ability to discern the differences between images comes naturally to human beings, this task is difficult to quantify, however many surrogate metrics like Structural Similarity Index (SSIM) and Peak Signal to Noise Ratio (PSNR) and many other metrics have been developed that help us make informed decisions on the quality of images. In our work, we utilize PSNR and SSIM to quantitatively measure the performance and also use underwater color image quality evaluation metric (UCIQE) introduced by authors in [21]. In this section we briefly outline the various metrics that are used for evaluation.

1. Peak Signal to Noise Ratio - The Peak Signal to Noise Ratio (PSNR) measures the ratio of maximum possible amount of signal and the corrupting noise that affects the fidelity of the signal. It is calculated as follows,

$$PSNR(I_i, \hat{I}_i) = 10 \log \frac{(max_{I_i})^2}{1/N \sum_{i=0}^{N} (I_i - \widehat{I_i})} \tag{5}$$

One of the important objectives of enhancing underwater images is to denoise them, we use PSNR to report the noise level of the enhanced images.

2. Structural Similarity Index - The structural similarity index (SSIM) is used to quantify the similarity between two images. We use this metric to measure how similar is the enhanced image to the ground truth image. It is calculated as follows,

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C1)(2\sigma_{xy} + C2)}{(\mu_x^2 + \mu_y^2 + C1)(\sigma_x^2 + \sigma_y^2 + C2)} \quad (6)$$

3. Underwater Color Image Quality - We use the underwater color image quality metric proposed by Yang et al [21] to provide a better evaluation of the enhanced images. The proposed metric combines criteria like chroma, contrast and saturation to measure how effective is the enhancement. It is calculated as follows,

$$UCIQ = c_1\sigma_c + c_2 contrast + c_3\mu_s \quad (7)$$

where c_1, c_2, c_3 are the weighted co-efficients and σ_c and μ_s are the standard deviation of chroma and average of saturation respectively. The values of these co-efficients have been experimentally found to be $c_1 = 0.4680$, $c_2 = 0.2745$ and $c_3 = 0.2576$. We use this equation to calculate how enhanced our images are. The results are summarized in the next section.

5.3. Results

In this section we demonstrate our proposed approach on various datasets and report the performance using quantitative evaluation metrics like PSNR, SSIM and UCIQ. In figure 3 we have performed style transfer on a paired dataset. As it is evident, we have tried to enhance/restore the underwater image by performing a realistic style transfer. The presence of wavelet pooling and un-pooling allows to independently control how much stylization can be done, by allowing us to apply the whitening and color transforms (WCT) at various stages of the model architecture. We report the results of WCT performed at encoder, decoder, high frequency skip connections stage and also a power set of all these combined. The PSNR and SSIM values are summarised in table 1. Its clear that performing style transfer at the decoder stage allows us to restore underwater image having higher structural similarity with the ground truth image and also reduced noise as reported by the PSNR values. Next we experiment our framework on the TURBID dataset [5], which consists of a reference image and corresponding images of varying degree of degradation. In Figure 5, the images are degraded by giving it a greenish hue, we try to restore two images of varying level of degradation, as evident from the SSIM value, our framework has

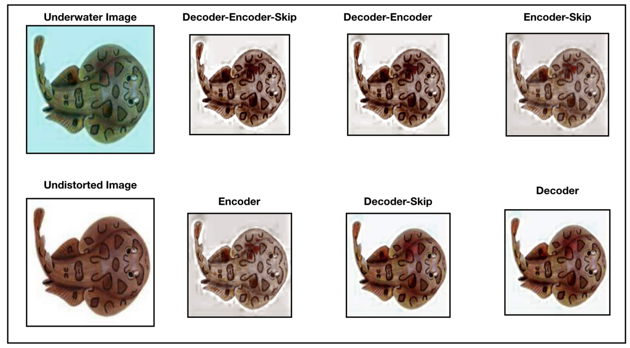

Figure 3. We restore/enhance underwater image consisting of a marine animal obscured by the greenish,blue hue. Performing stylization at the decoder stage allows us to obtain enhanced images which have higher structural similarity with the ground truth and also reduced noise level.

enhanced both the images. In figure 6 and figure 7 we experiment on few other underwater images, its clearly visible that the restored images have high content fidelity and are color corrected. The intricate color details and sharp features of the marine environment are completely preserved. Another little tweak is to include semantic segmentation maps for both the content and style images, so that style transfer occurs for the corresponding parts of the image. We also demonstrate our framework by stylizing a video, we use a stock youtube video that shows exploring the depth of the great barrier reef, we use an arbitrary undistorted style image of another coral reef. Figure 8 shows the video stylization results. The video result can be found at this link `https://youtu.be/NwjQ_Q1da2s`

6. Conclusion

In this paper we have proposed a novel deep learning framework augmented by wavelet pooling and unpooling as its network components to solve the problem of underwater image enhancement. We transform underwater image enhancement problem to photo-realistic style transfer problem and we achieve better results compared to those in literature. Our proposed method helps in recovering highly degraded images and helps achieve low noise and overall better global contrast while retaining the sharp features which are obfuscated by the backscattering of light underwater. We have demonstrated our results on various underwater image datasets and show that we achieve state-of-the-art results compared to works in literature [6, 2] as characterized by the PSNR and SSIM values. We have also demonstrated our proposed methodology by stylizing a video.

Table 1. Quantitative evaluation between the ground truth image and stylised images.

Image (GT vs)	PSNR/SSIM Figure 3		PSNR/SSIM Figure 4	
Decoder-Encoder-Skip	28.31	0.7398	29.13	0.7850
Decoder-Encoder	28.32	0.7401	29.17	0.7866
Encoder-Skip	27.91	0.7715	29.00	0.7519
Encoder	27.90	0.7718	29.00	0.7536
Decoder-Skip	29.93	0.9126	29.42	0.7927
Decoder	29.94	0.9127	29.47	0.7949

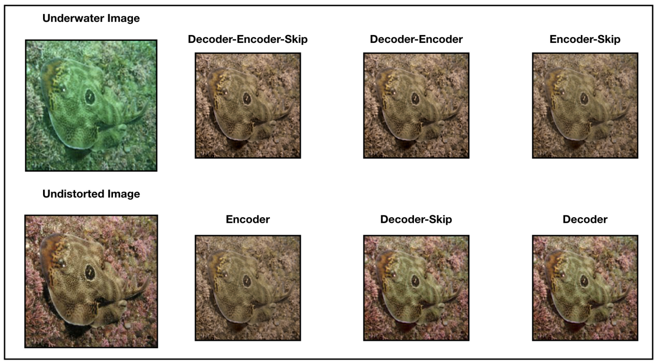

Figure 4. The restored image shows enhanced global contrast and low noise content. Its evident from the stylized images that intricate details of the marine animal and its environment is perfectly captured. Notice the surrounding marine environment covered in pinkish hue which is perfectly restored in our results.

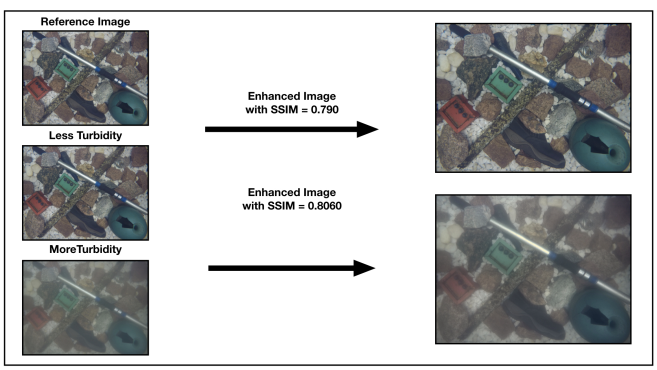

Figure 5. We experiment our framework with the TURBID dataset which has a reference image and corresponding degraded images of varying degree. We try to enhance the least turbid image and the most turbid image. SSIM suggests that our framework was successful in enhancing the images.

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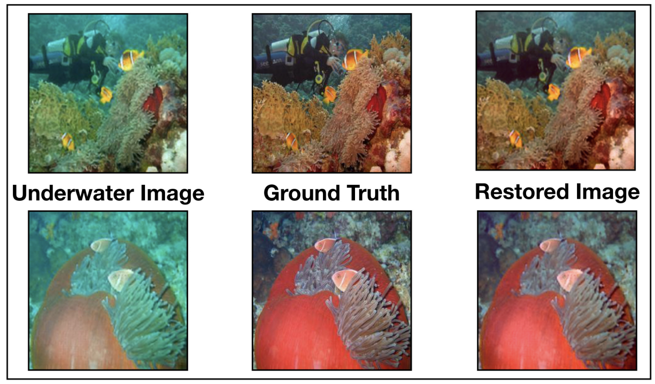

Figure 6. We perform stylization on few more paired underwater image datasets. The first column are underwater images, the second columns are the ground truth images and the third column are enhanced images. Its clear that the enhanced images are highly similar to the ground truth images, we quantify the high quality of the image using UCIQE [21] and we obtain a value of 35.6268. The higher the UCIQE value the better the enhancement.

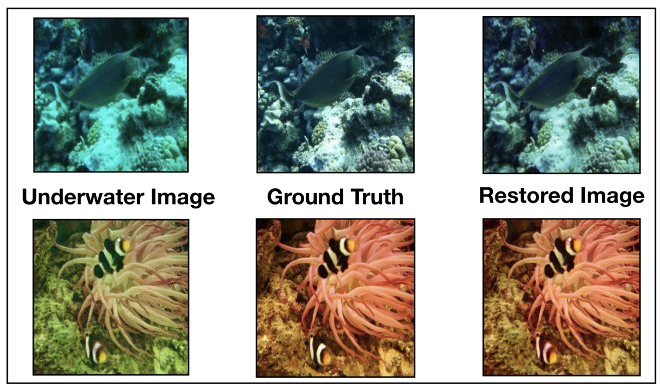

Figure 7. It is evident from the images above that the enhanced images have high content, color and contrast fidelity. The sharp features of various marine animals are preserved. UCIQE = 31.611.

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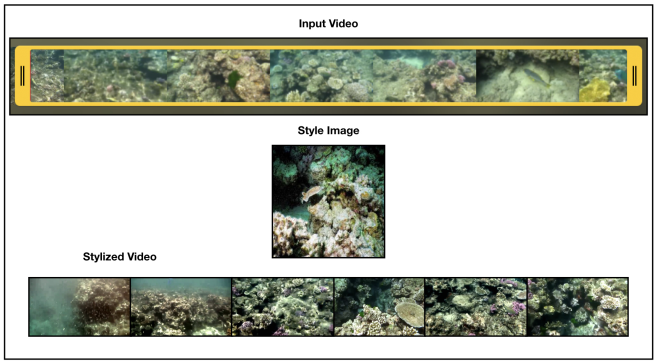

Figure 8. We have performed video stylization on a stock video obtained from youtube. The video consists of underwater exploration of the great barrier reef. We have used an arbitrary but related image of coral as style image. The output video is visually pleasing as well as the water effects are removed.

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