Supplementary Materials: Infinitesimal Drift Diffeomorphometry Models for Population Shape Analysis

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A. Current Matching Norm on Surfaces

A template subcortical gray matter structure is represented by a discrete triangulated surface, a set of n points and a triangulation of n_f faces, the j-th face consists of three ordered points from q_1 with indices denoted $f(j,1), f(j,2), f(j,3)$. An invariant cost function to sampling is based on current matching which is minimized when surfaces and normals are close. We define face centers $c(j) = [q_1(f(j,1)), q_1(f(j,2)), q_1(f(j,3))]/3$ and area weighted normals $A(j) = [q_1(f(j,2)) - q_1(f(j,1))] \times [q_1(f(j,3)) - q_1(f(j,1))]/2$ for $\times$ the cross product in $\mathbb{R}^3$ and $j \in \{1, \ldots, n_f\}$. The same notation is used for target surfaces, which may have a different number of faces or vertices:

$$\|S - S'\|^2 = \frac{1}{2\sigma^2} \left(\sum_{i,j=1}^{n_f} A^T(i) K(c(i), c(j)) A(j) - 2 \sum_{i=1}^{n} \sum_{j=1}^{n_{f'}} A^T(i) K(c(i), c'(j)) A'(j) + \sum_{i,j=1}^{n_{f'}} A'^T(i) K(c'(i), c'(j)) A'(j) \right) \quad (1)$$

with K a kernel defined similarly to that above.