Supplementary Materials
for
Guaranteed Matrix Completion under Multiple Linear Transformations

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1. Proofs

1.1. Proof of Lemma 1

Lemma 1. *Assume there exists a dual certificate* $\mathbf{\Lambda}$ *and the perturbation* $\mathbf{H}$ *obeying* $\mathcal{P}_\Omega(\mathbf{H}) = 0$*. Then the inequality*

$$\begin{aligned}&\|\mathcal{Q}(\mathbf{M}_0+\mathbf{H})\|_* \\ &\geq \|\mathcal{Q}(\mathbf{M}_0)\|_* + (1-\|\mathbf{R}_{\mathbf{\Lambda}}\|_2)\,\|\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\mathbf{H})\|_*\end{aligned}$$

obeys.

Proof. At the beginning, we can easily get the sub-gradient of $f(\mathbf{X}) = \|\mathcal{Q}(\mathbf{X})\|_*$ by the chain rule for differentiation. Assuming arbitrary $\mathbf{Z} \in \partial\|\mathcal{Q}(\mathbf{M}_0)\|_*$, due to the convexity of the nuclear norm, the following inequality holds

$$\|\mathcal{Q}(\mathbf{M}_0+\mathbf{H})\|_* \geq \|\mathcal{Q}(\mathbf{M}_0)\|_* + <\mathbf{Z},\mathbf{H}> . \tag{1}$$

Due to the definition of $\mathbf{\Lambda}$, $\mathbf{Z}$ satisfies the following equation:

$$\mathbf{Z} = \mathbf{\Lambda} - \mathcal{Q}^\star(\mathbf{R}_{\mathbf{\Lambda}}) + \mathcal{Q}^\star\mathcal{P}_{\mathbb{T}^\perp}(\mathbf{Z}). \tag{2}$$

Combing the two formulas above, it can be obtained that

$$\begin{aligned}\|\mathcal{Q}(\mathbf{M}_0+\mathbf{H})\|_* \geq &\|\mathcal{Q}(\mathbf{M}_0)\|_* + <\mathbf{\Lambda},\mathbf{H}> \\ &+ <\mathcal{P}_{\mathbb{T}^\perp}(\mathbf{Z}) - \mathbf{R}_{\mathbf{\Lambda}}, \mathcal{P}_{\mathbb{T}^\perp}(\mathcal{Q}(\mathbf{H}))> .\end{aligned} \tag{3}$$

Owing to the arbitrary picking of $\mathbf{Z}$ from the sub-gradient of f, there must exists a specific $\hat{\mathbf{Z}}$ such that

$$<\mathcal{P}_{\mathbb{T}^\perp}(\mathbf{Z}), \mathcal{P}_{\mathbb{T}^\perp}(\mathcal{Q}(\mathbf{H}))> = \|P_{\mathbb{T}^\perp}(\mathcal{Q}(\mathbf{H}))\|_*. \tag{4}$$

Furthermore, by using the duality between the spectral and nuclear norm, we have

$$<\mathbf{R}_{\mathbf{\Lambda}}, \mathcal{P}_{\mathbb{T}^\perp}(\mathcal{Q}(\mathbf{H}))> \leq \|\mathbf{R}_{\mathbf{\Lambda}}\|_2 \|P_{\mathbb{T}^\perp}(\mathcal{Q}(\mathbf{H}))\|_*. \tag{5}$$

Combing the formulas (3)-(5) and the fact that $<\mathbf{\Lambda},\mathbf{H}> = 0$, we can obtain the result shown in Lemma 1. □

1.2. Proof of Theorem 1

Assumption 1. *Let* $\mathbb{N}_\mathcal{Q}$ *and* $\mathbb{N}_\Omega$ *denote the null of the linear transformations* $\mathcal{Q}$ *and* $\mathcal{P}_\Omega$*, respectively. We assume that the relation* $\mathbb{N}_\mathcal{Q} \cap \mathbb{N}_\Omega = \{0\}$ *obeys.*

Theorem 1 (Error bound for a single $\mathcal{Q}$)**.** *With Assumption 1, and suppose the additional assumptions:*

i) there exists a dual certificate obeying $\|\mathbf{R}_{\mathbf{\Lambda}}\|_2 < 1$*;*

ii) $\exists p > 0, s.t. \mathcal{P}_\mathbb{T}\mathcal{Q}\mathcal{P}_\Omega\mathcal{Q}^\star\mathcal{P}_\mathbb{T} \succeq p\mathcal{I}$

iii) The product $[\mathcal{Q}]_{\langle 2\rangle} \cdot [\mathcal{Q}]^\star_{\langle 2\rangle}$ *is a diagonal matrix.*

Then the solution of

$$\min_{\mathbf{X}\in\mathbb{R}^{m_1\times m_2}} \|\mathcal{Q}(\mathbf{X})\|_* \quad s.t. \|\mathcal{P}_\Omega(\mathbf{X}) - \mathcal{P}_\Omega(\mathbf{Y})\|_F \leq \delta,$$

$\hat{\mathbf{M}}$ *obeys*

$$\begin{aligned}&\|\hat{\mathbf{M}} - \mathbf{M}_0\|_F \\ &\leq 2\delta \cdot \frac{cond(\mathcal{Q})}{1-\|\mathbf{R}_{\mathbf{\Lambda}}\|_2}\sqrt{\frac{\min\{n_1,n_2\}(p + \|\,[\mathcal{Q}]_{\langle 2\rangle}\,\|_2^2)}{p}}.\end{aligned} \tag{6}$$

Proof. Let $\hat{\mathbf{M}} - \mathbf{M}_0 = \mathbf{H}$, then

$$\|\mathbf{H}\|_F^2 = \|\mathbf{H}_\Omega + \mathbf{H}_{\Omega^c}\|_F^2 = \|\mathbf{H}_\Omega\|_F^2 + \|\mathbf{H}_{\Omega^c}\|_F^2, \tag{7}$$

where $\mathbf{H}_\Omega = \mathcal{P}_\Omega(\mathbf{H})$ and $\mathbf{H}_{\Omega^c} = (\mathcal{I} - \mathcal{P}_\Omega)(\mathbf{H})$. For $\mathbf{H}_\Omega$, we have

$$\begin{aligned}&\|\mathbf{H}_\Omega\|_F \\ &= \|\mathcal{P}_\Omega(\hat{\mathbf{M}}) - \mathcal{P}_\Omega(\mathbf{H}_{\Omega^c})\|_F \\ &= \|(\mathcal{P}_\Omega(\hat{\mathbf{M}}) - \mathcal{P}_\Omega(\mathbf{Y})) - (\mathcal{P}_\Omega(\mathbf{M}_0) - \mathcal{P}_\Omega(\mathbf{Y}))\|_F \\ &\leq \|\mathcal{P}_\Omega(\hat{\mathbf{M}}) - \mathcal{P}_\Omega(\mathbf{Y})\|_F + \|\mathcal{P}_\Omega(\mathbf{M}_0) - \mathcal{P}_\Omega(\mathbf{Y})\|_F \\ &\leq 2\delta.\end{aligned} \tag{8}$$

On the other side, for $\mathbf{H}_{\Omega^c}$, we split $\|\mathbf{H}_{\Omega^c}\|_F$ into two parts by the null space of $\mathbf{Q}$, *i.e.*

$$\begin{aligned}
&\|\mathbf{H}_{\Omega^c}\|_F^2 \\
&= \|\mathcal{P}_{\mathbb{N}_\mathcal{Q}}(\mathbf{H}_{\Omega^c}) + \mathcal{P}_{\mathbb{N}_\mathcal{Q}^c}(\mathbf{H}_{\Omega^c})\|_F^2 \\
&= \underbrace{\|\mathcal{P}_{\mathbb{N}_\mathcal{Q}}(\mathbf{H}_{\Omega^c})\|_F^2}_{item1.1} + \underbrace{\|\mathcal{P}_{\mathbb{N}_\mathcal{Q}^c}(\mathbf{H}_{\Omega^c})\|_F^2}_{tiem1.2}.
\end{aligned} \tag{9}$$

Due to Assumption 1, we can easily get that the item 1.1 equals zeros, *i.e.*,

$$\|\mathcal{P}_{\mathbb{N}_\mathcal{Q}}(\mathbf{H}_{\Omega^c})\|_F = 0. \tag{10}$$

Hence,

$$\mathcal{P}_{\mathbb{N}_\mathcal{Q}^c}(\mathbf{H}_{\Omega^c}) = \mathbf{H}_{\Omega^c}, \tag{11}$$

and we denote for brevity that

$$\|\widetilde{\mathbf{H}}\|_F := \|\mathbf{H}_{\Omega^c}\|_F. \tag{12}$$

Assume the truncated singular value decomposition (SVD) of the unfolding $[\mathcal{Q}]_{\langle n\rangle} = \mathbf{U}_\mathcal{Q}\mathbf{D}_\mathcal{Q}\mathbf{V}_\mathcal{Q}^T$ in which only the singular vectors related to non-zero vectors are kept. Then we have the following inequalities

$$\begin{aligned}
\|\widetilde{\mathbf{H}}\|_F &= \|\mathbf{V}_\mathcal{Q}^T\widetilde{\mathbf{H}}\|_F \\
&= \|\mathbf{D}_\mathcal{Q}^{-1}\mathbf{D}_\mathcal{Q}\mathbf{V}_\mathcal{Q}^T\widetilde{\mathbf{H}}\|_F \\
&\leq \|\mathbf{D}_\mathcal{Q}^{-1}\|_2\|\mathbf{D}_\mathcal{Q}\mathbf{V}_\mathcal{Q}^T\widetilde{\mathbf{H}}\|_F \\
&= \|\mathbf{D}_\mathcal{Q}^{-1}\|_2\|\mathbf{U}_\mathcal{Q}\mathbf{D}_\mathcal{Q}\mathbf{V}_\mathcal{Q}^T\widetilde{\mathbf{H}}\|_F \\
&= \sigma_{min}([\mathcal{Q}]_{\langle 2\rangle})^{-1}\|\mathcal{Q}(\widetilde{\mathbf{H}})\|_F,
\end{aligned} \tag{13}$$

where $\sigma_{min}([\mathcal{Q}]_{\langle 2\rangle})$ denotes the smallest non-zero singular value of $[\mathcal{Q}]_{\langle 2\rangle}$. In (13), the first equation holds because of (12), and the inequality holds owing to the definition of the matrix spectral norm. Next, we further split $\|\mathcal{Q}(\widetilde{\mathbf{H}})\|_F$ based on the tangent space $\mathbb{T}$. Then we have

$$\|\mathcal{Q}(\widetilde{\mathbf{H}})\|_F^2 = \underbrace{\|\mathcal{P}_\mathbb{T}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F^2}_{item2.1} + \underbrace{\|\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F^2}_{item2.2}. \tag{14}$$

We first bound the item 2.2 in the following proof. According Lemma 1, we have

$$\|\mathcal{Q}(\mathbf{M}_0+\widetilde{\mathbf{H}})\|_* \geq \|\mathcal{Q}(\mathbf{M}_0)\|_* + (1-\|\mathbf{R}_\mathbf{\Lambda}\|_2)\|P_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_*. \tag{15}$$

Furthermore, since $\hat{\mathbf{M}}$ is the optimal solution of (3), we have

$$\begin{aligned}
\|\mathcal{Q}(\mathbf{M}_0)\|_* &\geq \|\mathcal{Q}(\hat{\mathbf{M}})\|_* \\
&= \|\mathcal{Q}(\mathbf{M}_0 + \mathbf{H}_\Omega + \widetilde{\mathbf{H}})\|_* \\
&\geq \|\mathcal{Q}(\mathbf{M}_0 + \widetilde{\mathbf{H}})\|_* - \|\mathcal{Q}(\mathbf{H}_\Omega)\|_*
\end{aligned} \tag{16}$$

Combing (15) and (16), we get

$$\begin{aligned}
\|\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_* &\leq \frac{1}{(1-\|\mathbf{R}_\mathbf{\Lambda}\|_2)}\|\mathcal{Q}(\mathbf{H}_\Omega)\|_* \\
&\leq \frac{\sqrt{\min\{n_1,n_2\}}}{(1-\|\mathbf{R}_\mathbf{\Lambda}\|_2)}\|\mathcal{Q}(\mathbf{H}_\Omega)\|_F \\
&\leq \frac{\sqrt{\min\{n_1,n_2\}}}{(1-\|\mathbf{R}_\mathbf{\Lambda}\|_2)}\|\mathcal{Q}\|_2\|\mathbf{H}_\Omega\|_F.
\end{aligned} \tag{17}$$

By using the relationship between the nuclear norm and Frobenius norm and the inequalities (17), we have

$$\begin{aligned}
\|\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F &\leq \|\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_* \\
&\leq \frac{2\sqrt{\min\{n_1,n_2\}}}{1-\|\mathbf{R}_\mathbf{\Lambda}\|_2}\|\mathcal{Q}\|_2\delta.
\end{aligned} \tag{18}$$

For the item 2.1, we have

$$\begin{aligned}
\|\mathcal{P}_\Omega\mathcal{Q}^\star\mathcal{P}_\mathbb{T}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F^2 &= \left\langle \mathcal{P}_\Omega\mathcal{Q}^\star\mathcal{P}_\mathbb{T}\mathcal{Q}(\widetilde{\mathbf{H}}), \mathcal{P}_\Omega\mathcal{Q}^\star P_\mathbb{T}\mathcal{Q}(\widetilde{\mathbf{H}})\right\rangle \\
&= \left\langle \mathcal{P}_\mathbb{T}\mathcal{Q}P_\Omega\mathcal{Q}^\star\mathcal{P}_\mathbb{T}\mathcal{Q}(\widetilde{\mathbf{H}}), \mathcal{P}_\mathbb{T}\mathcal{Q}(\widetilde{\mathbf{H}})\right\rangle \\
&\geq p\|\mathcal{P}_\mathbb{T}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F^2,
\end{aligned} \tag{19}$$

where the inequality holds because of the second assumption in the theorem. For the left side of the equation (19), we have

$$\begin{aligned}
\|\mathcal{P}_\Omega\mathcal{Q}^\star\mathcal{P}_\mathbb{T}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F &= \|\mathcal{P}_\Omega\mathcal{Q}^\star(\mathcal{I}-\mathcal{P}_{\mathbb{T}^\perp})\mathcal{Q}(\widetilde{\mathbf{H}})\|_F \\
&\leq \|\mathcal{P}_\Omega\mathcal{Q}^\star\mathcal{Q}(\widetilde{\mathbf{H}})\|_F + \|\mathcal{P}_\Omega\mathcal{Q}^\star\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F \\
&= \|\mathcal{P}_\Omega\mathcal{Q}^\star\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F \\
&\leq \|\mathcal{Q}^\star\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F \\
&\leq \|\mathcal{Q}\|_2\|\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F,
\end{aligned} \tag{20}$$

where the second equation holds because of the third assumption of the theorem. Hence, the item 2.1 can be bounded by

$$\|\mathcal{P}_\mathbb{T}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F \leq \frac{1}{\sqrt{p}}\|\mathcal{Q}\|_2\|\mathcal{P}_{\mathbb{T}^\perp}\mathcal{Q}(\widetilde{\mathbf{H}})\|_F. \tag{21}$$

As the result, combing (7), (8), (18) and (20), we can get a total upper bound of the reconstruction error as given in the theorem. □

2. Proof of Theorem 2

Theorem 2. *With the assumptions in Theorem 1 for the concatenation $\widetilde{\mathcal{Q}}$, and further assume that the tuning parameter satisfies $\lambda > \|\mathcal{P}_\Omega(\mathbf{H})\|_2/\sqrt{\min\{m_1,m_2\}}$. Then the reconstruction error of*

$$\min_{\mathbf{X}\in\mathbb{R}^{m_1\times m_2}} \frac{1}{2}\|\mathcal{P}_\Omega(\mathbf{X})-\mathcal{P}_\Omega(\mathbf{Y})\|_F^2+\lambda\sum_{i\in[K]}\|\mathcal{Q}_i(\mathbf{X})\|_*.$$

is bounded by

$$\begin{aligned}&\|\hat{\mathbf{M}}-\mathbf{M}_0\|_F\\ &\le 8\lambda\left(\min\{m_1,m_2\}+\sum_{i\in[K]}\sqrt{\min\{n_1^{(i)},n_2^{(i)}\}}\|\left[\mathcal{Q}_i\right]_{<2>}\|_2\right)\\ &\cdot\frac{cond(\widetilde{\mathcal{Q}})\cdot\min\{\prod n_1^{(i)},\prod n_2^{(i)}\}(p+\|\left[\widetilde{\mathcal{Q}}\right]_{<2>}\|_2)}{p(1-\|\mathbf{R}_{\mathbf{\Lambda}}\|_2)^2}.\end{aligned} \tag{22}$$

Proof. Let $\hat{\mathbf{M}}$ be the optimal solution of

$$\min_{\mathbf{X}\in\mathbb{R}^{m_1\times m_2}} \frac{1}{2}\|\mathcal{P}_\Omega(\mathbf{X})-\mathcal{P}_\Omega(\mathbf{Y})\|_F^2+\lambda\sum_{i\in[K]}\|\mathcal{Q}_i(\mathbf{X})\|_*. \tag{23}$$

Then we can naturally obtain the following inequality

$$\begin{aligned}&\frac{1}{2}\|\mathcal{P}_\Omega(\hat{\mathbf{M}})-\mathcal{P}_\Omega(\mathbf{Y})\|_F^2+\lambda\sum_{i\in[K]}\|\mathcal{Q}_i(\hat{\mathbf{M}})\|_*\\ &\le\frac{1}{2}\|\mathcal{P}_\Omega(\mathbf{M}_0)-\mathcal{P}_\Omega(\mathbf{Y})\|_F^2+\lambda\sum_{i\in[K]}\|\mathcal{Q}_i(\mathbf{M}_0)\|_*\end{aligned}. \tag{24}$$

In addition, we have

$$\begin{aligned}&\|\mathcal{P}_\Omega(\hat{\mathbf{M}})-P_\Omega(\mathbf{Y})\|_F^2\\ &=\|\mathcal{P}_\Omega(\hat{\mathbf{M}})-\mathcal{P}_\Omega(\mathbf{M}_0)+\mathcal{P}_\Omega(\mathbf{M}_0)-\mathcal{P}_\Omega(\mathbf{Y})\|_F^2\\ &=\|\mathcal{P}_\Omega(\hat{\mathbf{M}})-\mathcal{P}_\Omega(\mathbf{M}_0)\|_F^2+\|\mathcal{P}_\Omega(\mathbf{Y})-\mathcal{P}_\Omega(\mathbf{M}_0)\|_F^2\\ &-2\left\langle\mathcal{P}_\Omega(\hat{\mathbf{M}})-\mathcal{P}_\Omega(\mathbf{M}_0),\mathcal{P}_\Omega(\mathbf{Y})-\mathcal{P}_\Omega(\mathbf{M}_0)\right\rangle.\end{aligned} \tag{25}$$

Combing (24) and (25), and assuming that $\|\mathbf{M}\|_Q := \sum_{i\in[K]}\|\mathcal{Q}_i(\mathbf{M})\|$ for brevity, the following equality holds

$$\begin{aligned}&\frac{1}{2}\|\mathcal{P}_\Omega(\hat{\mathbf{M}})-\mathcal{P}_\Omega(\mathbf{M}_0)\|_F^2\\ &\le\left\langle\mathcal{P}_\Omega(\hat{\mathbf{M}})-\mathcal{P}_\Omega(\mathbf{M}_0),\mathcal{P}_\Omega(\mathbf{Y})-\mathcal{P}_\Omega(\mathbf{M}_0)\right\rangle\\ &+\lambda\left(\|\mathbf{M}_0\|_Q-\|\hat{\mathbf{M}}\|_Q\right).\end{aligned} \tag{26}$$

Assume assume the error $\mathbf{E}=\hat{\mathbf{M}}-\mathbf{M}_0$. The partial reconstruction error can be bounded by the following inequalities

$$\begin{aligned}&\frac{1}{2}\|\mathcal{P}_\Omega(\mathbf{E})\|_F^2\\ &\le\langle\mathcal{P}_\Omega(\mathbf{E}),\mathcal{P}_\Omega(\mathbf{H})\rangle+\lambda\|\mathbf{E}\|_Q\\ &=\langle\mathbf{E},\mathcal{P}_\Omega(\mathbf{H})\rangle+\lambda\|\mathbf{E}\|_Q\\ &\le\|\mathbf{E}\|_*\|\mathcal{P}_\Omega(\mathbf{H})\|_2+\lambda\|\mathbf{E}\|_Q.\end{aligned} \tag{27}$$

In (27), the first inequality holds owing to the triangle inequality, and the second inequality holds because of the dual relationship between the matrix spectral and nuclear norm. By using the assumption in the theorem that $\lambda > \|\mathcal{P}_\Omega(\mathbf{H})\|_2/\sqrt{\min\{m_1,m_2\}}$, we have

$$\begin{aligned}&\frac{1}{2}\|P_\Omega(\mathbf{E})\|_F^2\\ &\le\lambda\min\{m_1,m_2\}\|\mathbf{E}\|_F+\lambda\sum_{i\in[K]}\|\mathcal{Q}_i(\mathbf{E})\|_*\\ &\le\lambda\min\{m_1,m_2\}\|\mathbf{E}\|_F+\\ &\qquad\lambda\sum_{i\in[K]}\sqrt{\min\{n_1^{(i)},n_2^{(i)}\}}\|\mathcal{Q}_i(\mathbf{E})\|_F\\ &\le\lambda\Big(\min\{m_1,m_2\}+\\ &\qquad\sum_{i\in[K]}\sqrt{\min\{n_1^{(i)},n_2^{(i)}\}}\|\left[\mathcal{Q}_i\right]_{<2>}\|_2\Big)\|\mathbf{E}\|_F\end{aligned}. \tag{28}$$

By using Theorem 1 in the paper, we have

$$\begin{aligned}&\|\mathbf{E}\|_F^2\le 4\|P_\Omega(\mathbf{E})\|_F^2\cdot\\ &\frac{cond(\widetilde{\mathcal{Q}})\cdot\min\{\prod n_1^{(i)},\prod n_2^{(i)}\}(p+\|\left[\widetilde{\mathcal{Q}}\right]_{<2>}\|_2)}{p(1-\|\mathbf{R}_{\mathbf{\Lambda}}\|_2)^2}.\end{aligned} \tag{29}$$

Combing (28) and (29), we can the final results. The proof is completed. ☐

3. Additional materials of the experiment

3.1. Influences by λ

In this section, we discuss how the completion performance is influenced by the tuning parameter λ in the experiment (see Fig. 2). Specifically, we test our method with same configuration given in the paper but choose different λ . As to the observation, we use "URC" as the missing pattern with different observation percentage $perc = 0.1, 0.3, 0.5, 0.7, 0.9$. As shown in Fig. 2, although the overfitting phenomena becomes serious if we choose too small λ, but the performance decrease slowly if we choose larger λ, specially when the observation percentage is small. It implies that the completion performance is robust to λ with a large variable range.

Figure 1. 12 images used as the dataset to evaluate the performance of the proposed method.

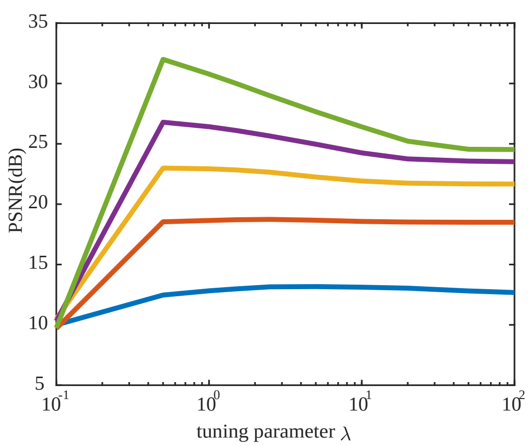

Figure 2. Performance for image inpainting with different . Different lines in the figure represent different observation percentage $perc = 0{:}1; 0{:}3; 0{:}5; 0{:}7; 0{:}9$. It can be seen that the performance with different is stable when is large enough (> 1). The overfitting phenomena becomes serious if we choose small , but the performance decrease slowly if we choose larger , specially when the observaton percentage is small.