# CONSAC: Robust Multi-Model Fitting by Conditional Sample Consensus Supplementary Material

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# Appendix

This appendix contains additional implementation details (Sec. A) which may be helpful for reproducing our results. Sec. B provides additional details about the datasets presented and used in our paper. In Sec. C, we show additional details complementing our experiments shown in the paper.

# **A. Implementation Details**

In Alg. 1, we present the CONSAC algorithm in another form, in addition to the description in Sec. 3 of the main paper, for ease of understanding. A list of all user definable parameters and the settings we used in our experiments is given in Tab. 1.

Algorithm 1 CONSAC

```
Input: \mathcal{Y} – set of observations, w – network parameters
Output: \hat{\mathcal{M}} – multi-hypothesis
\mathcal{P} \leftarrow \emptyset
for i \leftarrow 1 to P do
         \mathcal{M} \leftarrow \varnothing
         \mathbf{s} \gets \mathbf{0}
         for m \leftarrow 1 to M do
                  \mathcal{H} \leftarrow \emptyset
                  for s \leftarrow 1 to S do
                            Sample a minimal set of observations
                               \{\mathbf{y}_1,\ldots,\mathbf{y}_C\} with \mathbf{y} \sim p(\mathbf{y}|\mathbf{s};\mathbf{w}).
                            \mathbf{h} \leftarrow f_{\mathsf{S}}(\{\mathbf{y}_1, \dots, \mathbf{y}_C\})
                            \mathcal{H} \leftarrow \mathcal{H} \cup \{\mathbf{h}\}
                   end
                   \hat{\mathbf{h}} \leftarrow \arg \max_{\mathbf{h} \in \mathcal{H}} g_{\mathsf{s}}(\mathbf{h}, \mathcal{Y}, \mathcal{M})
                   \mathcal{M} \leftarrow \mathcal{M} \cup \{\hat{\mathbf{h}}\}
                  \mathbf{s} \leftarrow \max_{\hat{\mathbf{h}} \in \mathcal{M}} g_{\mathsf{y}}(\mathcal{Y}, \hat{\mathbf{h}})
         end
         \mathcal{P} \leftarrow \mathcal{P} \cup \{\mathcal{M}\}
end
\hat{\mathcal{M}} \leftarrow \operatorname{arg\,max}_{\mathcal{M} \in \mathcal{P}} g_{\mathsf{m}}(\mathcal{M}, \mathcal{Y})
```

			VP	homography
			estimation	estimation
training	learning rate		$10^{-4}$	$2 \cdot 10^{-6}$
	batch size	B	16	1
	batch normalisation		yes	no
	epochs		400	100
	inlier threshold	au	$10^{-3}$	$10^{-4}$
	IMR weight	$\kappa$	$10^{-2}$	$10^{-2}$
	observations per scene	$ \mathcal{Y} $	256	256
	number of instances	M	3	6
	single-instance samples	S	2	2
	multi-instance samples	P	2	2
	sample count	K	4	8
test	inlier threshold	au	$10^{-3}$	$10^{-4}$
	inlier thresh. (selection)	$\theta$		$3 \cdot 10^{-3}$
	inlier cutoff (selection)	Θ		6
	observations per scene	$ \mathcal{Y} $	variable	
	number of instances	M	6	6
	single-instance samples	S	32	100
	multi-instance samples	P	32	100
	EM iterations		10	10
	EM standard deviation	$\sigma$	$10^{-8}$	$10^{-9}$

Table 1: **User definable parameters** of CONSAC and the values we chose for our experiments on vanishing point estimation and homography estimation. We distinguish between values used during training and at test time. Mathematical symbols refer to the notation used either in the main paper or in this supplementary document.

# A.1. Neural Network

We use a neural network similar to PointNet [20] and based on [4, 28] for prediction of conditional sampling weights in CONSAC. Fig. 1 gives an overview of the architecture. Observations  $\mathbf{y} \in \mathcal{Y}$ , e.g. line segments or feature point correspondences, are stacked into a tensor of size  $D \times |\mathcal{Y}| \times 1$ . Note that the size of the tensor depends on the number of observations per scene. The dimensionality D of



Figure 1: **CONSAC neural network architecture** used for all experiments. We stack observations  $\mathcal{Y}$ , e.g. line segments or point correspondences (*not* an image), and state s into a tensor of size  $(D + 1) \times |\mathcal{Y}| \times 1$ , and feed it into the network. The network is composed of linear  $1 \times 1$  convolutional layers interleaved with instance normalisation [24], batch normalisation [9] and ReLU [6] layers which are arranged as residual blocks [7]. Only using  $1 \times 1$  convolutions, the network is order invariant w.r.t. observations  $\mathcal{Y}$ . The architecture is based on [4, 28].

each observation y is application specific. The current state s contains a scalar value for each observation and is hence a tensor of size  $1 \times |\mathcal{Y}| \times 1$ . The input of the network is a concatenation of observations  $\mathcal{Y}$  and state s, *i.e.* a tensor of size  $(D+1) \times |\mathcal{Y}| \times 1$ . After a single convolutional layer  $(1 \times 1,$ 128 channels) with ReLU [6] activation function, we apply six residual blocks [7]. Each residual block is composed of two series of convolutions  $(1 \times 1, 128$  channels), instance normalisation [24], batch normalisation [9] (optional) and ReLU activation. After another convolutional layer  $(1 \times 1,$ 1 channel) with sigmoid activation, we normalise the outputs so that the sum of sampling weights equals one. Only using  $1 \times 1$  convolutions, this network architecture is order invariant w.r.t. observations  $\mathcal{Y}$ . We implement the architecture using PyTorch [19] version 1.2.0.

#### A.1.1 Training Procedure

We train the neural network using the Adam [11] optimiser and utilise a cosine annealing learning rate schedule [13]. We clamp losses to a maximum absolute value of 0.3 in order to avoid divergence caused by large gradients resulting from large losses induced by poor hypothesis samples.

Number of Observations In order to keep the number of observations  $|\mathcal{Y}|$  constant throughout a batch, we sample a fixed number of observations from all observations of a scene during training. At test time, all observations are used.

**Pseudo Batches** During training, we sample *P* multihypotheses  $\mathcal{M}$ , from which we select the best multihypothesis  $\hat{\mathcal{M}}$  for each set of input observations  $\mathcal{Y}$  within a batch of size B. To approximate the expectation of our training loss (see Sec. 3.2 of the main paper), we repeat this process K times, to generate K samples of selected multi-hypotheses  $\mathcal{M}$  for each  $\mathcal{Y}$ . We generate each multihypothesis  $\mathcal{M}$  by sequentially sampling S single-instance hypotheses h and selecting the best one, conditioned on a state s. The state s varies between these innermost sampling loops, since we compute s based on all previously selected single instance hypotheses h of a multi-hypothesis  $\mathcal{M}$ . Because s is always fed into the network alongside observations  $\mathcal{Y}$ , we have to run  $P \cdot K$  forward passes for each batch. We can, however, parallelise these passes by collating observations and states into a tensor of size  $P \times K \times B \times (D+1) \times |\mathcal{Y}|$ . We reshape this tensor so that it has size  $B^* \times (D+1) \times |\mathcal{Y}|$  with an effective pseudo batch size  $B^* = P \cdot K \cdot B$ , in order to process all samples in parallel while using the same neural network weights for each pass within  $B^*$ . This means that sample sizes P and K are subject to both time and hardware memory constraints. We observe, however, that small sample sizes during training are sufficient in order to achieve good results using higher sample sizes at test time.

**Inlier Masking Regularisation** For self-supervised training, we multiply the inlier masking regularisation (IMR) term  $\ell_{im}$  (cf. Sec. 3.2.2 in the main paper) with a factor  $\kappa$  in order to regulate its influence compared to the regular self-supervision loss  $\ell_{self}$ , *i.e.*:

$$\ell = \ell_{\mathsf{self}} + \kappa \cdot \ell_{\mathsf{im}} \tag{1}$$

# **A.2. Scoring Functions**

In order to gauge whether an observation y is an inlier of model instance h, we utilise a soft inlier function adapted



Figure 2: Visualisation of the angle  $\alpha$  used for the vanishing point estimation residual function  $r(\mathbf{y}, \mathbf{h})$ .

from [3]:

$$g_{i}(\mathbf{y}, \mathbf{h}) = 1 - \sigma(\beta r(\mathbf{y}, \mathbf{h}) - \beta \tau), \qquad (2)$$

with inlier threshold  $\tau$ , softness parameter  $\beta = 5\tau^{-1}$ , a task-specific residual function  $r(\mathbf{y}, \mathbf{h})$  (see Sec. A.3 for details), and using the sigmoid function:

$$\sigma(x) = \frac{1}{1 + e^{-x}} \,. \tag{3}$$

The multi-instance scoring function  $g_m$ , which we use to select the best muti-hypothesis, *i.e.* hypothesis of multiple model instances  $\hat{\mathcal{M}} = {\{\hat{\mathbf{h}}_1, \dots, \hat{\mathbf{h}}_M\}}$ , from a pool of multi-instance hypotheses  $\mathcal{P} = {\mathcal{M}_1, \dots, \mathcal{M}_P}$ , counts the joint inliers of all models in a multi-instance:

$$g_{\mathsf{m}}(\mathcal{M}, \mathcal{Y}) = \sum_{\mathbf{y} \in \mathcal{Y}} \max_{\mathbf{h} \in \mathcal{M}} g_{\mathsf{i}}(\mathbf{y}, \mathbf{h}) \,. \tag{4}$$

The single instance scoring function  $g_s$ , which we use for selection of single model instances h given the set of previously selected model instances  $\mathcal{M}$ , is a special case of the multi-instance scoring function  $g_m$ :

$$g_{\mathsf{s}}(\mathbf{h}, \mathcal{Y}, \mathcal{M}) = g_{\mathsf{m}}(\mathcal{M} \cup \{\mathbf{h}\}, \mathcal{Y}) \,. \tag{5}$$

#### A.3. Residual Functions

**Line Fitting** For the line fitting problem, each observation is a 2D point in homogeneous coordinates  $\mathbf{y} = (x \, y \, 1)^{\mathsf{T}}$ , and each model is a line in homogeneous coordinates  $\mathbf{h} = \frac{1}{\|(n_1 \, n_2)\|} (n_1 \, n_2 \, d)^{\mathsf{T}}$ . We use the absolute point-to-line distance as the residual:

$$r(\mathbf{y}, \mathbf{h}) = |\mathbf{y}^{\mathsf{T}} \mathbf{h}| \,. \tag{6}$$

**Vanishing Point Estimation** Observations  $\mathbf{y}$  are given by line segments with start point  $\mathbf{p}_1 = (x_1 y_1 1)^T$  and end point  $\mathbf{p}_2 = (x_2 y_2 1)^T$ , and models are vanishing points  $\mathbf{h} = (x y 1)^T$ . For each line segment  $\mathbf{y}$ , we compute the corresponding line  $\mathbf{l}_{\mathbf{y}} = \mathbf{p}_1 \times \mathbf{p}_2$  and the centre point  $\mathbf{p}_c = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2)$ . As visualised by Fig. 2, we define the residual via the cosine of the angle  $\alpha$  between  $\mathbf{l}_{\mathbf{y}}$  and the constrained line  $\mathbf{l}_c = \mathbf{h} \times \mathbf{p}_c$ , *i.e.* the line connecting the vanishing point with the centre of the line segment:

$$r(\mathbf{y}, \mathbf{h}) = 1 - \cos \alpha = 1 - \frac{|\mathbf{l}_{\mathbf{y}, 1:2}^{\mathsf{T}} \mathbf{l}_{c, 1:2}|}{\|\mathbf{l}_{\mathbf{y}, 1:2}\| \|\mathbf{l}_{c, 1:2}\|}.$$
 (7)



Figure 3: Line fitting: we show examples from the synthetic dataset we used to train CONSAC on the line fitting problem. Each scene consists of four lines placed at random, with points sampled along them, perturbed by Gaussian noise and outliers. Cyan = ground truth lines.

**Homography Estimation** Observations  $\mathbf{y}$  are given by point correspondences  $\mathbf{p}_1 = (x_1 y_1 1)^T$  and  $\mathbf{p}_2 = (x_2 y_2 1)^T$ , and models are plane homographies  $\mathbf{h} = \mathbf{H}^{3\times3}$  which shall map  $\mathbf{p}_1$  to  $\mathbf{p}_2$ . We compute the symmetric squared transfer error:

$$r(\mathbf{y}, \mathbf{h}) = \|\mathbf{p}_1 - \mathbf{p}_1'\|^2 + \|\mathbf{p}_2 - \mathbf{p}_2'\|^2,$$
 (8)

with  $\mathbf{p}_2' \propto \mathbf{H}\mathbf{p}_1$  and  $\mathbf{p}_1' \propto \mathbf{H}^{-1}\mathbf{p}_2$ .

#### **B.** Dataset Details and Analyses

#### **B.1.** Line Fitting

For training CONSAC on the line fitting problem, we generated a synthetic dataset of 10000 scenes. Each scene consists of four lines placed at random within a  $\{0, 1\} \times \{0, 1\}$  square. For each line, we randomly define a line segment with a length of 30 - 100% of the maximum length of the line within the square. Then, we randomly sample 40 - 100 points along the line segment and perturb them by Gaussian noise  $\mathcal{N} \sim (0, \sigma^2)$ , with  $\sigma \in (0.007, 0.008)$  sampled uniformly. Finally, we add 40 - 60% outliers via random uniform sampling. Fig. 3 shows a few examples from this dataset.

For evaluation, we use the synthetic *stair4*, *star5* and *star11* scenes from [23], which were also used by [2]. As Fig. 4 shows, each scene consists of 2D points forming four,



Figure 4: Line fitting: we use the synthetic *stair4* (left), *star5* (middle) and *star11* (right) scenes from [23], which were also used by [2], in our experiments.

five or eleven line segments. The points are perturbed by Gaussian noise ( $\sigma = 0.0075$ ) and contain 50 - 60% outliers.

## **B.2.** Vanishing Point Estimation

**NYU-VP** In Fig. 5 (top), we show a histogram of the number of vanishing points per image in our new NYU-VP dataset. In addition, we show a few example images for different numbers of vanishing points. NYU-VP solely consists of indoor scenes.

**YUD+** In Fig. 5 (bottom), we show a histogram of the number of vanishing points per image in our new YUD+ dataset extension. By comparison, the original YUD [5] contains exactly three vanishing point labels for each of the 102 scenes. YUD contains both indoor and outdoor scenes.

#### **B.3.** Homography Estimation

For self-supervised training for the task of homography estimation, we use SIFT [14] feature correspondences extracted from the structure-from-motion scenes of [8, 22, 27]. Specifically, we used the outdoor scenes Buckingham, Notredame, Sacre Coeur, St. Peter's and Reichstag from [8], Fountain and Herzjesu from [22], and 16 indoor scenes from SUN3D [27]. We use the SIFT correspondences computed and provided by Brachmann and Rother [4], and discard suspected gross outliers with a matching score ratio greater than 0.9. As this dataset is imbalanced in the sense that some scenes contain significantly more image pairs than others - for St. Peter's we have 9999 image pairs, but for *Reichstag* we only have 56 – we apply a rebalancing sampling during training: instead of sampling image pairs uniformly at random, we uniformly sample one of the scenes first, and then we sample an image pair from within this scene. This way, each scene is sampled during training at the same rate. During training, we augment the data by randomly flipping all points horizontally or vertically, and shifting and scaling them along both axes independently by up to  $\pm 10\%$  of the image width or height.

# **C. Additional Experimental Results**

# C.1. Line Fitting

Sampling Efficiency In order to analyse the efficiency of the conditional sampling of CONSAC compared to a Sequential RANSAC, we computed the F1 score w.r.t. estimated model instances on the stair4, star5 and star11 line fitting scenes from [23] for various combinations of singleinstance samples S and multi-instance samples P. As Fig. 6 shows, CONSAC achieves higher F1 scores with fewer hypotheses on stair4 and star5. As we trained CONSAC on data containing only four line segments, while star5 depicts five lines, this demonstrates that CONSAC is able to generalise beyond the number of model instances it has been trained for. On star11, which contains eleven lines, it does not perform as well, suggesting that this generalisation may not extend arbitrarily beyond numbers of instances CON-SAC has been trained on. In practice, however, our realworld experiments on homography estimation and vanishing point estimation show that it is sufficient to simply train CONSAC on a reasonably large number of instances in order to achieve very good results.

**Sampling Weights Throughout Training** We looked at the development of sampling weights as neural network training progresses, using *star5* as an example. As Fig. 7 shows, sampling weights are randomly – but not uniformly – distributed throughout all instance sampling steps before training has begun. At 1000 iterations, we observe that the neural network starts to focus on different regions of the data throughout the instance sampling steps. From thereon, this focus gets smaller and more accurate as training progresses. After 100000 iterations, the network has learned to focus on points mostly belonging to just one or two true line segments.

#### C.2. Vanishing Point Estimation

**Evaluation Metric** We denote ground truth VPs of an image by  $\mathcal{V} = \{\mathbf{v}_1, \dots, \mathbf{v}_M\}$  and estimates by  $\hat{\mathcal{V}} = \{\hat{\mathbf{v}}_1, \dots, \hat{\mathbf{v}}_N\}$ . We compute the error between two particular VP instances via the angle  $e(\mathbf{v}, \hat{\mathbf{v}})$  between their corresponding directions in 3D using camera intrinsics K:

$$e(\mathbf{v}, \hat{\mathbf{v}}) = \arccos \frac{\left| \left( \mathbf{K}^{-1} \mathbf{v} \right)^{\mathsf{T}} \mathbf{K}^{-1} \hat{\mathbf{v}} \right|}{\left| \left| \mathbf{K}^{-1} \mathbf{v} \right| \left| \cdot \left| \left| \mathbf{K}^{-1} \hat{\mathbf{v}} \right| \right| \right|}.$$
 (9)

We use this error to define the cost matrix C:  $C_{ij} = e(\mathbf{v}_i, \hat{\mathbf{v}}_j)$  in Sec. 5.2.1 of the main paper.

**Results** For vanishing point estimation, we provide recall curves for errors up to  $10^{\circ}$  in Fig. 8 for our new NYU-VP dataset, for our YUD+ dataset extension, as well as the original YUD [5]. We compare CONSAC with the robust



Figure 5: **Vanishing points per scene:** Histograms showing the numbers of vanishing point instances per image for our new NYU-VP dataset (top) and our YUD+ dataset extension (bottom), in addition to a few example images. We illustrate the vanishing points present in each example via colour-coded line segments.



Figure 6: Line fitting: Using the *stair4* (top), *star5* (middle) and *star11* (bottom) line fitting scenes from [23], we compute the F1 scores for various combinations of single-instance samples S (abscissa) and multi-instance samples P (ordinate) and plot them as a heat map. We compare CON-SAC (left) with Sequential RANSAC (right). Magenta = low, cyan = high F1 score.



Figure 7: Line fitting: We show how the sampling weights at each instance sampling step develop as neural network training progresses, using the *star5* line fitting scene from [23] as an example. Each row depicts the sampling weights used to sample the eventually selected best multi-hypothesis  $\hat{\mathcal{M}}$ . Top to bottom: training iterations 0 - 100000. Left to right: model instance sampling steps 1 - 5. Sampling weights: Blue = low, white = high.

multi-model fitting approaches T-Linkage [15], Sequential RANSAC [25], Multi-X [1], RPA [16] and RansaCov [17], as well as the task-specific vanishing point estimators of Zhai et al. [29], Simon et al. [21] and Kluger et al. [12]. We selected the result with the median area under the curve (AUC) of five runs for each method. CONSAC does not find more vanishing points within the 10° range than state-of-the-art vanishing point estimators, indicated by similar



Figure 8: Vanishing point estimation: Recall curves for errors up to 10° for all methods which we considered in our experiments. We selected the result with the *median* AUC out of five runs for each method. Robust estimators are represented with solid lines, task-specific VP estimators with dashed lines. **Top:** Results on our new NYU-VP dataset. **Middle:** Results on our new YUD+ dataset extension. **Bottom:** Results on the original YUD [5].

	no. of	CONSACS	MCT [18]	Sequential
	planes	CONSAC-S		RANSAC
barrsmith	2	2.07	11.29	12.95
bonhall	6	16.63	29.29	20.43
bonython	1	0.00	2.42	0.00
elderhalla	2	4.39	21.41	16.36
elderhallb	3	11.69	20.31	18.67
hartley	2	2.94	15.19	9.38
johnsona	4	14.48	18.77	28.04
johnsonb	6	19.17	33.87	27.46
ladysymon	2	2.95	16.46	3.80
library	2	1.21	14.79	11.35
napiera	2	2.72	21.32	11.66
napierb	3	6.72	16.83	21.24
neem	3	2.74	14.36	14.44
nese	2	0.00	12.83	0.47
oldclass.	2	1.69	15.20	1.32
physics	1	0.00	3.21	0.00
sene	2	0.40	4.80	2.00
unihouse	5	8.84	34.10	10.69
unionhouse	1	0.30	1.51	1.51
averag	e	5.21	16.21	11.14

Table 2: **Homography estimation:** Misclassification errors (in %, average over five runs) for all homography estimation scenes of AdelaideRMF [26].

recall values at  $10^{\circ}$ . However, it does estimate vanishing points more accurately on NYU-VP and YUD+, as the high recall values for low errors (<  $4^{\circ}$ ) show. On YUD [5], CONSAC achieves similar or slightly worse recall. Compared to other robust estimators, however, CONSAC performs better than all methods on all datasets across the whole error range. In Fig. 10, we show additional qualitative results from the NYU-VP dataset, and in Fig. 11, we show additional qualitative results from the YUD+ dataset.

## C.3. Homography Estimation

We provide results computed on AdelaideRMF [26] for all scenes seperately. In Fig. 9, we compare CONSAC-S – *i.e.* CONSAC trained in a self-supervised manner – to Progressive-X [2], Multi-X [1], PEARL [10], RPA [16], RansaCov [17] and T-Linkage [15]. We adapted the graph directly from [2]. CONSAC-S achieves state-of-the-art performance on 13 of 19 scenes. Tab. 2 compares CONSAC-S with MCT [18] and Sequential RANSAC. We computed results for MCT using code provided by the authors, and used our own implementation for Sequential RANSAC, since no results obtained using the same evaluation protocol (average over five runs) were available in previous works. In Fig. 12, we show additional qualitative results from the AdelaideRMF [26] dataset.



Figure 9: **Homography estimation:** Misclassification errors (in %, average over five runs) for all homography estimation scenes of AdelaideRMF [26]. Graph adapted from [2].

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Figure 10: Three qualitative examples for VP estimation with CONSAC on our NYU-VP dataset. For each example we show the original image, extracted line segments, line assignments to ground truth VPs, and to final estimates in the first row. In the second and third row, we visualise the generation of the multi-hypothesis  $\hat{\mathcal{M}}$  eventually selected by CONSAC. The second row shows the sampling weights per line segment which were used to generate each hypothesis  $\hat{\mathbf{h}} \in \hat{\mathcal{M}}$ . The third row shows the resulting state s. (Blue = low, white = high.) Between rows two and three, we indicate the individual VP errors. The checkerboard pattern and "—" entries indicate instances for which no ground truth is available. The last example is a failure case, where only two out of four VPs were correctly estimated.



Figure 11: Three qualitative examples for VP estimation with CONSAC on the YUD+ dataset. For each example we show the original image, extracted line segments, line assignments to ground truth VPs, and to final estimates in the first row. In the second and third row, we visualise the generation of the multi-hypothesis  $\hat{\mathcal{M}}$  eventually selected by CONSAC. The second row shows the sampling weights per line segment which were used to generate each hypothesis  $\hat{\mathbf{h}} \in \hat{\mathcal{M}}$ . The third row shows the resulting state s. (Blue = low, white = high.) Between rows two and three, we indicate the individual VP errors. The checkerboard pattern and "—" entries indicate instances for which no ground truth is available. The last example is a failure case, where only two out of four VPs were correctly estimated.



Figure 12: Three qualitative examples for homography estimation with CONSAC-S on the AdelaideRMF [26] dataset. For each example we show the original images, points with ground truth labels, final estimates, and the misclassification error (ME) in the first row. In the second and third row, we visualise the generation of the multi-hypothesis  $\hat{\mathcal{M}}$  eventually selected by CONSAC. The second row shows the sampling weights per point correspondence which were used to generate each hypothesis  $\hat{\mathbf{h}} \in \hat{\mathcal{M}}$ . The third row shows the resulting state s. (Blue = low, white = high.) The checkerboard pattern indicates instances which were discarded by CONSAC in the final instance selection step.