

# Softmax Splatting for Video Frame Interpolation

## — Supplementary Material —

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Please note that the section numbering within this supplementary document has manually been adjusted to match the relevant sections in the main paper.

### 3.1. Forward Warping via Softmax Splatting

Our proposed softmax splatting shares resemblance to the softmax function and is hence invariant to translations  $\beta$  with respect to  $Z$  which can easily be shown as follows.

$$\text{let } \mathbf{u} = \mathbf{p} - (\mathbf{q} + F_{0 \rightarrow t}[\mathbf{q}]) \quad (1)$$

$$b(\mathbf{u}) = \max(0, 1 - |\mathbf{u}_x|) \cdot \max(0, 1 - |\mathbf{u}_y|) \quad (2)$$

$$I_t^\sigma[\mathbf{p}] = \frac{\sum_{\forall \mathbf{q} \in I_0} \exp(Z[\mathbf{q}] + \beta) \cdot b(\mathbf{u}) \cdot I_0[\mathbf{q}]}{\sum_{\forall \mathbf{q} \in I_0} \exp(Z[\mathbf{q}] + \beta) \cdot b(\mathbf{u})} \quad (3)$$

$$= \frac{\sum_{\forall \mathbf{q} \in I_0} \exp(Z[\mathbf{q}]) \cdot \exp(\beta) \cdot b(\mathbf{u}) \cdot I_0[\mathbf{q}]}{\sum_{\forall \mathbf{q} \in I_0} \exp(Z[\mathbf{q}]) \cdot \exp(\beta) \cdot b(\mathbf{u})} \quad (4)$$

$$= \frac{\sum_{\forall \mathbf{q} \in I_0} \exp(Z[\mathbf{q}]) \cdot b(\mathbf{u}) \cdot I_0[\mathbf{q}]}{\sum_{\forall \mathbf{q} \in I_0} \exp(Z[\mathbf{q}]) \cdot b(\mathbf{u})} \quad (5)$$

This property is particularly important when mapping multiple pixels to the same location. For example, if  $Z$  represents depth then moving then changing  $Z' = Z + \beta$  should not affect the way the mapping ambiguity is resolved.

### 4.3. Quantitative Evaluation

As shown in Table 1 as well as Table 2 and summarized in Table 3, our proposed approach ranks first in the relevant Middlebury benchmark for optical flow [1]. Specifically, our approach achieves an average rank of 2.5 in terms of the interpolation error, compared to 5.4 achieved by the second highest ranked method. Similarly, in terms of the

normalized interpolation error, the average rank of our approach and the second highest ranked method are 2.0 and 10.8, respectively. Note that the second-best method differs, the second best method in terms of the interpolation error has not yet been published while DAIN [2] is the second best method in terms of the normalized interpolation error. These results demonstrate that our images synthesis approach, empowered by softmax plating, is able to consistently produce high-quality interpolation results.

Since common datasets to evaluate frame interpolation are subject to a relatively low resolution, we additionally incorporated 4K video clips from Xiph. These video clips are commonly used to assess video compression and contain challenging scenarios, such as significant bokeh that mixes foreground and background regions or fine-details that are reflective and subject to sudden brightness changes. For the purpose of evaluating frame interpolation techniques, we selected the eight 4K video clips with the most amount of inter-frame motion. Please see our main paper for more details where we state the average metrics. For completeness, we show the per-clip metrics in Tables 4–6.

### 4.4. Qualitative Evaluation

Since videos are at the heart of our approach, we provide a qualitative comparison in the supplementary video. We additionally provide still results on challenging examples from DAVIS [7] in Figures 1–3. These support the findings of the quantitative evaluation and show difficult scenarios where our proposed approach produces high-quality results whereas competing techniques are subject to visual artifacts. Please note that for methods with models trained using different losses, we only show the results for the models that focus on perceptual quality.



	Boxing		Crosswalk		Driving		Market-1		Market-2		RitualDance		Square		Tango	
	2K	"4K"	2K	"4K"	2K	"4K"	2K	"4K"	2K	"4K"	2K	"4K"	2K	"4K"	2K	"4K"
	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR
	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
SepConv - $\mathcal{L}_1$ [6]	36.75	33.35	36.34	32.79	34.95	33.32	32.03	31.21	36.49	34.62	28.15	23.12	38.63	36.16	34.85	31.90
SepConv - $\mathcal{L}_F$ [6]	36.54	33.15	35.69	32.10	34.84	33.24	31.69	30.68	36.26	34.27	27.95	23.06	38.41	35.84	34.43	31.08
ToFlow [8]	36.75	33.52	33.54	31.42	34.84	33.38	30.87	29.41	34.24	30.30	28.11	22.61	38.87	36.59	34.24	28.67
CyclicGen [4]	36.51	32.95	33.73	31.37	34.74	33.46	30.02	28.69	29.89	27.91	28.21	22.95	37.44	35.13	33.45	29.60
CtxSyn - $\mathcal{L}_{Lap}$ [5]	37.41	33.55	38.14	34.03	34.92	32.71	32.93	31.97	38.33	37.18	28.47	23.08	39.35	37.09	36.16	34.21
CtxSyn - $\mathcal{L}_F$ [5]	36.68	32.88	37.40	33.01	34.56	32.45	32.20	31.10	37.94	36.62	28.24	23.10	38.87	36.61	35.36	33.10
DAIN [2]	37.74	34.75	38.81	35.90	35.14	33.60	33.06	31.99	38.03	36.49	29.16	23.91	39.50	37.00	36.14	34.28
Ours - $\mathcal{L}_{Lap}$	<u>38.48</u>	<u>35.40</u>	38.71	33.69	<u>35.68</u>	<u>33.82</u>	<u>33.34</u>	<u>32.16</u>	<u>39.61</u>	<u>37.86</u>	<u>29.20</u>	<u>23.67</u>	<u>41.23</u>	<u>38.00</u>	<u>36.73</u>	<u>34.16</u>
Ours - $\mathcal{L}_F$	37.45	34.08	37.15	32.36	35.15	33.15	32.16	30.95	39.06	36.64	28.72	23.29	40.65	37.04	35.57	32.51

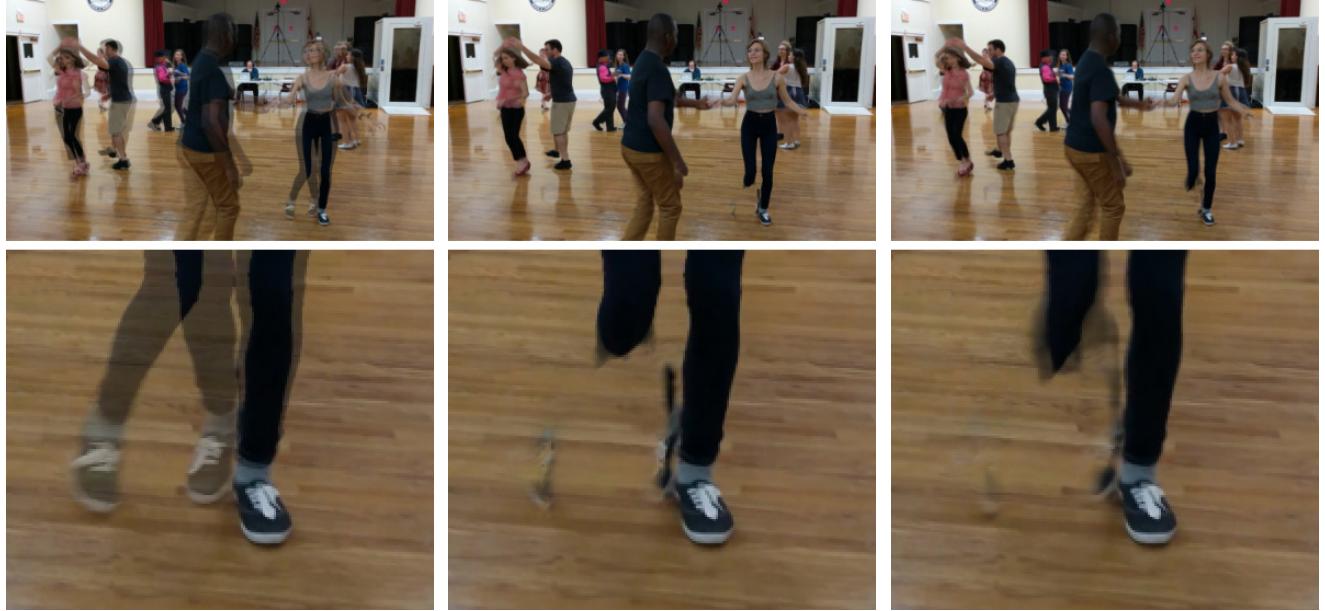
Table 4: Quantitative comparison in terms of PSNR on the eight 4K clips from Xiph with the most inter-frame motion.

	Boxing		Crosswalk		Driving		Market-1		Market-2		RitualDance		Square		Tango	
	2K	"4K"														
	SSIM															
	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
SepConv - $\mathcal{L}_1$ [6]	0.946	0.903	0.937	0.890	0.939	0.916	0.823	0.752	0.963	0.925	0.926	0.858	0.978	0.952	0.919	0.844
SepConv - $\mathcal{L}_F$ [6]	0.940	0.886	0.926	0.871	0.936	0.912	0.805	0.718	0.960	0.914	0.917	0.846	0.976	0.947	0.906	0.812
ToFlow [8]	0.947	0.900	0.925	0.874	0.935	0.917	0.805	0.703	0.947	0.834	0.928	0.852	0.978	0.953	0.915	0.816
CyclicGen [4]	0.946	0.894	0.924	0.867	0.930	0.916	0.764	0.658	0.847	0.749	0.923	0.850	0.972	0.938	0.904	0.815
CtxSyn - $\mathcal{L}_{Lap}$ [5]	0.952	0.908	0.944	0.898	0.940	0.917	0.841	0.771	0.972	0.944	0.932	0.866	0.980	0.956	0.928	0.863
CtxSyn - $\mathcal{L}_F$ [5]	0.934	0.862	0.926	0.858	0.934	0.905	0.804	0.715	0.968	0.931	0.919	0.842	0.977	0.947	0.904	0.796
DAIN [2]	0.956	0.915	0.948	<u>0.905</u>	0.942	0.921	0.846	0.770	0.972	0.945	0.939	0.879	0.981	0.958	0.932	0.866
Ours - $\mathcal{L}_{Lap}$	<u>0.958</u>	<u>0.922</u>	<u>0.951</u>	<u>0.905</u>	<u>0.947</u>	<u>0.924</u>	<u>0.853</u>	<u>0.782</u>	<u>0.975</u>	<u>0.949</u>	<u>0.944</u>	<u>0.888</u>	<u>0.984</u>	<u>0.962</u>	<u>0.936</u>	<u>0.872</u>
Ours - $\mathcal{L}_F$	0.936	0.874	0.921	0.850	0.936	0.907	0.797	0.706	0.970	0.925	0.923	0.848	0.980	0.946	0.902	0.793

Table 5: Quantitative comparison in terms of SSIM on the eight 4K clips from Xiph with the most inter-frame motion.

	Boxing		Crosswalk		Driving		Market-1		Market-2		RitualDance		Square		Tango	
	2K	"4K"														
	LPIPS															
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
SepConv - $\mathcal{L}_1$ [6]	0.037	0.133	0.094	0.208	0.038	0.061	0.156	0.331	0.026	0.106	0.100	0.239	0.012	0.040	0.071	0.234
SepConv - $\mathcal{L}_F$ [6]	0.021	0.057	0.046	0.111	0.037	0.055	0.096	0.187	0.017	0.062	0.063	0.171	0.010	0.027	0.037	0.109
ToFlow [8]	0.035	0.088	0.088	0.152	0.047	0.062	0.139	0.238	0.027	0.118	0.075	0.189	0.013	0.038	0.063	0.171
CyclicGen [4]	0.039	0.090	0.085	0.142	0.068	0.067	0.185	0.271	0.108	0.169	0.077	0.177	0.029	0.060	0.073	0.159
CtxSyn - $\mathcal{L}_{Lap}$ [5]	0.053	0.172	0.103	0.215	0.040	0.074	0.185	0.354	0.017	0.073	0.086	0.223	0.012	0.045	0.085	0.244
CtxSyn - $\mathcal{L}_F$ [5]	0.021	0.056	0.031	0.082	0.038	0.058	0.078	0.154	0.013	0.036	0.061	0.174	0.009	0.022	0.031	0.066
DAIN [2]	0.063	0.161	0.139	0.214	0.045	0.073	0.200	0.329	0.020	0.088	0.091	0.200	0.013	0.051	0.105	0.243
Ours - $\mathcal{L}_{Lap}$	0.078	0.226	0.177	0.322	0.053	0.095	0.273	0.455	0.022	0.118	0.105	0.238	0.013	0.067	0.131	0.349
Ours - $\mathcal{L}_F$	<u>0.016</u>	<u>0.043</u>	<u>0.026</u>	<u>0.079</u>	<u>0.033</u>	<u>0.053</u>	<u>0.074</u>	<u>0.148</u>	<u>0.010</u>	<u>0.034</u>	<u>0.045</u>	<u>0.136</u>	<u>0.006</u>	<u>0.018</u>	<u>0.024</u>	<u>0.059</u>

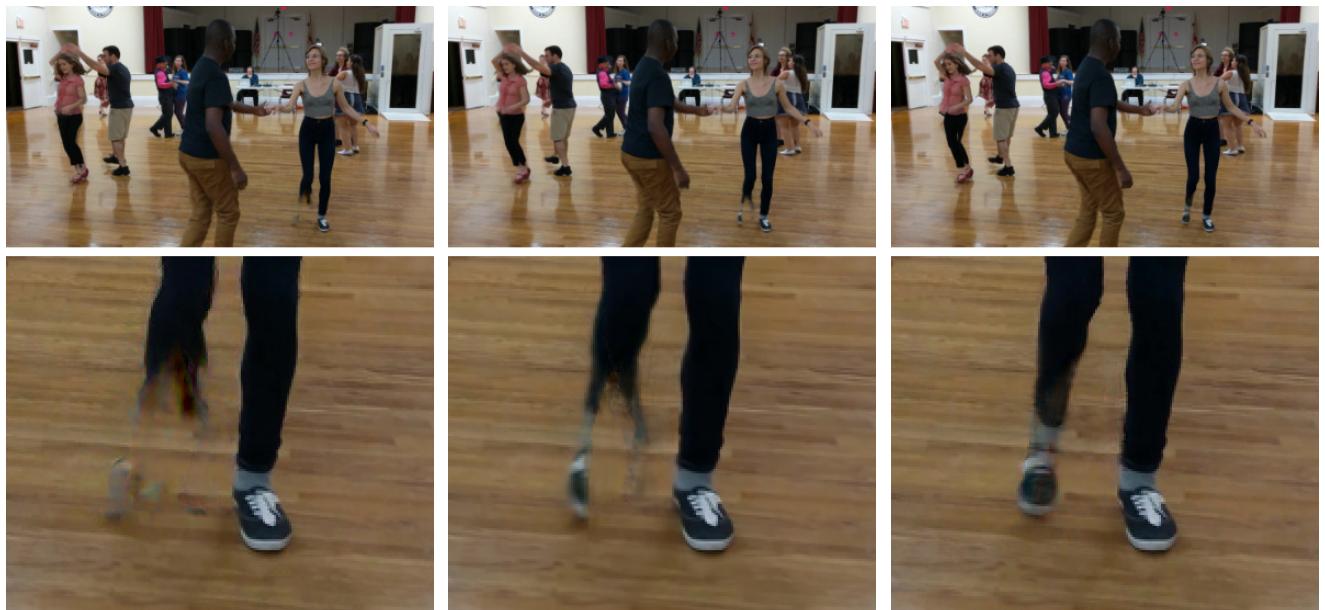
Table 6: Quantitative comparison in terms of LPIPS on the eight 4K clips from Xiph with the most inter-frame motion.



Overlaid Input

ToFlow [8]

CyclicGen [4]

CtxSyn -  $\mathcal{L}_F$  [5]

DAIN [2]

Ours -  $\mathcal{L}_F$ 

Figure 1: Qualitative comparison, please also consider our supplementary video demo to see this example in motion.

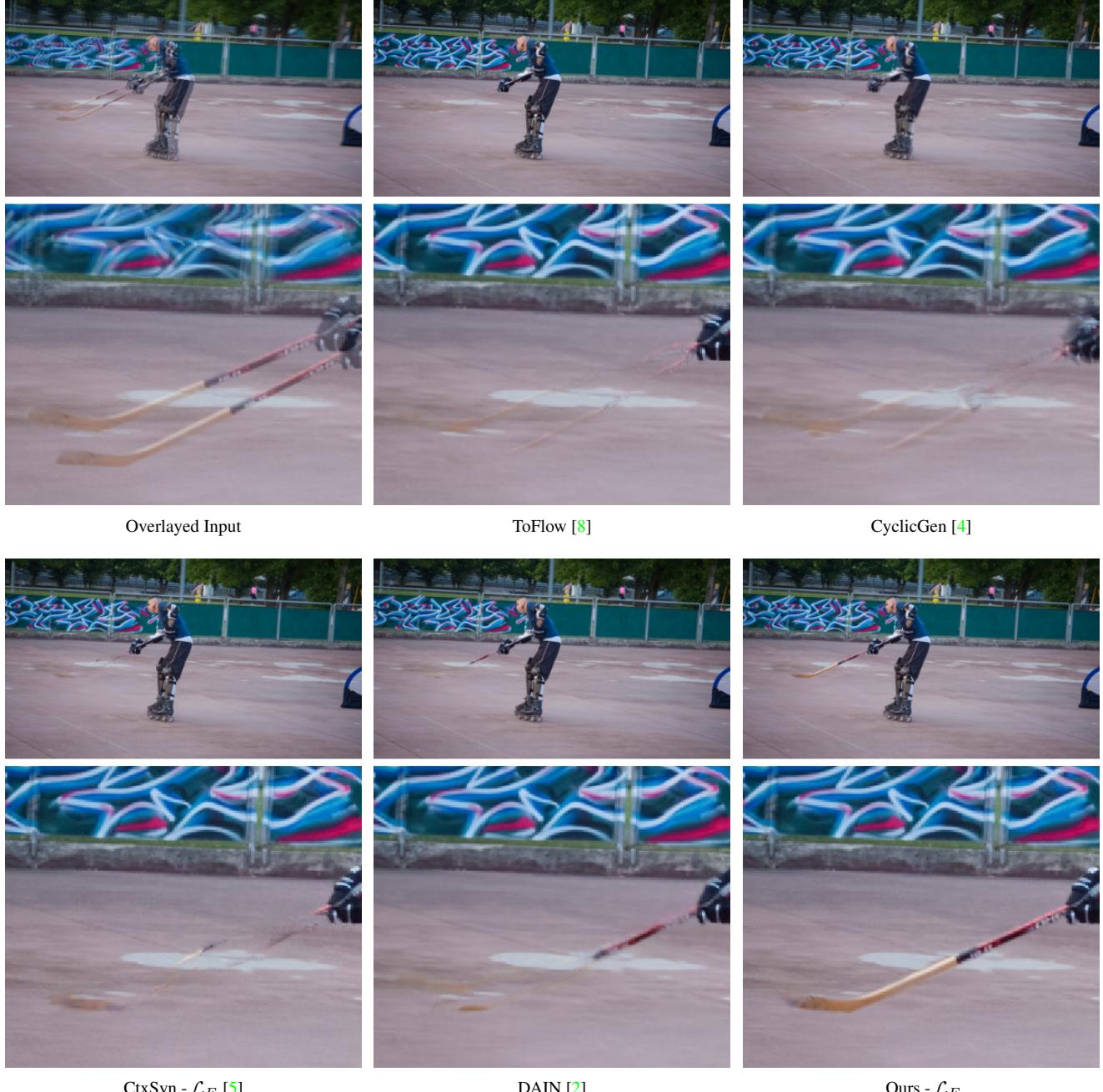


Figure 2: Qualitative comparison, please also consider our supplementary video demo to see this example in motion.

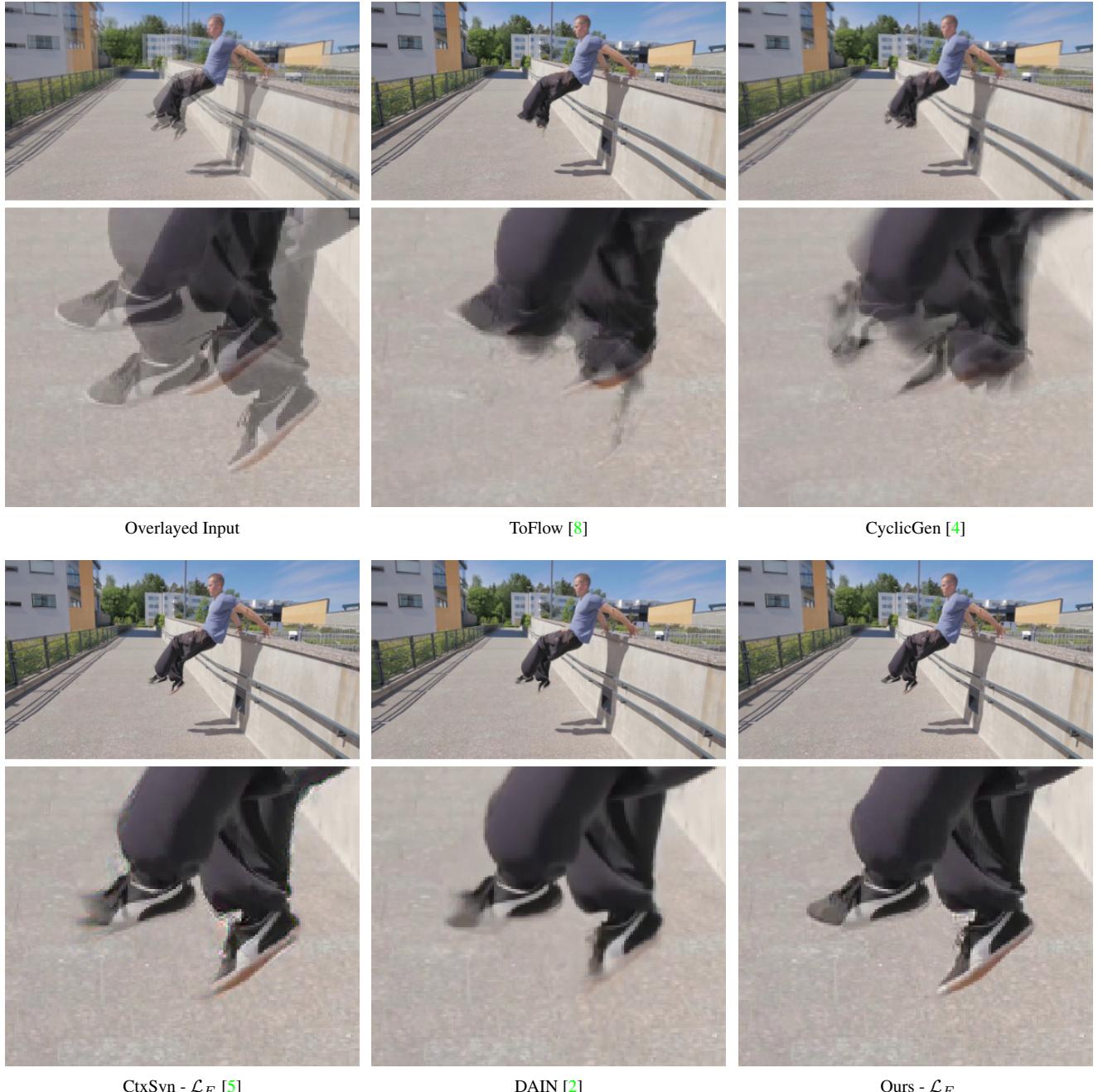


Figure 3: Qualitative comparison, please also consider our supplementary video demo to see this example in motion.

## References

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