

# Deblurring Natural Image Using Super-Gaussian Fields

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**Abstract.** Blind image deblurring is a challenging problem due to its ill-posed nature, of which the success is closely related to a proper image prior. Although a large number of sparsity-based priors, such as the sparse gradient prior, have been successfully applied for blind image deblurring, they inherently suffer from several drawbacks, limiting their applications. Existing sparsity-based priors are usually rooted in modeling the response of images to some specific filters (e.g., image gradients), which are insufficient to capture the complicated image structures. Moreover, the traditional sparse priors or regularizations model the filter response (e.g., image gradients) independently and thus fail to depict the long-range correlation among them. To address the above issues, we present a novel image prior for image deblurring based on a Super-Gaussian field model with adaptive structures. Instead of modeling the response of the fixed short-term filters, the proposed Super-Gaussian fields capture the complicated structures in natural images by integrating potentials on all cliques (e.g., centring at each pixel) into a joint probabilistic distribution. Considering that the fixed filters in different scales are impractical for the coarse-to-fine framework of image deblurring, we define each potential function as a super-Gaussian distribution. Through this definition, the partition function, the curse for traditional MRFs, can be theoretically ignored, and all model parameters of the proposed Super-Gaussian fields can be data-adaptively learned and inferred from the blurred observation with a variational framework. Extensive experiments on both blind deblurring and non-blind deblurring demonstrate the effectiveness of the proposed method.

## 1 Introduction

Image deblurring involves the estimation of a sharp image when given a blurred observation. Generally, this problem can be formalized as follows:

$$\mathbf{y} = \mathbf{k} \otimes \mathbf{x} + \mathbf{n}, \quad (1)$$

where the blurred image  $\mathbf{y}$  is generated by convolving the latent image  $\mathbf{x}$  with a blur kernel  $\mathbf{k}$ ,  $\otimes$  denotes the convolution operator, and  $\mathbf{n}$  denotes the noise corruption. When the kernel  $\mathbf{k}$  is unknown, the problem is termed blind image deblurring (BID), and conversely, the non-blind image deblurring (NBID). It has been shown that both of these two problems are highly ill-posed. Thus, to obtain meaningful solutions, appropriate priors on latent image  $\mathbf{x}$  is necessary.

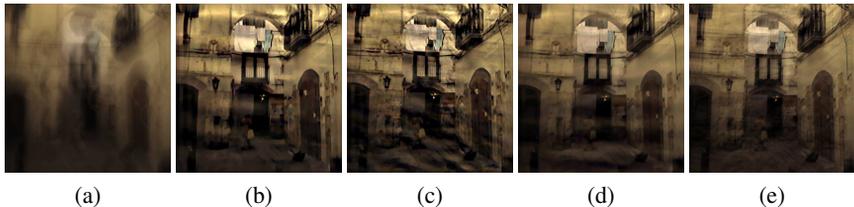


Fig. 1: Deblurring results (and PSNR values) of a challenging example from [1]. From left to right: Blurred image (22.386), Cho and Lee [2] (22.611), Xu and Jia [3] (22.782), Pan et al. [4] (26.259), Ours (27.5427).

It has been found that the statistics of image response to specific filters can well depict the underlying structures as priors. One of the most representative examples is the sparse character in natural gradient domain, which conveys the response of an image to some basic filters, e.g.,  $[-1, 1]$ , and represents the locally spatial coherence of natural images. Inspired by this, extensive prevailing methods [5–13] have developed various sparse priors or regularizations to emphasize the sparsity on the latent images in gradient domain for deblurring. A brief review will be introduced in Section 2. Although these methods have made such remarkable progress, their performance still need to be improved to satisfy the requirement of real applications, especially when handling some challenging cases. This is caused by two aspects of inherent limitations in these prior models. 1) Image gradient only records the response of the image to several basic filters, which are insufficient to capture structures more complicated than local coherence. In general, those complicated structures often benefit recovering more details in the deblurred results. 2) Most of existing sparse priors (e.g., Laplace prior) or regularizations (e.g.,  $\ell_p$  norm,  $0 \leq p \leq 1$ ) model the gradient on each pixel independently, and thus fails to depict the long-range correlation among pixels, such as non-local similarity or even more complex correlation. Failure to consider such kind of correlation often results in some unnatural artifacts in the deblurred results, as shown in Fig. 1.

To simultaneously address these two problems, we propose to establish an appropriate image prior with the high order Markov random fields (MRFs) model. This is motivated by the two advantages of MRFs. First, MRFs can learn an ensemble of filters to determine the statistic distribution of images, which is sufficient to capture the complicated image structures. Second, MRFs integrates the potential defined on each clique (i.e., centering at each pixel) into a probabilistic joint distribution, which is able to capture the long-range correlation among pixels potentially.

However, traditional MRFs models (e.g., Fields of experts (FoE) [14]) cannot be directly embedded into the commonly used BID framework which estimates the blur kernel in a coarse-to-fine scheme. Due to the intractable partition function, those models often learn parameters from an external image database, which results in the response of the latent image to those learned filters distributing differently across various scales and thus failing to be well depicted by the same MRFs model. For example, the learned filters lead to a heavy-tailed sparse distribution on the fine-scale while a Gaussian dis-

tribution on the coarse scale. That is one of the inherent reasons for why MRFs model is rarely employed for BID.

To overcome this difficulty, we propose a novel MRFs based image prior, termed super-Gaussian fields (SGF), where each potential is defined as a super-Gaussian distribution. By doing this, the partition function, the curse for traditional MRFs, can be theoretically ignored during parameter learning. With this advantage, the proposed MRF model can be seamlessly integrated into the coarse-to-fine framework, and all model parameters, as well as the latent image, can be data-adaptively learned and inferred from the blurred observation with a variational framework. Compared with prevailing deblurring methods on extensive experiments, the proposed method shows obvious superiority under both BID and NBID settings.

## 2 Related Work

### 2.1 Blind Image Deblurring

Due to the pioneering work of Fergus et al. [5] that imposes sparsity on image in the gradient spaces, sparse priors have attracted attention [5–13]. For example, a mixture of Gaussian models is early used to chase the sparsity due to its excellent approximate capability [5, 6]. A total variation model is employed since it can encourage gradient sparsity [7, 8]. A student-t prior is utilized to impose the sparsity [9]. A super-Gaussian model is introduced to represent a general sparse prior [10]. Those priors are limited by the fact that they are related to the  $l_1$ -norm. To relax the limitation, many the  $l_p$ -norm (where  $p < 1$ ) based priors are introduced to impose sparsity on image [11–13]. For example, Krishnan et al. [11] propose a normalized sparsity prior ( $l_1/l_2$ ). Xu et al. [12] propose a new sparse  $l_0$  approximation. Ge et al. [13] introduce a spike-and-slab prior that corresponds to the  $l_0$ -norm. However, all those priors are limited by the fact that they assume the coefficients in the gradient spaces are mutually independent.

Besides the above mentioned sparse priors, a family of blind deblurring approaches explicitly exploits the structure of edges to estimate the blur kernel [2, 3, 15–19]. Joshi et al. [16] and Cho et al. [15] rely on restoring edges from the blurry image. However, they fail to estimate the blur kernel with large size. To remedy it, Cho and Lee [2] alternately recover sharp edges and the blur kernel in a coarse-to-fine fashion. Xu and Jia [3] further develop this work. However, these approaches heavily rely on empirical image filters. To avoid it, Sun et al. [17] explore the edges of natural images using learned patch prior. Lai et al. [18] predict the edges by learned prior. Zhou and Komodakis [19] detect edges using a high-level scene-specific prior. All those priors only explore the local patch in the latent image but neglect the global characters.

Rather than exploiting edges, there are many other priors. Komodakis and Paragios [20] explore the quantized version of the sharp image by a discrete MRF prior. Their MRF prior is different with the proposed SG-FoE prior that is a continuous MRF prior. Michaeli and Irani [21] seek sharp image by the recurrence of small image patches. Gong et al. [22, 23] hire a subset of the image gradients for kernel estimation. Pan et al. [4] and Yan et al. [24] explore dark and bright pixels for BID, respectively. Besides, deep learning based methods have been adopted recently [25, 26].

## 2.2 High-Order MRFs for Image Modeling

Since gradient filters only model the statistics of first derivatives in the image structure, high-order MRF generalizes traditional based-gradient pairwise MRF models, e.g., cluster sparsity field [27], by defining linear filters on large maximal cliques. Based on the Hammersley-Clifford theorem [28], high-order MRF can give the general form to model image as follows:

$$p(\mathbf{x}; \Theta) = \frac{1}{Z(\Theta)} \prod_{c \in C} \prod_{j=1}^J \phi(\mathbf{J}_j \mathbf{x}_c), \quad (2)$$

where  $C$  are the maximal cliques,  $\mathbf{x}_c$  are the pixels of clique  $c$ ,  $\mathbf{J}_j$  are the linear filters and  $j = 1, \dots, J$ ,  $Z(\Theta)$  is the partition function with parameters  $\Theta$  that depend on  $\phi$  and  $\mathbf{J}_j$ ,  $\phi$  are the potentials. In contrast to previous high-order MRF in which the model parameters are hand-defined, FoE [14], a class of high-order MRF, can learn the model parameters from an external database, and hence has attracted high attention in image denoising [29, 30], NBID [31] and image super resolution [32].

## 3 Image Modeling with Super-Gaussian Fields

In this section, we first figure out the reason why traditional high-order MRFs models cannot be directly embedded into the coarse-to-fine deblurring framework. To this end, we comprehensively investigate a typical high-order MRF model, Gaussian scale mixture-FoE model (GSM-FoE) [29], and finally find out its inherent limitation. Then, we propose a super-Gaussian Fields based image prior model and analyze its properties.

### 3.1 Blind Image Deblurring with GSM-FoE

According to [29], GSM-FoE follows the general MRFs form in (2) and defines each potential with GSM as follows:

$$\phi(\mathbf{J}_j \mathbf{x}_c; \alpha_{j,k}) = \sum_{k=1}^K \alpha_{j,k} \mathcal{N}(\mathbf{J}_j \mathbf{x}_c; 0, \eta_j / s_k), \quad (3)$$

where  $\mathcal{N}(\mathbf{J}_j \mathbf{x}_c; 0, \eta_j / s_k)$  denotes the Gaussian probability density function with zero mean and variance  $\eta_j / s_k$ .  $s_k$  and  $\alpha_{j,k}$  denote the scale and weight parameters, respectively. It has been shown that GSM-FoE can well depict the sparse and wide heavy-tailed distributions [29]. Similar as most previous MRFs models, the partition function  $Z(\Theta)$  for GSM-FoE is generally intractable since it requires integrating over all possible images. However, evaluating  $Z(\Theta)$  is necessitated to learn all the model parameters, e.g.,  $\{\mathbf{J}_j\}$  and  $\{\alpha_{j,k}\}$  ( $\eta_j$  and  $s_k$  are generally constant). To sidestep this difficulty, most MRFs models including GSM-FoE turn to learn model parameters by maximizing the likelihood in (2) on an external image database [14, 29, 30], and then apply the learned model in the following applications, e.g., image denoising, super-resolution etc.

However, the pre-learned GSM-FoE cannot be directly employed to BID. This is because of that BID commonly adopts a coarse-to-fine framework, while the responses

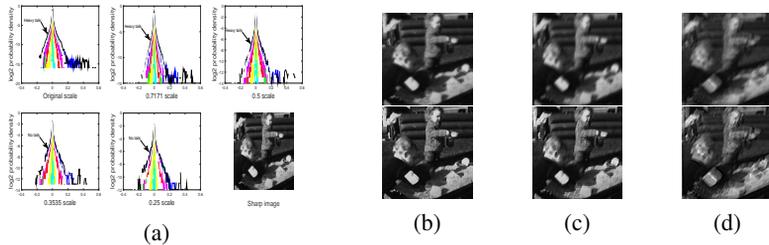


Fig. 2: (a) The 8 distributions with different colors of outputs by applying the 8 learned filters from [29] to the sharp image (the bottom right in (a)) at different scales. The 0.7171, 0.5, 0.3536 and 0.25 denote different downsampling rates. (b)-(d) The top: Blurred images with different kernel size (Successively,  $13 \times 13$ ,  $19 \times 19$ ,  $27 \times 27$ ). The bottom: Corresponding deblurred images using GSM-FoE.

of the latent image to these learned filters in the pre-learned GSM-FoE often express different distributions across various scales and thus fails to be well fitted by the same GSM-FoE prior. To illustrate this point clearly, we apply the learned filters in GSM-FoE to an example image and show the responses of an image across various scales in Fig. 2a. We can find that the response obtained in the fine scale (e.g., the original scale) exhibits obvious sparsity as well as heavy tails, while the response obtained in more coarse scales (e.g., 0.3536 and 0.25, the down-sampling rates) exhibits a Gaussian-like distribution. Thus, the Gaussian-like response in coarse scale cannot be well fitted by the GSM-FoE which prefers to fitting sparse and heavy-tailed distribution. A similar observation is also reported in [29]. To further demonstrate the negative effect of such kind of distribution mismatch on BID, we embed the pre-learned GSM-FoE prior into the Bayesian MMSE framework introduced in the following Section 4.1 to deal with an example image blurred with different kernel sizes. The deblurred results are shown in Fig. 2b-2d. Generally, a blurred image with larger kernel size requires deblurring at the coarser scale. For example, deblurring image with  $13 \times 13$  kernel requires deblurring at 0.5 scale and obtains a good result, since the filter response exhibits sparsity and heavy tails at 0.5 scale shown as in 2a. However, deblurring image with  $19 \times 19$  kernel obtains an unsatisfactory result, since it requires deblurring at 0.3536 scale where the filter response mismatches the sparse and heavy-tailed distribution depicted by GSM-FoE shown as 2a. In addition, more artifacts are generated in the deblurred results when the kernel size is  $27 \times 27$ , since it requires deblurring at 0.25 scale which produces more serious distribution mismatch.

### 3.2 Super-Gaussian Fields

To overcome the distribution mismatch problem of pre-learned MRFs model and embed it into the coarse-to-fine deblurring framework, we propose a novel MRFs prior model, termed super-Gaussian fields (SGF), which defines each potential in (2) as a super-Gaussian distribution [10, 33] as follows:

$$\phi(\mathbf{J}_j \mathbf{x}_c) = \max_{\gamma_{j,c} \geq 0} \mathcal{N}(\mathbf{J}_j \mathbf{x}_c; 0, \gamma_{j,c}), \quad (4)$$

where  $\gamma_{j,c}$  denotes the variance. Similar to GSM, SG also can depict sparse and heavy-tailed distributions [33]. Different from GSM-FoE and most MRFs models, the partition function in super-Gaussian fields can be ignored during parameter learning. More importantly, with such an advantage, it is possible to learn its model parameters directly from the blurred observation in each scale, and thus the proposed super-Gaussian fields can be seamlessly embedded into the coarse-to-fine deburring framework. In the following, we give the theoretical results to ignore the partition function in details.

*Property 1.* The potential  $\phi$  of SGF is related to  $\mathbf{J}_j$  and  $\mathbf{x}_c$ , but not  $\gamma_{j,c}$ . Hence, the partition function  $Z(\Theta)$  of SGF just depends on the linear filters  $\mathbf{J}_j$ .

*Proof.* As shown in (4),  $\gamma_{j,c}$  can be determined by  $\mathbf{J}_j$  and  $\mathbf{x}_c$ . Hence, the potential  $\phi$  in (4) is related to only  $\mathbf{J}_j$  and  $\mathbf{x}_c$ . Furthermore, because  $Z(\Theta) = \int \prod_{c \in C} \prod_{j=1}^J \phi(\mathbf{J}_j \mathbf{x}_c) d\mathbf{x}$ , the partition function  $Z(\Theta)$  just depends on the linear filters  $\mathbf{J}_j$  once the integral is done. Namely,  $\Theta = \{\mathbf{J}_j | j = 1, \dots, J\}$ .

*Property 2.* Given any set of  $J$  orthonormal vectors  $\{\mathbf{V}_{\mathbf{J}_j}\}, \{\mathbf{V}_{\mathbf{J}'_j}\}, \mathbf{V}_{\mathbf{J}_j}$  denote the vectored version of the linear filters  $\mathbf{J}_j$  ( $\mathbf{V}_{\mathbf{J}_j}$  is the vector formed through the concatenation of vectors of  $\mathbf{J}_j$ ), for the partition function of SGF:  $Z(\{\mathbf{V}_{\mathbf{J}_j}\}) = Z(\{\mathbf{V}_{\mathbf{J}'_j}\})$ .

To proof *Property 2*, we first introduce the following theory,

**Theorem 1 ([30])** Let  $E(\mathbf{V}_{\mathbf{J}_j}^T \mathbf{T}_x)$  be an arbitrary function of  $\mathbf{V}_{\mathbf{J}_j}^T \mathbf{T}_x$  and define  $Z(\mathbf{V}) = \int e^{-\sum_j E(\mathbf{V}_{\mathbf{J}_j}^T \mathbf{T}_x)} d\mathbf{x}$ , where  $\mathbf{T}_x$  denotes the Toeplitz (convolution) matrix (e.g.,  $\mathbf{J}_j \otimes \mathbf{x} = \mathbf{V}_{\mathbf{J}_j}^T \mathbf{T}_x$ ) with  $\mathbf{x}$ . Then  $Z(\mathbf{V}) = Z(\mathbf{V}')$  for any set of  $J$  orthonormal vectors  $\{\mathbf{V}_{\mathbf{J}_j}\}, \{\mathbf{V}'_{\mathbf{J}_j}\}$ .

*Proof (to Property 2).* Since the partition function  $Z(\Theta)$  of SGF just depends on the linear filters  $\{\mathbf{J}_j\}$  as mentioned in *Property 1* and the potential  $\phi$  in (4) also perfectly meets the form of  $E(\mathbf{V}_{\mathbf{J}_j}^T \mathbf{T}_x)$  in **Theorem 1**, it is easy to proof *Property 2*.

Based on *Property 1*, we do not need to evaluate the partition function  $Z(\Theta)$  of SGF to straightforward update  $\gamma_{j,c}$ , since  $Z(\Theta)$  do not depend on  $\gamma_{j,c}$ . Further, based on *Property 2*, we can also do not need evaluate  $Z(\Theta)$  of SGF to update  $\mathbf{J}_j$ , if we limit updating  $\mathbf{J}_j$  in the orthonormal space.

## 4 Image Deblurring with the Proposed SGF

In this section, we first propose a iterative method with SGF to handle BID in an coarse-to-fine scheme. We then show how to extend the proposed to non-blind image deblurring and non-uniform blind image deblurring.

### 4.1 Blind Image Deblurring with SGF

Based on the proposed SGF, namely (2) and (4), we propose a novel approach for BID in this section. In contrast to some existing methods which can only estimate the blur kernel, our approach can simultaneously recover latent image and the blur kernel. We will further discuss it in Section 4.2.

**Recovering Latent Image** Given the blur kernel, a conventional approach to recover latent image is Maximum a Posteriori (MAP) estimation. However, MAP favors the no-blur solution due to the influence of image size [34]. To overcome it, we introduce Bayesian MMSE to recover latent image. MMSE can eliminate the influence by integration on image as follows [35]:

$$\hat{\mathbf{x}} = \arg \min_{\tilde{\mathbf{x}}} \int \|\tilde{\mathbf{x}} - \mathbf{x}\|^2 p(\mathbf{x}|\mathbf{y}, \mathbf{k}, \mathbf{J}_j, \gamma_{j,c}) d\mathbf{x} = E(\mathbf{x}|\mathbf{y}, \mathbf{k}, \mathbf{J}_j, \gamma_{j,c}), \quad (5)$$

which is equal to the mean of the posterior distribution  $p(\mathbf{x}|\mathbf{y}, \mathbf{k}, \mathbf{J}_j, \gamma_{j,c})$ . Computing the posterior distribution is general intractable. Conventional approaches that resort to sum-product belief propagation or sampling algorithms often face with high computational cost. To reduce the computational burden, we use a variational posterior distribution  $q(\mathbf{x})$  to approximate the true posterior distribution  $p(\mathbf{x}|\mathbf{y}, \mathbf{k}, \mathbf{J}_j, \gamma_{j,c})$ . The variational posterior  $q(\mathbf{x})$  can be found by minimizing the Kullback-Leibler divergence  $KL(q(\mathbf{x})|p(\mathbf{x}|\mathbf{y}, \mathbf{k}, \mathbf{J}_j, \gamma_{j,c}))$ . This optimization is equivalent to the maximization of the lower bound of the free energy:

$$\max_{q(\mathbf{x}), \mathbf{k}, \mathbf{J}_j, \gamma_{j,c}} F = \max_{q(\mathbf{x}), \mathbf{k}, \mathbf{J}_j, \gamma_{j,c}} \int q(\mathbf{x}) \log p(\mathbf{x}, \mathbf{y}|\mathbf{k}, \mathbf{J}_j, \gamma_{j,c}) d\mathbf{x} - \int q(\mathbf{x}) \log q(\mathbf{x}) d\mathbf{x}. \quad (6)$$

Normally,  $p(\mathbf{x}, \mathbf{y}|\mathbf{k}, \mathbf{J}_j, \gamma_{j,c})$  in (6) should be equivalent to  $p(\mathbf{y}|\mathbf{x}, \mathbf{k}, \mathbf{J}_j, \gamma_{j,c})p(\mathbf{x})$ . We empirically introduce a weight parameter  $\lambda$  to regularize the influences of prior and likelihood similar to [14, 29]. In this case,  $p(\mathbf{x}, \mathbf{y}|\mathbf{k}, \mathbf{J}_j, \gamma_{j,c}) = p(\mathbf{y}|\mathbf{x}, \mathbf{k}, \mathbf{J}_j, \gamma_{j,c})p(\mathbf{x})^\lambda$ . Without loss of generality, we assume that the noise in (1) obeys i.i.d. Gaussian distribution with zero mean and  $\delta^2$  variance.

**Inferring  $q(\mathbf{x})$ :** Setting the partial differential of (6) with respect to  $q(\mathbf{x})$  to zero and omitting the details of derivation, we obtain:

$$-\log q(\mathbf{x}) = \frac{1}{2} \mathbf{x}^T \mathbf{A} \mathbf{x} - \mathbf{b}^T \mathbf{x}, \quad (7)$$

with  $\mathbf{A} = \delta^{-2} \mathbf{T}_k^T \mathbf{T}_k + \sum_j \lambda \mathbf{T}_{J_j}^T \mathbf{W}_j \mathbf{T}_{J_j}$ ,  $\mathbf{b} = \delta^{-2} \mathbf{T}_k^T \mathbf{y}$ , where the image  $\mathbf{x}$  is vectored here,  $\mathbf{W}_j$  denote the diagonal matrices with  $\mathbf{W}_j(i, i) = \gamma_{j,c}^{-1}$  where  $i$  is the index over image pixels and corresponds to the center of clique  $c$ ,  $\mathbf{T}_k$  and  $\mathbf{T}_{J_j}$  denote the Toeplitz (convolution) matrix with the filter  $\mathbf{k}$  and  $\mathbf{J}_j$ , respectively. Similar to [10, 6], to reduce computational burden, the mean  $\langle \mathbf{x} \rangle$  of  $q(\mathbf{x})$  can be found by the linear system  $\mathbf{A} \langle \mathbf{x} \rangle = \mathbf{b}$ , where  $\langle * \rangle$  refers to the expectation of  $*$ , and the covariance  $\mathbf{A}^{-1}$  of  $q(\mathbf{x})$  that will be used in (9) is approximated by inverting only the diagonals of  $\mathbf{A}$ .

**Learning  $\mathbf{J}_j$ :** Although  $\mathbf{J}_j$  is related to the intractable partition function  $Z(\Theta)$ , based on *Property 2*, we can limit learning  $\mathbf{J}_j$  in the orthonormal space where  $Z(\{\mathbf{J}_j\})$  is constant. For that, we can easily define a set  $\{\mathbf{B}_j\}$  and then consider all possible rotations of a single basis set of filters  $\mathbf{B}_j$ . That is, if we use  $\mathbf{B}$  to denote a matrix whose  $j$ -th column is  $\mathbf{B}_j$  and  $\mathbf{R}$  to denote any orthogonal matrix, then  $Z(\mathbf{B}) = Z(\mathbf{R}\mathbf{B})$ . Consequently, we can give the solution of updating  $\mathbf{J}_j$  by maximizing (6) under the condition

that  $\mathbf{R}$  is any orthogonal matrix as follows:

$$\mathbf{R}_j = \text{eig min}(\mathbf{B}^T \langle \mathbf{T}_x \mathbf{W}_j \mathbf{T}_x^T \rangle \mathbf{B}) \quad , \quad \mathbf{V}_{\mathbf{J}_j} = \mathbf{B} \mathbf{R}_j, \quad (8)$$

where  $\text{eig min}(\ast)$  denotes the eigenvector of  $\ast$  with minimal eigenvalue,  $\mathbf{T}_x$  denotes the Toeplitz (convolution) matrix with  $\mathbf{x}$ . We require that  $\mathbf{R}_j$  be orthogonal to the previous columns  $\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_{j-1}$ .

**Learning  $\gamma_{j,c}$ :** By contrast to updating  $\mathbf{J}_j$ , updating  $\gamma_{j,c}$  is more straightforward because  $Z(\Theta)$  is not related to  $\gamma_{j,c}$  as mentioned in *Property 1*. We can easily give the solution of updating  $\gamma_{j,c}$  by setting the partial differential of (6) with respect to  $\gamma_{j,c}$  to zero, as follows:

$$\gamma_{j,c} = \langle (\mathbf{J}_j \mathbf{x}_c)^2 \rangle. \quad (9)$$

**Learning  $\delta^2$ :** Learning  $\delta^2$  is easily performed by setting the partial differential of (6) with respect to  $\delta^2$  to zero. However, this way is problematic because BID is an underdetermined problem where the size of the sharp image  $\mathbf{x}$  is larger than that of the blurred image  $\mathbf{y}$ . We introduce the hyper-parameter  $d$  to remedy it similar to [36] as follows:

$$\delta^2 = \frac{\langle (\mathbf{y} - \mathbf{k} \otimes \mathbf{x})^2 \rangle}{n} + d, \quad (10)$$

where  $n$  is the size of image.

**Recovering the Blur Kernel** Similar to existing approaches [6, 12, 17], given  $\langle \mathbf{x} \rangle$ , we obtain the blur kernel estimation by solving

$$\min_{\mathbf{k}} \|\nabla \mathbf{x} \otimes \mathbf{k} - \nabla \mathbf{y}\|_2^2 + \beta \|\mathbf{k}\|_2^2, \quad (11)$$

where  $\nabla \mathbf{x}$  and  $\nabla \mathbf{y}$  denote the latent image  $\langle \mathbf{x} \rangle$  and the blurred image  $\mathbf{y}$  in the gradient spaces, respectively. To speed up computation, FFT is used as derived in [2]. After obtaining  $\mathbf{k}$ , we set the negative elements of  $\mathbf{k}$  to 0, and normalize  $\mathbf{k}$ . The proposed approach is implemented in a coarse-to-fine manner similar to state-of-the-art methods. Algorithm 1 shows the pseudo-code of the proposed approach.

## 4.2 Extension to Other Deblurring Problems

In this section, we extend the above method to handle the other two deblurring problems, namely the non-uniform Blind deblurring and the non-blind deblurring.

**Non-uniform Blind Deblurring** The proposed approach can be extended to handle the non-uniform blind deblurring where the blur kernel varies across spatial domain [37, 38]. Generally, the non-uniform blind deblurring problem can be formulated as [37]:

$$\mathbf{V}_y = \mathbf{D} \mathbf{V}_x + \mathbf{V}_n, \quad \text{or} \quad \mathbf{V}_y = \mathbf{E} \mathbf{V}_k + \mathbf{V}_n, \quad (12)$$

**Algorithm 1** Pseudo-code of the propose approach**Input:** Blurred image  $\mathbf{y}$ **Output:** The blur kernel  $\mathbf{k}$ 

- 1: Initialize:  $\mathbf{k}$ ,  $\mathbf{x}$ ,  $\mathbf{J}_j$ ,  $\mathbf{B}$ ,  $\delta^2$ ,  $\gamma_{j,c}$ ,  $\lambda$ ,  $\beta$  and  $d$
- 2: **while** stopping criterion is not satisfied **do**
- 3:   Inferring latent image  $\mathbf{x}$ , learning filters  $\mathbf{J}_j$  and variances  $\gamma_{j,c}$  by (7)-(9)
- 4:   Update for blur kernel  $\mathbf{k}$  by (11)
- 5:   Learning for noise  $\delta^2$  by (10)
- 6: **end while**

where  $\mathbf{V}_y$ ,  $\mathbf{V}_x$  and  $\mathbf{V}_n$  denote the vectored forms of  $\mathbf{y}$ ,  $\mathbf{x}$  and  $\mathbf{n}$  in (1).  $\mathbf{D}$  is a large sparse matrix, where each row contains a local blur filter acting on  $\mathbf{V}_x$  to generate a blurry pixel and each column of  $\mathbf{E}$  contains a projectively transformed copy of the sharp image when  $\mathbf{V}_x$  is known.  $\mathbf{V}_k$  is the weight vector which satisfies  $\mathbf{V}_{k_t} \geq 0$  and  $\sum_t \mathbf{V}_{k_t} = 1$ . Based on (12), the proposed approach can handle the non-uniform blind deblurring problem by alternatively solving the following problems:

$$\max_{q(\mathbf{V}_x), \mathbf{D}, \mathbf{J}_j, \gamma_{j,c}} \int q(\mathbf{V}_x) \log q(\mathbf{V}_x) d\mathbf{V}_x - \int q(\mathbf{V}_x) \log p(\mathbf{V}_x, \mathbf{V}_y | \mathbf{D}, \mathbf{J}_j, \gamma_{j,c}) d\mathbf{V}_x, \quad (13)$$

$$\min_{\mathbf{V}_k} \|\nabla \mathbf{E} \mathbf{V}_k - \mathbf{V}_{\nabla y}\|_2^2 + \beta \|\mathbf{V}_k\|_1. \quad (14)$$

Here, (14) employs  $l_1$ -norm to encourage a sparse kernel as [37]. The optimal  $q(\mathbf{V}_x)$  in (13) can be computed by using formulas similar to (7)-(9) in which  $\mathbf{k}$  is replaced by  $\mathbf{D}$ . In addition, the efficient filter flow [39] is adopted to accelerate the implementation of the proposed approach.

**Non-blind Image Deblurring** Similar as most of previous non-blind image deblurring, the proposed approach can handle non-blind image deblurring by (7)-(9) with the kernel  $\mathbf{k}$  given beforehand.

## 5 Analysis

In this part, we demonstrate two properties of the proposed SGF in image deblurring. *1) The learned filters in SGF are sparse-promoting.* As mentioned in (8), the filters in SGF are estimated as the eigenvector of  $\langle \mathbf{T}_x \mathbf{W}_j \mathbf{T}_x^T \rangle$  with minimal eigenvalue, viz., the filters are the singular vector  $\langle \mathbf{T}_x (\mathbf{W}_j)^{\frac{1}{2}} \rangle$  with minimal singular value. This implies that the proposed method seeks filters  $\mathbf{J}_j$  which lead to  $\mathbf{V}_{\mathbf{J}_j}^T \mathbf{T}_x (\mathbf{W}_j)^{\frac{1}{2}}$  being sparse as possible.  $\mathbf{V}_{\mathbf{J}_j}$  denotes the vectorized  $\mathbf{J}_j$ . Since  $(\mathbf{W}_j)^{\frac{1}{2}}$  is a diagonal matrix which only scales each column of  $\mathbf{T}_x$ , the sparsity of  $\mathbf{V}_{\mathbf{J}_j}^T \mathbf{T}_x (\mathbf{W}_j)^{\frac{1}{2}}$  is mainly determined by  $\mathbf{V}_{\mathbf{J}_j}^T \mathbf{T}_x$ . Consequently, the proposed approach seeks filters  $\mathbf{J}_j$  which lead to the corresponding response  $\mathbf{V}_{\mathbf{J}_j} \mathbf{T}_x$  of the latent image being as sparse as possible. This can be further illustrated by the visual results in Fig. 3(a) where the distribution of image

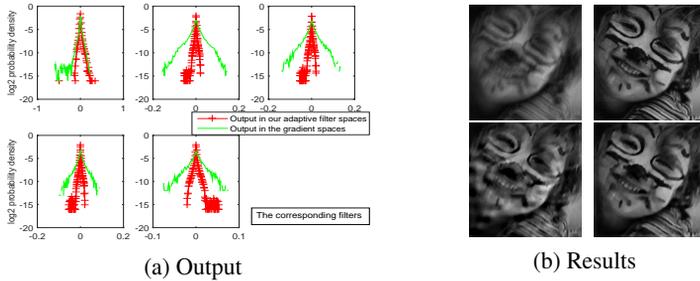


Fig. 3: Comparison of the outputs in the gradient spaces and our adaptive filter spaces at different scales. (a) The distributions of the filter outputs of sharp image (The top right in (b)) in gradient spaces and our adaptive spaces (by using filters corresponding to the bottom right in (a)) at different scales. From top to bottom and from left to right: the original, 0.7171 (sampling rate), 0.5, 0.3536, 0.25 scales. The bottom right is our final obtained filters corresponding to the different scales. (b) From top to bottom, from left to right: blurred image, sharp image, estimated latent images with basic filters and our adaptive filters.

response to these learned filters are plotted. 2) *These filters  $\mathbf{J}_j$ s learned in each scales are more powerful than the basic filters for image gradient which are extensively adopted in previous methods.* To illustrate this point, we compare the response of the latent image to these learned filters with image gradient in Fig. 3(a). It can be seen that those learned filters lead to more sparse response than that on gradients. With these two group of filters, we recover the latent image with the proposed approach. The corresponding deblurred results are shown in Fig. 3(b). We can find that these learned filters lead to more clear and sharp results. These results demonstrate that those learned filters are more powerful than the basic filters for image gradient.

## 6 Experiments

In this section, we illustrate the capabilities of the proposed method for blind, non-blind and non-uniform image deblurring. We first evaluate its performance for blind image deblurring on three datasets and some real images. Then, we evaluate its performance for non-blind image deblurring. Finally, we report results on blurred images undergoing non-uniform blur kernel.

**Experimental Setting:** In all experiments unless especially mentioned, we set  $\delta^2 = 0.002$ ,  $\beta = 20$ ,  $\gamma_{j,c} = 1e^{-3}$  and  $d = 1e^{-4}$ . To initialize the filters  $\mathbf{J}_j$ , we first down-sample all images (grayscale) from the dataset [40] to reduce noise, then train 8  $3 \times 3$  filters  $\mathbf{J}_j$  on the downsampling images using the method proposed in [29] as the initialization.  $\lambda$  is set as 1/8. To initialize basis set  $\mathbf{B}$ , we use the shifted versions of the whitening filter whose power spectrum equals the mean power spectrum of  $\mathbf{J}_j$  as suggested in [30]. We use the proposed non-blind approach in Section 4.2 to give the final sharp image unless otherwise mentioned. We implement the proposed method in Mat-

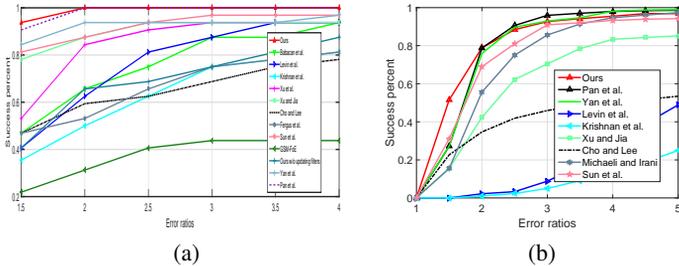


Fig. 4: Quantitative evaluations on dataset [41] (a) and dataset [17] (b).

lab and evaluate the performance on an Intel Core i7 CPU with 8GB of RAM. Our implementation processes images of  $255 \times 255$  pixels in about 27 seconds.

## 6.1 Experiments on Blind Image Deblurring

**Dataset from Levin et al. [41]:** The proposed method is first applied to a widely used dataset [41], which consists of 32 blurred images, corresponding to 4 ground truth images and 8 motion blur kernels. We compare it with state-of-the-art approaches [5–7, 2, 3, 11, 12, 17, 21, 24, 4]. To further verify the performance of GSM-FoE, we implement GSM-FoE for BID by integrating the pre-learned GSM-FoE prior into the Bayesian MMSE framework introduced in the following Section 4.1. We also verify the performance of the proposed method without updating filters to illustrate the necessity to update filters. For the fair comparison, after estimating blur kernels using different approaches, we use the nonblind deconvolution algorithm [42] with the same parameters in [6] to reconstruct the final latent image. The deconvolution error ratio, which measures the ratio between the Sum of Squared Distance (SSD) deconvolution error with the estimated and correct kernels, is used to evaluate the performance of different methods above. Fig. 4a shows the cumulative curve of error ratio. The results shows that the proposed method obtains the best performance in terms of success percent 100% under error ratio 2. More detailed results can be found in supplementary material.

**Dataset from Sun et al. [17]:** In a second set of experiments we use dataset from [17], which contains 640 images synthesized by blurring 80 natural images with 8 motion blur kernels borrowed from [41]. For fair comparison, we use the non-blind deconvolution algorithm of Zoran and Weiss [43] to obtain the final latent image as suggested in [17]. We compare the proposed approach with [6, 2, 3, 11, 17, 21]. Fig. 4b shows the cumulative curves of error ratio. Our results are visually competitive with others.

**Dataset from Köhler et al. [1]:** We further implement the proposed method on dataset, which is blurred by space-varying blur, borrowed from [1]. Although real images often exhibit spatially varying blur kernel, many approaches that assume shift-invariant blur kernel can perform well. We compare the proposed approach with [5, 2, 3, 11, 44, 45,

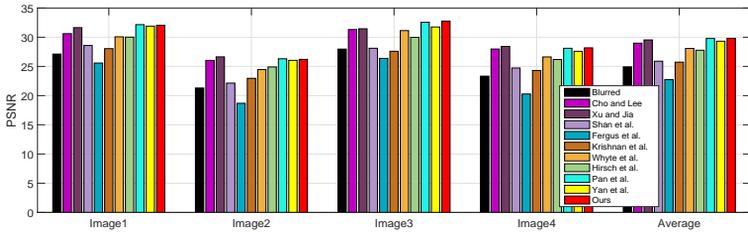


Fig. 5: Quantitative evaluations on Dataset [1]. Our results are competitive.

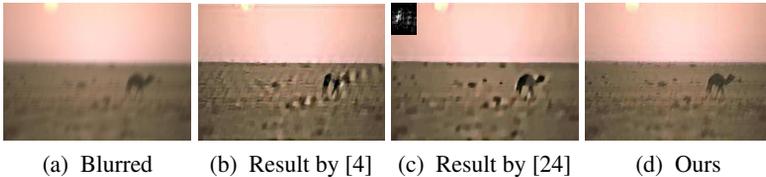


Fig. 6: A challenging example with sky patches.

38, 4]. The peak-signal-to-noise ratio (PSNR) is used to evaluate their performance. Fig. 5 shows the PSNRs of different approaches above. We can see that our results are superior to the state-of-the-art approaches.

**Comparison of the proposed approach, Pan et al. [4] and Yan et al. [24]:** Recently, the method in [4] based on dark channel prior shows state-of-the-art results. As shown in Figs. 4a and 5, the proposed method performs on par with the method in [4] on datasets Levin et al. [41] and Köhler et al. [1]. On the other hand, the method in [4] fails to handle blurred images which do not satisfy the condition of the dark channel prior, e.g., images with sky patches [46]. To a certain extent, Yan et al. [24] alleviate the limitation of the dark channel prior with bright pixels. However, the methods in Yan et al. [24] is still affected by complex brightness, as shown in Fig. 6.

**Real images:** We further test the proposed method using two real natural images. In Fig. 7 we show two comparisons on real photos with unknown camera shakes. For blurred image (a), Xu et al. [12] and the proposed method produce high-quality images. Further, for the blurred image (e), the proposed method produces sharper edges around the texts than Xu and Jia [3] and Sun et al. [17].

## 6.2 Experiments on Non-blind Image Deblurring

We also use the dataset from [41] to verify the performance of the proposed approach on non-blind image deblurring where the blur kernel is given by reference to [41]. Here, we set  $\delta^2 = 1e^{-4}$  and the remaining parameters are initialized as mentioned above. We compare the proposed method against Levin et al. [42] with the same parameters in [6], Krishnan and Fergus [47], Zoran and Weiss [43] and Schmidt et al. [31]. Schmidt et

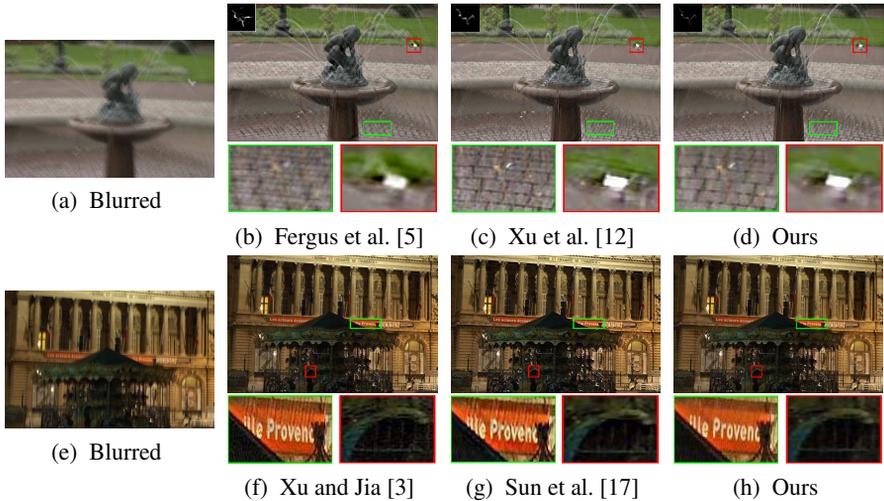


Fig. 7: Two example images with unknown camera shake from [5] and [3].

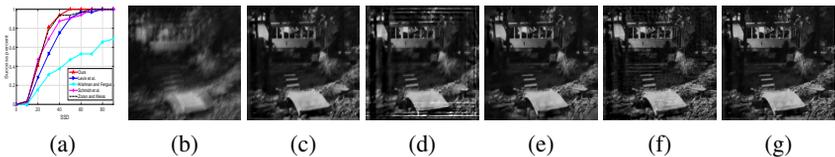


Fig. 8: Quantitative and qualitative evaluation on dataset [41]. (a) Cumulative histograms of SSD. (b)-(h) show a challenging example. From left to right: Blurred SS-D:574.38, Levin et al. [42] SSD:73.51, Krishnan and Fergus [47] SSD:182.41, Zoran and Weiss [43] SSD:64.03, Schmidt et al. [31] SSD:64.35, Ours SSD:44.61.

al. [31] and the proposed method are based on high-order MRFs. The difference is that Schmidt et al. [31] use GSM-FoE model but we use the proposed SGF. The SSD is also used to evaluate the performance of different methods.

Fig. 8 shows the cumulative curve of SSD and deblurred results by different approaches on a challenging example. We can see that Zoran and Weiss [43] and the proposed produce competitive results. Additionally, as shown in Table 1, compared with Zoran and Weiss [43], the proposed method obtains lower the average SSD and less run time. Further, compared with GSM-FoE based Schmidt et al. [31], our SG-FoE acquires better results and requires more less run time.

### 6.3 Experiments on Non-uniform Image Deblurring

In the last experiment, we evaluate the performance of the proposed approach on blurred images with non-uniform blur kernel.  $\beta$  in (14) is set as 0.01 for non-uniform deblurring. Again, initializing  $\gamma_{j,c} = 1e^{-3}$ ,  $\mathbf{J}_j$ ,  $\lambda$  and  $B$  are the same as blind deblurring. We compare the proposed method with Whyte et al. [37] and Xu et al. [12]. Fig. 9

	Levin et al. [42]	Zoran and Weiss [43]	Schmidt et al. [31]	Krishnan and Fergus [47]	Ours
Average SSD	30.20	24.60	25.43	82.35	<b>21.77</b>
Time (s)	109	3093	>10000	<b>6</b>	485

Table 1: Average SSD and run time on dataset [41].

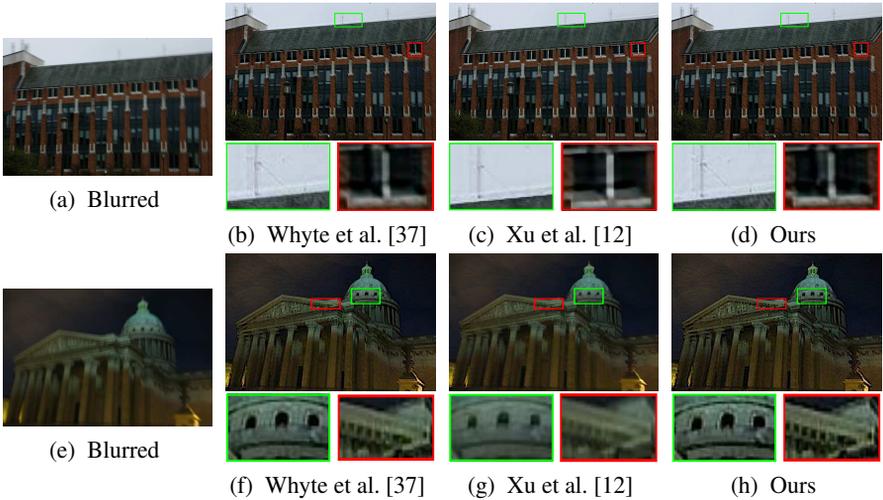


Fig. 9: Non-uniform blind deblurring results.

shows two real natural images with non-uniform blur kernel and deblurred results. The proposed method generates images with fewer artifacts and more details.

## 7 Conclusions

To capture the complicated image structures for image deblurring, we analyze the reason why traditional high-order MRFs model fails to handle BID in a coarse-to-fine scheme. To overcome this problem, we propose a novel super-Gaussian fields model. This model contains two exciting properties, *Property 1* and *Property 2* introduced in Section 3.1, so that the partition function can be theoretically ignored during parameter learning. With this advantage, the proposed MRF model has been integrated into blind, non-blind and non-uniform blind image deblurring framework. Extensive experiments demonstrate the effectiveness of the proposed method. In contrast to previous fixed gradient based approaches, the proposed method explores sparsity in adaptive sparse-promoting filter spaces so that it dramatically performs well. It is interesting to exploit adaptive sparse-promoting filter spaces by other methods for BID in the future.

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