Supplementary Material for Making Minimal Solvers for Absolute Pose Estimation Compact and Robust

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1. Elimination Ideal

To find the new constraints on the camera matrix,

$$\begin{aligned}
p_{21}p_{31} + p_{22}p_{32} + p_{23}p_{33} &= 0 \\
p_{11}p_{31} + p_{12}p_{32} + p_{13}p_{33} &= 0 \\
p_{11}p_{21} + p_{12}p_{22} + p_{13}p_{23} &= 0 \\
p_{11}^2 + p_{12}^2 + p_{13}^2 - p_{21}^2 - p_{22}^2 - p_{23}^2 &= 0 \\
p_{13}^2p_{32} - p_{21}^2p_{32} - p_{22}^2p_{32} - p_{12}p_{13}p_{33} - p_{22}p_{23}p_{33} &= 0 \\
p_{12}p_{13}p_{32} + p_{22}p_{23}p_{32} - p_{12}^2p_{33} + p_{21}^2p_{33} + p_{23}^2p_{33} &= 0 \\
p_{11}p_{13}p_{32} + p_{21}p_{23}p_{32} - p_{11}p_{12}p_{33} - p_{21}p_{22}p_{33} &= 0 \\
p_{13}^2p_{31} - p_{22}^2p_{31} + p_{21}p_{22}p_{32} - p_{11}p_{13}p_{33} &= 0 \\
p_{12}p_{13}p_{31} + p_{22}p_{23}p_{31} - p_{11}p_{12}p_{33} - p_{21}p_{22}p_{33} &= 0
\end{aligned}$$

the following Macaulay2 [2] code was used:

```
KK = ZZ / 30097;
R = KK[p_{1,1}..p_{3,3},r_{1,1}..r_{3,3},f,s]
-- we can ignore translation since it poses
-- no extra constraint on P

P =  map(R^3,R^3,(i,j) -> p_{i+1,j+1});
Q =  map(R^3,R^3,(i,j) -> r_{i+1,j+1});
K = matrix({{f,0,0},{0,f,0},{0,0,1}});
eye = matrix({{s,0,0},{0,s,0},{0,0,s}});

eqs = flatten(P-K*Q | transpose(Q)*Q-eye);

I = saturate(ideal eqs, s);

elim_vars = {f,s, r_{1,1},r_{1,2},r_{1,3},
                  r_{2,1},r_{2,2},r_{2,3},
                  r_{3,1},r_{3,2},r_{3,3}};
J = eliminate(I,elim_vars);
```

2. Additional Figures from the Experiments

2.1. Noise Experiment

Figure 1 shows the errors in the radial distortion for the noise experiment in Section 4.2.

2.2. Evaluation on Real Images

Figure 2 and Figure 3 shows histograms of the relative errors for the *Rotunda* and *Graffiti* datasets. Figure 4 and Figure 5 shows example images from the datasets.

References

[1] M. Bujnak, Z. Kukelova, and T. Pajdla. New efficient solution to the absolute pose problem for camera with unknown focal length and radial distortion. In *Asian Conference on Computer Vision*, pages 11–24. Springer, 2010. 2

[2] D. R. Grayson and M. E. Stillman. Macaulay 2, a software system for research in algebraic geometry, 2002. 1

[3] K. Josephson and M. Byröd. Pose estimation with radial distortion and unknown focal length. In *Computer Vision and Pattern Recognition, 2009. CVPR 2009. IEEE Conference on*, pages 2419–2426. IEEE, 2009. 2

[4] Z. Kukelova, M. Bujnak, and T. Pajdla. Real-time solution to the absolute pose problem with unknown radial distortion and focal length. In *Proceedings of the IEEE International Conference on Computer Vision*, pages 2816–2823, 2013. 2

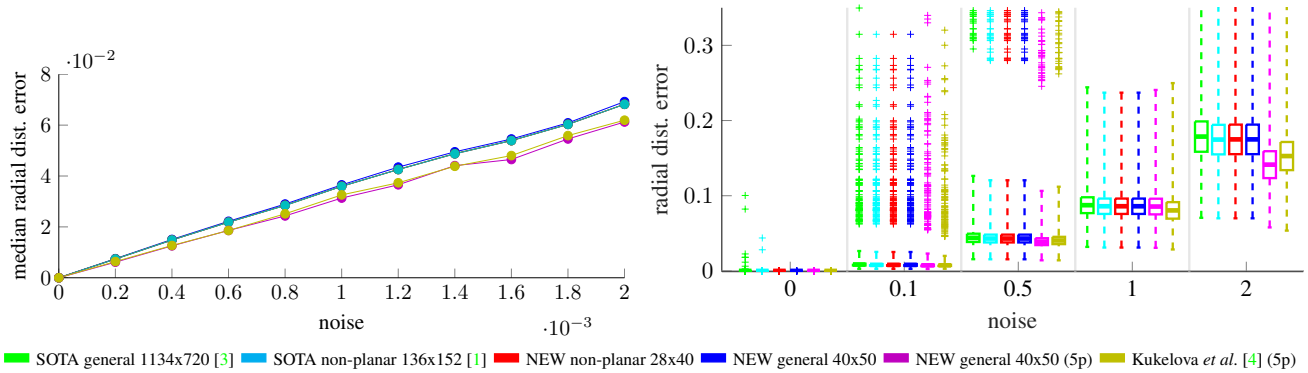

Figure 1. Comparison of the errors of radial distortion estimated by different solvers for varying levels of noise. The ground truth values were set to $f_{gt} = 1.5$ and $k_{gt} = -0.4$. *Left:* Median radial distortion errors. *Right:* Boxplot of radial distortion errors.

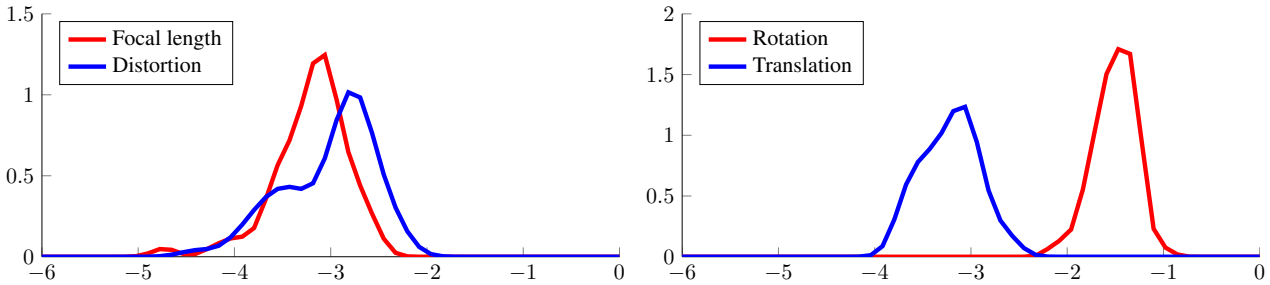

Figure 2. Errors ($\log_{10}$) for the *Rotunda* dataset. The measured errors are the relative focal length error $|f - f_{GT}|/f_{GT}$, the distortion error $|k - k_{GT}|$, the rotation error in degrees and the relative translation error $\|\boldsymbol{t} - \boldsymbol{t}_{GT}\|/\|\boldsymbol{t}_{GT}\|$.

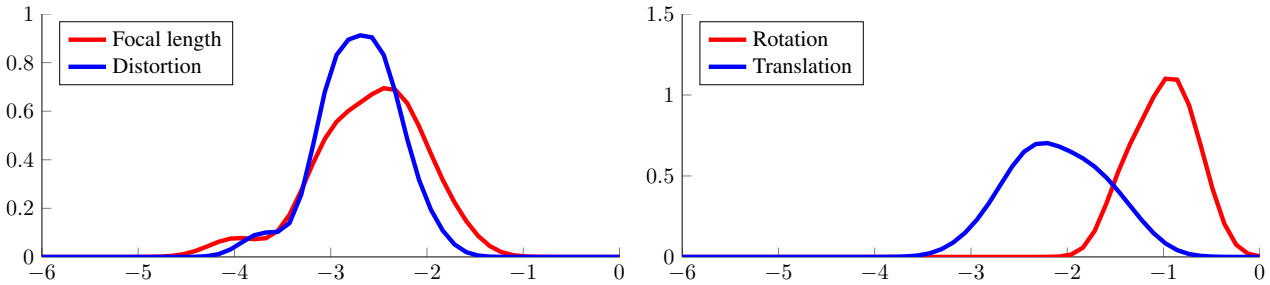

Figure 3. Errors ($\log_{10}$) for the *Graffiti* dataset. The measured errors are the relative focal length error $|f - f_{GT}|/f_{GT}$, the distortion error $|k - k_{GT}|$, the rotation error in degrees and the relative translation error $\|\boldsymbol{t} - \boldsymbol{t}_{GT}\|/\|\boldsymbol{t}_{GT}\|$.

Figure 4. Some examples of the images in the *Rotunda*. *Left:* Original images. *Right:* Undistorted images.

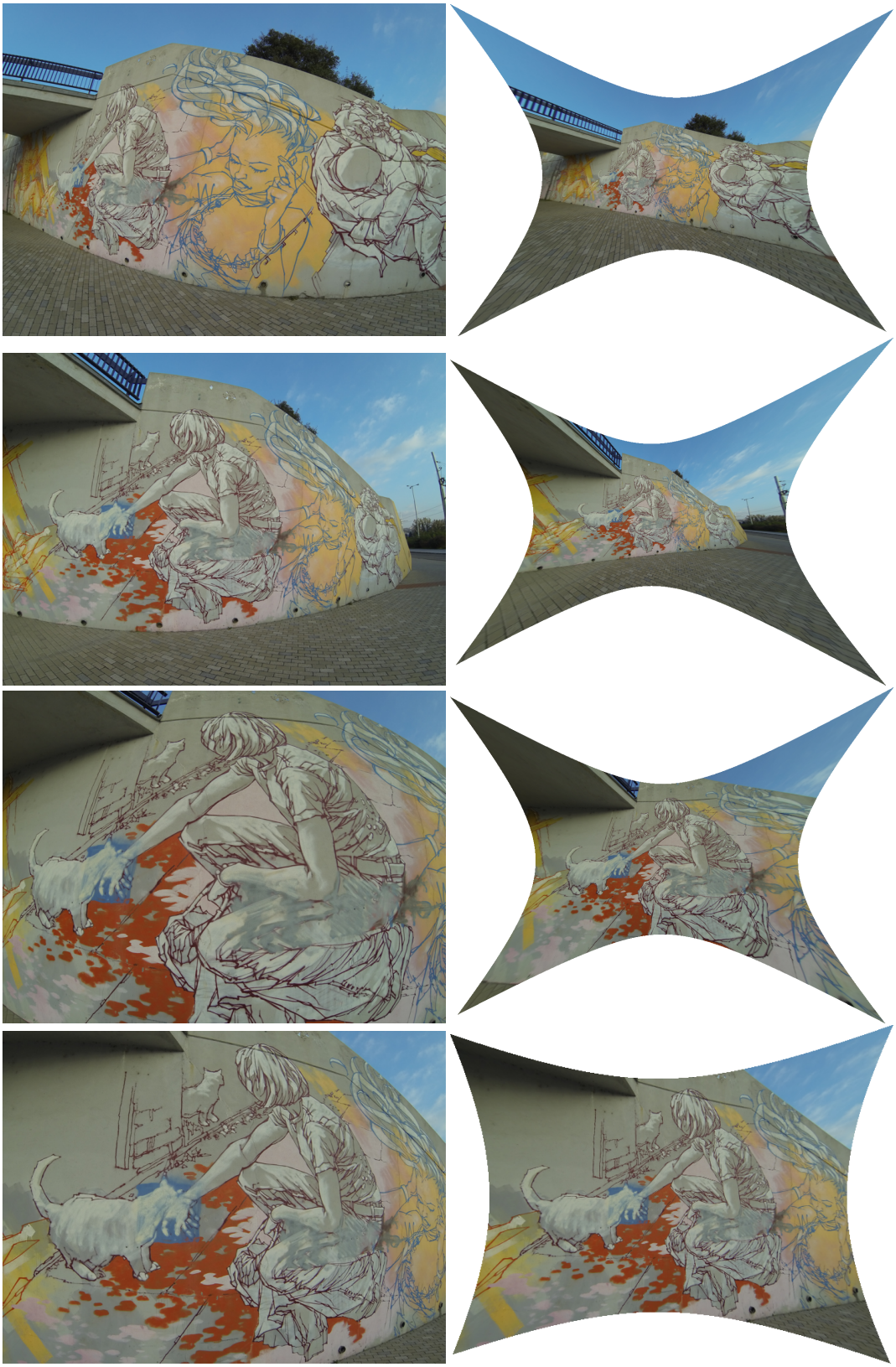

Figure 5. Some examples of the images in the *Graffiti*. *Left:* Original images. *Right:* Undistorted images.