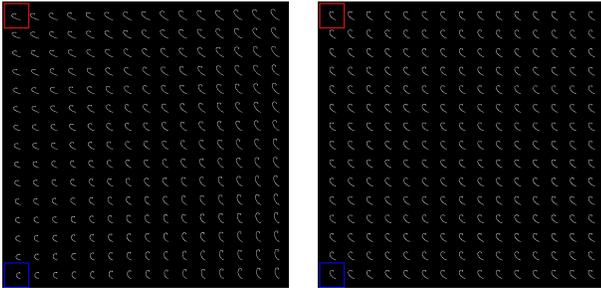


# Going Unconstrained with Rolling Shutter Deblurring (Supplementary Material)

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(a) With inplane rotations (b) Without inplane rotations

Figure S1. Full set of PSFs of Fig. 2(b) illustrating the effect of inplane rotations for wide-angle setting (29 mm) using [6].

We begin by revisiting the problem motivation. This is followed by section S2 which contains our proof for claim 2 (section 3). Section S3 gives implementation details, and section S4 is devoted to additional evaluations.

## S1. Problem Motivation (illustrative)

As mentioned in section 1 in the main paper, our method advances the state-of-the-art in RS deblurring, as it can deal with wide-angle configuration, unconstrained ego-motion and unconstrained shutter, *without* the need for timing information. Here, we further elaborate the significance of these problems that we have addressed in our work.

**Unconstrained Focal-length:** The PSFs provided in Fig. 2(b), which illustrates the importance of inplane rotations for wide-angle systems, is created using a focal length of 29 mm and real hand-held trajectory #39 in [6]. The full set of PSFs is provided in Fig. S1. We give in Fig. S2 focal-length settings of some popular CMOS imaging devices. It is clearly evident from the figure that wide-angle configurations are indeed important in photography (and predominant in cell-phones and drone cameras). However, the state-of-the-art RS-BMD [11] works only for narrow-angle settings. Hence, it is important to accommodate wide-angle settings.

**Unconstrained Ego-motion:** Even though a polynomial function can reasonably model human camera shake, RS

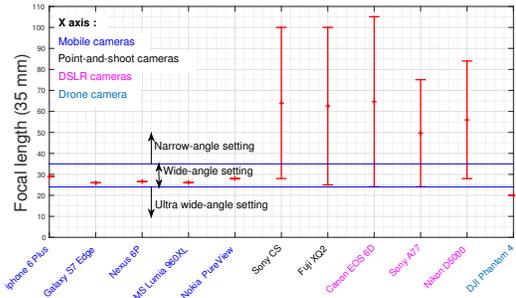


Figure S2. Focal lengths of some popular CMOS devices. Note the wide-angle setting predominant in cell-phone and drone cameras.

blur also exists in images captured by drones, street view cars, *etc.*, wherein the ego-motion is seldom regular [11]. Fig. S3 illustrates this fact with an under-damped response of a robotic system (which we employed in Figs. 5(d-f) using [4]). Also given is the approximation using a fourth order polynomial (as used in state-of-the-art RS-BMD [11]). From the plot it is clear that the polynomial model is *unable* to adequately capture the motion, thus underscoring the need for handling unconstrained ego-motion.

**RS timing information:** Both shutter speed ( $t_e$ ) and inter-row delay ( $t_r$ ) are required a priori in state-of-the-art RS-BMD [11] to fragment the motion trajectory for each image-row. Getting  $t_r$  from a camera requires processing of videos taken using the same camera setting (section 5.2 in [11]). Deriving both  $t_e$  and  $t_r$  *without* the meta-data and camera information further escalates the difficulty. In contrast, our method does *not* need any a priori timing information. Note that we estimate the value  $t_r/t_e$  for the RS prior in Eq. (9) solely from image intensities as discussed in section 4.4.

## S2. Proof of Claim 2

**Claim 2:** *The prior which restricts drifting of TSFs between blocks (in Eq. (9)) is a convex function in  $\mathbf{w}$ , and can be represented as a norm of matrix vector multiplication, i.e., as  $\|\mathbf{G}\mathbf{w}\|_2^2$ , with sparse  $\mathbf{G}$ .*

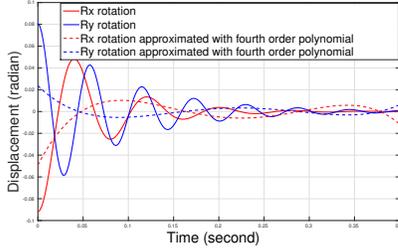


Figure S3. Vibrational ego-motion of a robotic system (using [4]). The polynomial model is *inadequate* to capture these trajectories.

To prove this, we draw from the following well-known properties of convex function [1] which are a linear function is always convex (prop. 1), composition of convex functions is always convex (prop. 2), and non-negative sum of convex functions is convex (prop. 3).

**Proof:** Considering  $n_b$  number of image blocks and each block-MDF  $w_i$  having length  $l$ , an individual additive component in our RS prior (in Eq. (9)) can be represented as  $\|\Gamma(r_b(\mathbf{j} - \mathbf{i} + 1)) \cdot \mathbf{S}_{(i,j)} \mathbf{w}\|_2^2$ , where  $\mathbf{S}_{(i,j)}$  is a matrix of dimension  $l \times n_b \cdot l$ , with all zeros except two scaled identity matrices of dimension  $l \times l$  corresponding to  $i$ th TSF (with scale 1) and  $j$ th TSF (with scale  $-1$ ). Therefore, the term  $\{\Gamma(r_b(\mathbf{j} - \mathbf{i} + 1)) \cdot \mathbf{S}_{(i,j)} \mathbf{w}\}$  is a linear function in  $\mathbf{w}$ . Since  $\|\Gamma(r_b(\mathbf{j} - \mathbf{i} + 1)) \cdot \mathbf{S}_{(i,j)} \mathbf{w}\|_2^2$  is a composite of squared  $L_2$  norm (which is convex) of a linear function in  $\mathbf{w}$ , each additive component is convex (props. 1 and 2). Resultantly, the sum of all additive components in Eq. (9), *i.e.*,  $\text{prior}(\mathbf{w})$ , is a convex function in  $\mathbf{w}$  (prop. 3).

Also,  $\text{prior}(\mathbf{w})$  can be represented as  $\|\mathbf{G}\mathbf{w}\|_2^2$ , where matrix  $\mathbf{G}$  is obtained by vertically concatenating matrices  $\{\Gamma(r_b(\mathbf{j} - \mathbf{i} + 1)) \cdot \mathbf{S}_{(i,j)}\}$  corresponding to the individual additive component in RS prior. Since  $\mathbf{S}_{(i,j)}$  is a sparse matrix,  $\mathbf{G}$  will also be sparse. Hence proved. ■

### S3. Implementation Details

We implemented our algorithm in MATLAB. We empirically set 7 scales, each with 7 iterations, in our scale-space framework (section 4). The blurred image in the  $i$ th scale is formed by downscaling the input image by a factor of  $(1/\sqrt{2})^{i-1}$ . To start the alternative minimization, the coarsest scale MDFs are initialized with Kronecker delta. For ego-motion estimation (section 4.2), we consistently used the RS-prior regularization ( $\alpha$  in Eq. (13)) in level  $i$  as  $2^{7-i}$  (so that the RS prior can cope with the increasing image size, and thus the data fidelity magnitude  $\|\mathbf{F}\mathbf{w} - \nabla\mathbf{B}\|_2^2$ , in finer levels). We used the MDF regularization  $\beta'$  (in Eq. (13)) as 0.01. For latent image estimation (section 4.3), we used  $R = 48$  such that each image-patch is square, and with 6 patches along the shorter dimension and 8 along the longer dimension. For the Richardson-Lucy deconvolution

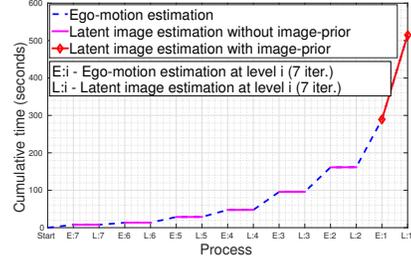


Figure S4. Cumulative time for different processes. Note the computational gains of the prior-less RS-EFF based image estimation.

(employed in the last iteration of the finest level), we used a total number of 30 iterations. For the selection of block-size (section 4.4), we used an initial block-size  $r_0$  as 145, and a downscaling factor of 2 (*i.e.*,  $M_0 = M/2$  and  $N_0 = N/2$ ).

Running time reported in Table 1 is obtained on the same system with an Intel Xeon processor with 32 GB memory. We found that for deblurring an  $800 \times 800$  RGB image (of maximum blur-length of 30 pixels), our unoptimized MATLAB implementation took about 9 minutes. Fig. S4 provides a detailed break-up of the time taken for each estimation step. In fact, observe that a large fraction of the total time is utilized for latent image estimation in the *final* iteration which involves a costly image-prior (see section 4.3). This underscores the importance of our efficient prior-less estimation in the initial iterations derived from RS-EFF (Eq. (14)).

### S4. Additional Evaluations

We provide in Fig. S5 iteration-by-iteration results to illustrate how the algorithm works. In Fig. S6, we give full images corresponding to the patches of synthetic experiment results provided in Figs. 5(a-i). In Figs. S7-S11, we give additional evaluations for the real RS-BMD examples provided in Figs. 7 & 8. These include SIV-BMD [2] and RS rectification followed with SIV-BMD [2] (as reported in [11]), and state-of-the-art CCD-BMD [9]. We also consider BMD *without* our RS prior to illustrate the ego-motion ambiguity in RS-BMD. For low-light case, we consider [5] that specifically addresses low-light BMD (albeit for CCD cameras). The codes for [5], [11] and [9] are downloaded from the author's website and executed using default parameters. Additional examples under different lighting condition and for wide-angle settings are given in Fig. S12.

For sake of completeness; we provide GS deblurring comparisons with state-of-the-art CCD-BMD methods of [9, 8, 10, 13, 12] and [3] in Figs. S13 & S14. We evaluated on the examples from the dataset of [7] and [9] using their reported results. The results show that our method works equally well for CCD cameras and importantly, *without* warranting any prior knowledge of the shutter.

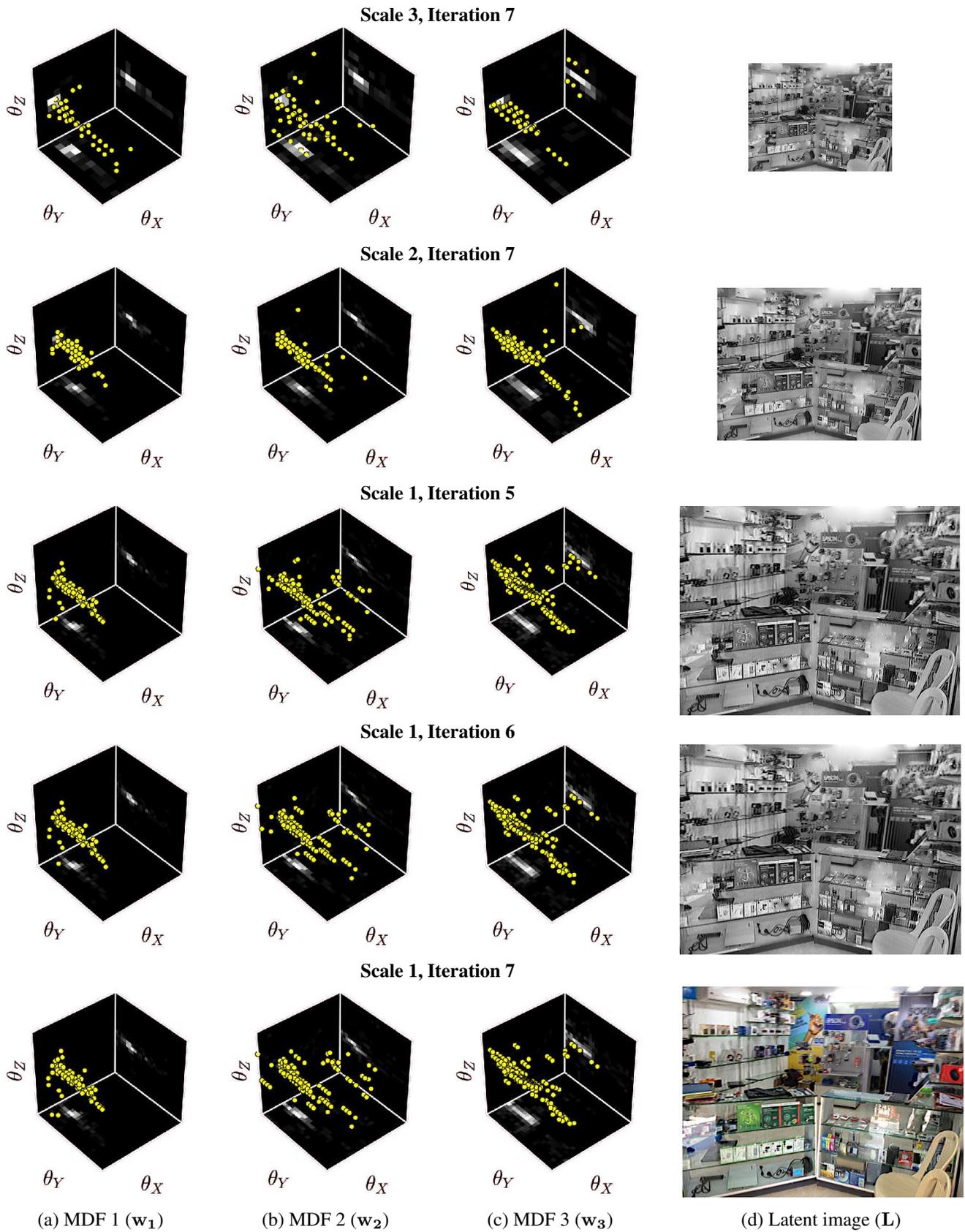


Figure S5. Iteration-by-iteration results of the alternative minimization of block-wise MDFs and latent image: (a-c) Estimated block-wise MDFs and (d) Estimated latent image. Notice the variation in block-wise MDFs, which depicts the characteristic of RS blur (as shown in Fig. 3). Also, observe the convergence of the block-wise MDFs through iteration 5 to 7 in the finest image scale (last three rows).

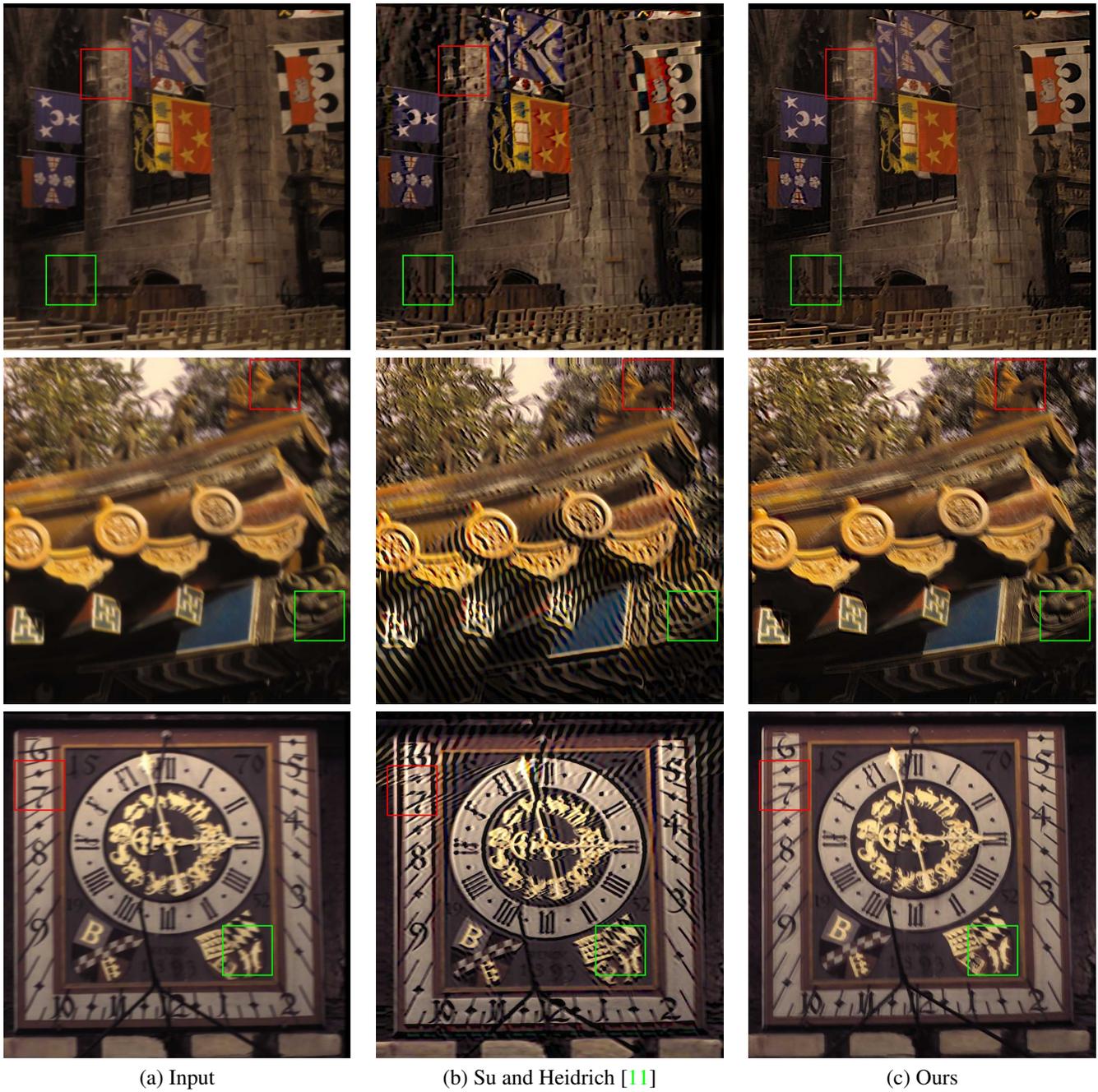


Figure S6. Full-sized images corresponding to the image patches given in Figs. 2(a-i): First row gives a case of wide-angle system (Figs. 2(a-c)), second row gives a case of vibratory motion (Figs. 2(d-f)), and third row gives a case of CCD-blur (Figs. 2(g-i)). (Best viewed on high-resolution display with zoom-in corresponding to an  $800 \times 800$  image size.)

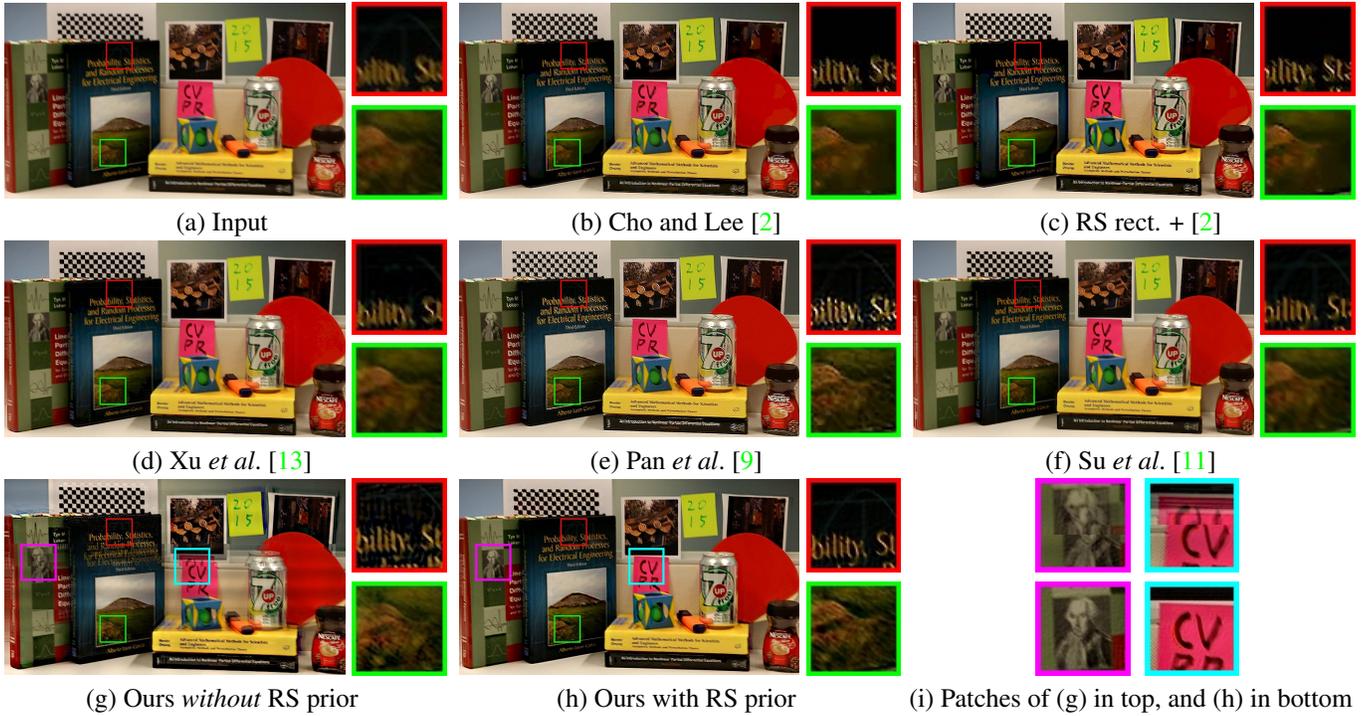


Figure S7. Detailed comparisons for RS narrow-angle example in dataset [11] (Fig. 7-top-row). Note the effect of incoherent combination due to the block shift-ambiguity (section 3, claim 1) in (i)-first row, which is successfully suppressed by our RS prior ((i)-second row).

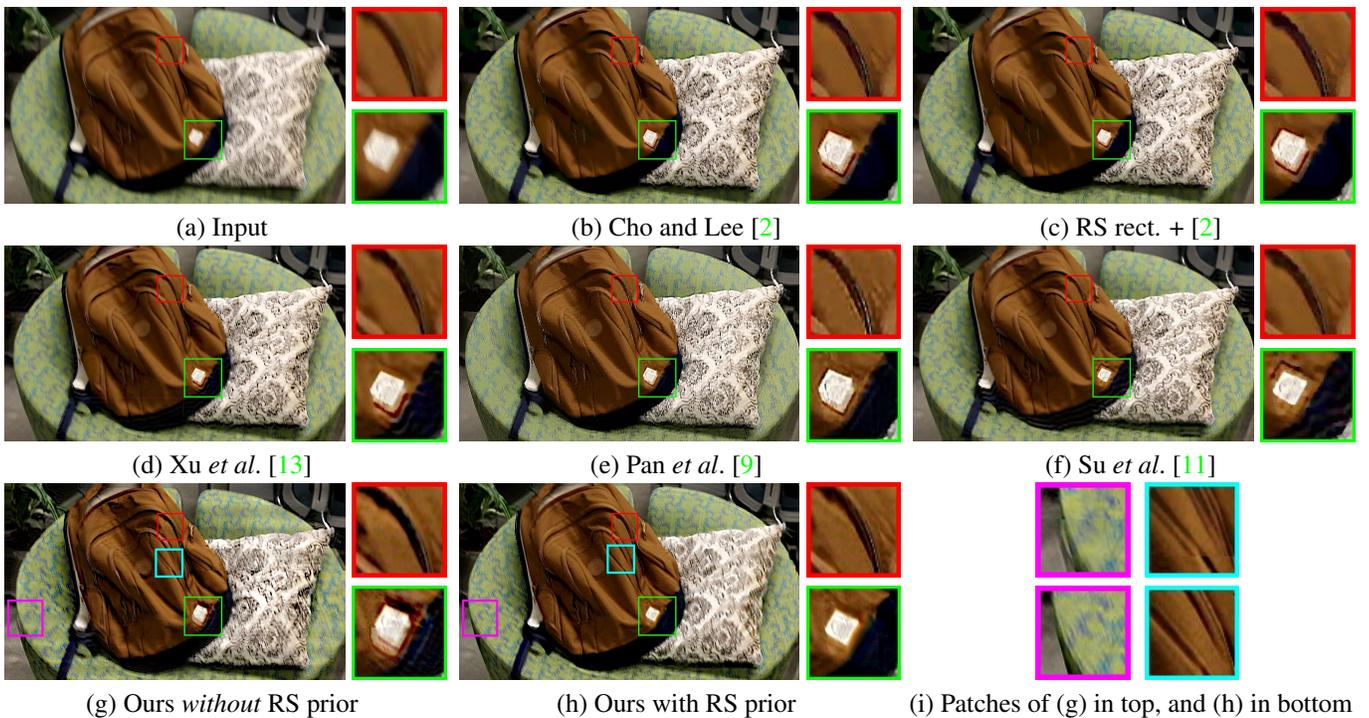


Figure S8. Detailed comparisons for RS narrow-angle example in dataset [11] (Fig. 7-second-row). Our method recovers finer details (see bag-zipper in patch 1), and deblur with negligible ringing artefacts (see bag-badge in patch 2), as compared to competing methods.

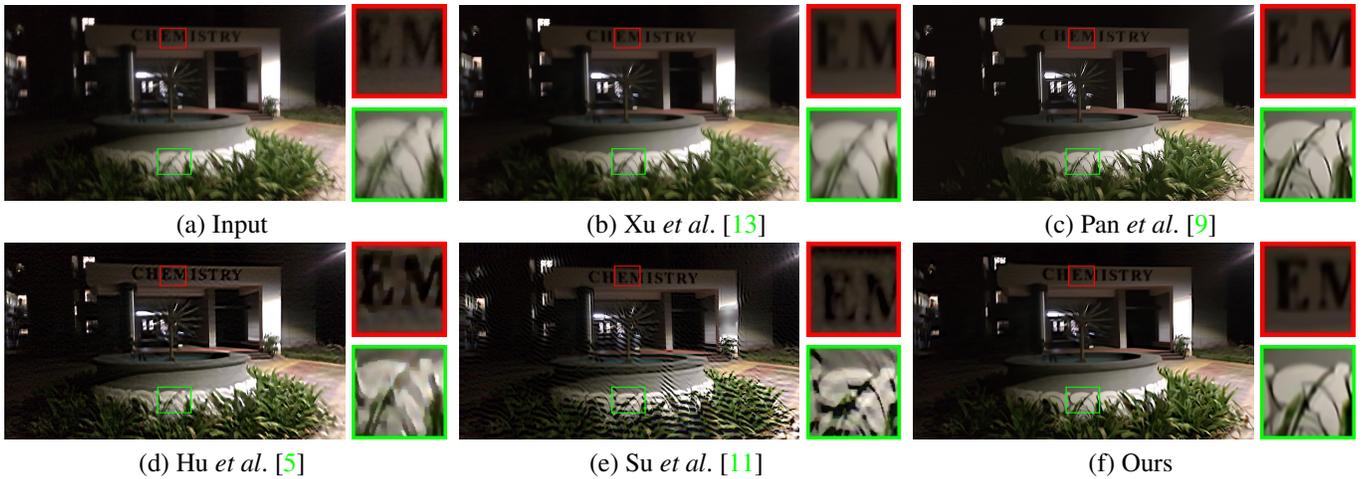


Figure S9. Detailed comparisons for RS wide-angle example (Fig. 8-first row). In contrast to competing methods, our method models the RS ego-motion better (observe the residual blur in the letters, and the repeated occurrence of the longest grass leaf in (c)).

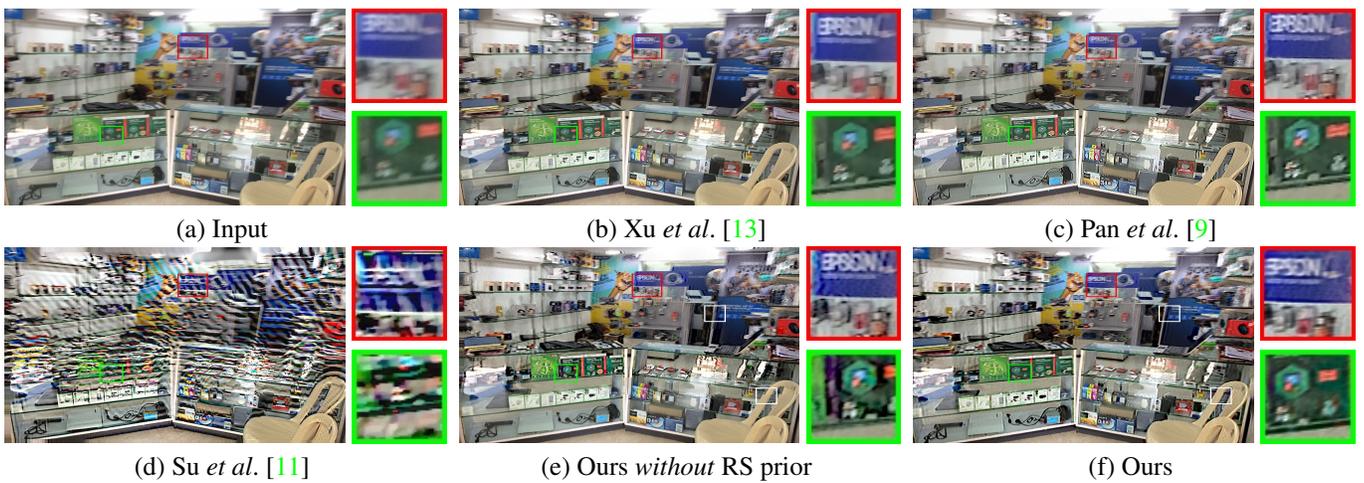


Figure S10. Comparisons for RS wide-angle case (Fig. 8-second row). White boxes in images (e) and (f) show the effect of RS prior.

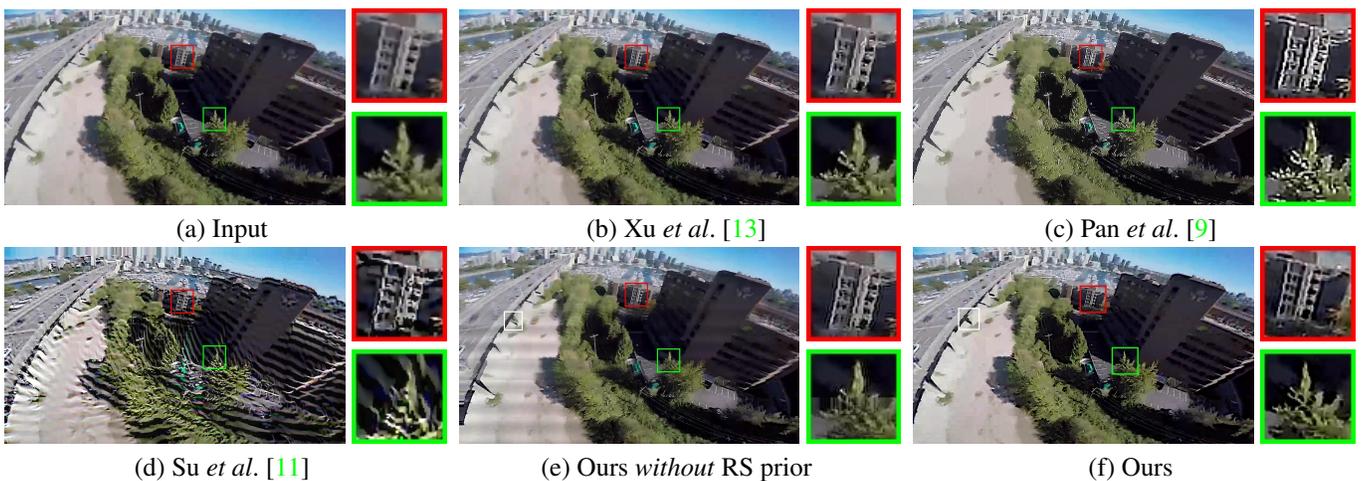


Figure S11. Comparisons for RS wide-angle example (Fig. 8-third row). White box in images (e) and (f) shows the effect of RS prior.

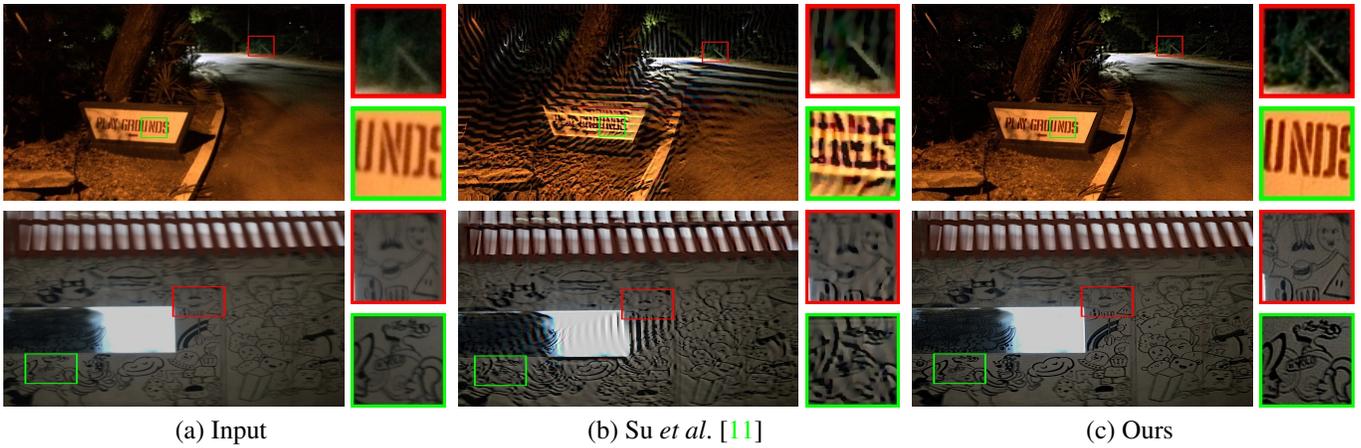


Figure S12. Additional RS comparisons with state-of-the-art RS-BMD method [11] under different lighting conditions and for wide-angle settings. Note the inefficacy of the competing method in dealing with wide angle systems.

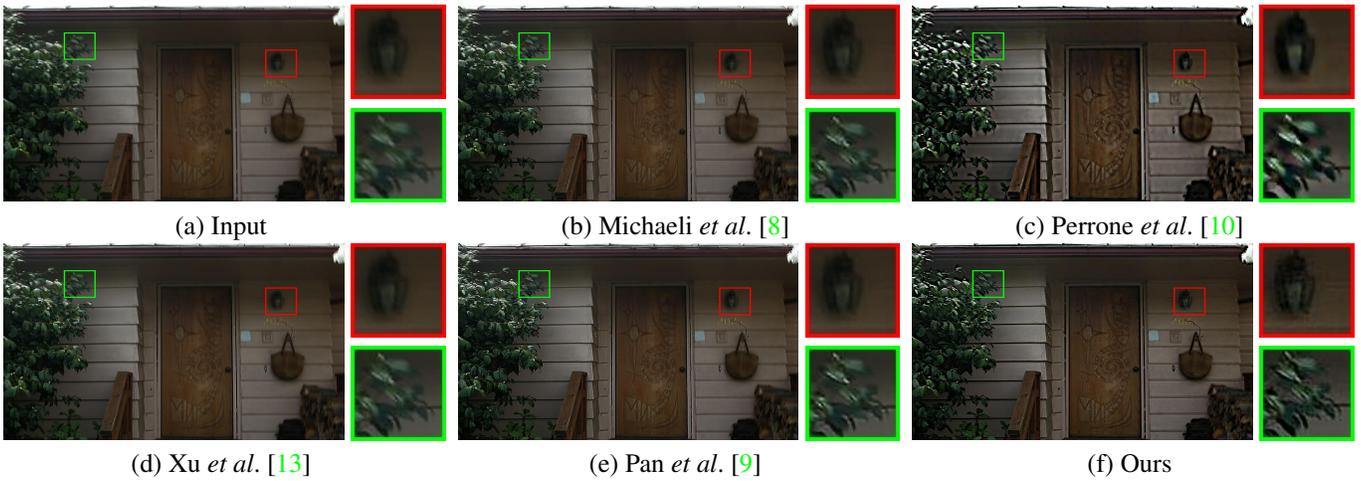


Figure S13. Comparisons for CCD blur example in dataset [7]. Our result is comparable with [8, 10, 13] and [9].



Figure S14. Comparisons for CCD blur example in dataset [9]. Our result is comparable with [3, 12, 13] and [9].

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