A. Proofs

We begin by proving that D-optimizable functions are Lipschitz on the cube $\mathcal{C}^0$ as we claimed in the beginning of Subsection 3.2. In the remainder of the Section we restate and prove the theorems and lemmas stated in the main text.

Lemma 5. *If F is D-quasi optimizable in $\mathcal{C}^0$, then there exists $L > 0$ such that.*

$$F(x_1) - F(x_2) \leq L\|x_1 - x_2\|, \forall x_1, x_2 \in \mathcal{C}^0 \tag{A.1}$$

Proof. If $F(x_1) \leq F(x_2)$ then (A.1) holds for all $L \geq 0$. Now assume $F(x_1) > F(x_2)$ and let y_2 be the minimizer of $E(x_2, \cdot)$. Then due to the differentiability of $E(\cdot, y_2)$,

$$F(x_1) - F(x_2) \leq E(x_1, y_2) - E(x_2, y_2) \leq L\|x_1 - x_2\|$$

where L is the maximum of the norm of the gradient of $E(\cdot, y)$ over all $x \in \mathcal{C}^0$ and $y \in \mathcal{Y}$. □

Theorem 2. *There exist positive constants $C_1, \dots, C_4$, such that*

$$C_1\epsilon^{-D/2} \leq n_{\mathrm{BnB}} \leq C_2\epsilon^{-D}. \tag{14}$$

$$n_{\mathrm{qBnB}} \leq C_3\epsilon^{-D/2}. \tag{15}$$

Furthermore, if E has a finite number of minimizers $(x^*_\ell, y^*_\ell)_{\ell=1}^N$, and the Hessian of $E(\cdot, y^*_\ell)$ is strictly positive definite for all ℓ, then

$$n_{\mathrm{qBnB}} \leq C_4 \log_2(1/\epsilon). \tag{16}$$

Proof. **Part 1.** We begin with a general discussion of the complexity of Algorithm 1 in both the BnB and quasi-BnB version, and prove the upper bound on n_{BnB} and n_{qBnB} simultaneously. To do so we denote

$$\Delta_\alpha(\delta) = M\delta^\alpha, \alpha = 1, 2. \tag{A.2}$$

We denote by n_α the number of F-evaluations used by the algorithm in each case, so

$$n_1 = n_{\mathrm{BnB}}, n_2 = n_{\mathrm{qBnB}}.$$

For simplicity of notation we will use lb_i in the first part of the proof to denote both lower bounds and quasi lower bounds (in contrast with the qlb_i notation used elsewhere). We also note that while our bounds (18),(21) include besides $M\delta^2$ also higher terms in δ these terms can be absorbed into $M\delta^2$ with a larger value of M.

In the proof we will call g from Algorithm 1 the *generation* of the algorithm. We begin with showing that the algorithm necessarily terminates, and bounding the final value of g which we denote by g_f. First recall from Subsection 3.2 that if $\mathcal{C}_{h_i}(x_i) \in L_g$ contains a global minimum x_*, then its sub-cubes will always be added to L_{g+1}. Recall also that the global lower bound lb obtained by minimizing over all lb_i obtained from the cubes in L_g, is a global lower bound for F over $\mathcal{C}^0$. It follows that if the algorithm does terminate, then the output x_* is an ϵ-optimal solution as its difference from the minimum F^* satisfies

$$F(x_*) - F^* = \mathrm{ub} - F^* \leq \mathrm{ub} - \mathrm{lb} \leq \epsilon. \tag{A.3}$$

Now to bound g_f note that cubes in L_g have half-edge length $h(g) \equiv h_0 2^{-g}$. The algorithm must terminate once it visits all cubes of generation g whose edge length $h(g)$ satisfies

$$\Delta_\alpha(\sqrt{D}h(g)) \leq \epsilon. \tag{A.4}$$

This is because for all cubes $\mathcal{C}_{h_j}(x_j)$ in this generation,

$$\mathrm{ub} - \mathrm{lb}_j \leq \mathrm{ub}_j - \mathrm{lb}_j = \Delta_\alpha(\sqrt{D}h_j) \leq \epsilon.$$

and by taking the minimum over j we obtain that $\mathrm{ub} - \mathrm{lb} \leq \epsilon$.

Some algebraic manipulation shows that (A.4) occurs when

$$g_f = g_f(\epsilon) = \lceil 1/\alpha \log \frac{\bar{C}_\alpha}{\epsilon} \rceil \text{ where } \bar{C}_\alpha = M(\sqrt{D}h_0)^\alpha \tag{A.5}$$

and we use $\log = \log_2$ throughout this proof. The number of F evaluations n_α is bounded by the worst case scenario where all cubes need to be divided in all generations

$$n_\alpha \le \sum_{g=0}^{g_f} 2^{Dg} = 2^{Dg_f} \sum_{g=0}^{g_f} 2^{-Dg} \le 2^{Dg_f} \sum_{g=0}^{\infty} 2^{-Dg} = 2^{Dg_f} \frac{1}{1-2^D} \le^{(A.5)} \left[\frac{2^D}{1-2^D} \bar{C}_\alpha^{D/\alpha} \right] \epsilon^{-D/\alpha}.$$

this proves the upper bound on n_1, n_2.

Part 2. We now show a lower bound on n_{BnB}. Our first step is to show that the sub-cubes of a given cube have better=larger lower bounds. To see this let $\mathcal{C}^1 = \mathcal{C}_{h_1}(x_1)$ be a cube, and let $\mathcal{C}^2 = \mathcal{C}_{h_2}(x_2)$ be one of its sub-cubes. Then $h_2 = h_1/2$ and $|x_1 - x_2| = \sqrt{D} h_1/2$. It follows that, using the notation of (A.2) with $\alpha = 1$,

$$F(x_2) \ge F(x_1) - M\sqrt{D} h_1/2 \tag{A.6}$$

Now denoting by lb_1 and lb_2 the lower bounds computed for the cubes $\mathcal{C}^1$ and $\mathcal{C}^2$ respectively, we have

$$\mathrm{lb}_2 = F(x_2) - M\sqrt{D} h_1/2 \ge^{(A.6)} F(x_1) - M\sqrt{D} h_1 = \mathrm{lb}_1,$$

and so we have $\mathrm{lb}_2 \ \ge \mathrm{lb}_1$ as we stated.

Next note that by the quadratic bound (9) we have that for a global minimum x_* and $\eta = \sqrt{\epsilon/C}$,

$$F(x) - F(x_*) \le \epsilon, \text{ for all } x \in B_\eta(x_*). \tag{A.7}$$

Now, let g_F denote the value of $g_f(2\epsilon)$ from (A.5), for the case $\alpha = 1$. Recall that $g_f(2\epsilon)$ is defined as the first integer for which (A.4) holds, where ϵ is replaced by 2ϵ, and $\alpha = 1$. Thus for $g < g_F$ we have that

$$M\sqrt{D} h(g) > 2\epsilon \tag{A.8}$$

Let $\mathcal{C}_{h_i}(x_i)$ be a cube of generation g_0 containing x_*, where g_0 is large enough so that the diameter of the cube is smaller than η and thus it is contained in $B_\eta(x_*)$. This occurs for

$$g_0 = \lceil \log(\bar{C}/\sqrt{\epsilon}) \rceil, \text{ where } \bar{C} = 2h_0\sqrt{CD}.$$

Every sub-cube $\mathcal{C}_{h_j}(x_j)$ of $\mathcal{C}_{h_i}(x_i)$, from any generation $g_0 \le g < g_F$, satisfies

$$\mathrm{lb}_j = F(x_j) - M\sqrt{D} h_j <^{(A.8)} F(x_j) - 2\epsilon \le^{(A.7)} F(x_*) - \epsilon. \tag{A.9}$$

In particular it follows that the cube $\mathcal{C}_{h_i}(x_i)$, and all its sub-cubes, will be visited during the BnB search. This is because we saw that lower bounds improve by refinement, and so by (A.9) any cube from the earlier generations $g < g_0$ which contains $\mathcal{C}_{h_i}(x_i)$, also has lower bounds which are lower that the global minimum (by at least ϵ) and so such a cube would not be removed from the search..

We can now bound n_{BnB} by the number of subcubes of $\mathcal{C}_{h_i}(x_i)$ at the $g_F - 1$ generation alone:

$$n_{\mathrm{BnB}} \ge 2^{D(g_F - 1 - g_0)} \tag{A.10}$$

Now

$$g_F - 1 - g_0 = -1 + \lceil \log \frac{\bar{C}_1}{2\epsilon} \rceil - \lceil \log(\bar{C}/\sqrt{\epsilon}) \rceil \ge \log(\bar{C}_1/(2\bar{C}\sqrt{\epsilon})) - 2 = \log(\bar{C}_1/(8\bar{C}\sqrt{\epsilon})).$$

So returning to (A.10) we obtain

$$n_{\mathrm{BnB}} \ge 2^{D(g_f - 1 - g_0)} \ge \left(\frac{\bar{C}_1}{8\bar{C}} \right)^D \epsilon^{-D/2}.$$

Part 3. We now turn to prove the last part of the theorem. Let $\mathcal{J}(\ell)$ denote the set of indices k for which (x_ℓ^*, y_k^*) is a minimizer. Note the we always have that ℓ is in $\mathcal{J}_\ell$, and if $k \in \mathcal{J}(\ell)$ then $x_\ell^* = x_k^*$. Let m be half of the minimun over the minimal eigenvalue of the hessian of $E(\cdot, y_\ell^*)$ at x_ℓ^* for all ℓ. The assumption that E has a finite number N of minimizers x_ℓ^*, y_ℓ^*, with strictly positive definite hessian, implies that $m > 0$, and so for small enough positive η,

$$F(x) - F(x_\ell^*) = \min_{k \in \mathcal{J}(\ell)} E(x, y_k^*) - E(x_k^*, y_k^*) \ge m\|x - x_\ell^*\|^2, \ \forall 1 \le \ell \le N \text{ and } \forall x \in B_\eta(x_\ell^*). \tag{A.11}$$

The minimum of F on $\mathcal{C}^0 \setminus \bigcup_i B_{\eta/2}(x^*_\ell)$ is strictly larger than F^*. Therefore there exists some g_0 independent of ϵ, such that all cubes of generation g_0 which are not contained in one of the balls $B_\eta(x^*_\ell)$ will be removed in the g_0-th stage.

We now claim that for $g \geq g_0$, g-th generation cubes $\mathcal{C}_{h_i}(x_i)$ contained in one of the balls $B_\eta(x^*_\ell)$ will be removed if

$$\|x_i - x^*_\ell\|_\infty > \sqrt{\frac{2MD}{m}} h_i = \sqrt{\frac{2\Delta_*(\sqrt{D}h_i)}{m}}. \tag{A.12}$$

This is because

$$\begin{aligned}
\mathrm{qlb}_j &= F(x_i) - \Delta_*(\sqrt{D}h_i) \geq^{(A.11)} F(x^*_\ell) + m\|x_i - x^*_\ell\|^2 - \Delta_*(\sqrt{D}h_i) \\
&= \mathrm{ub} + (F(x^*_\ell) - \mathrm{ub}) + m\|x_i - x^*_\ell\|^2 - \Delta_*(\sqrt{D}h_i) \geq^{(*)} \mathrm{ub} + m\|x_i - x^*_\ell\|^2 - 2\Delta_*(\sqrt{D}h_i) \\
&\geq \mathrm{ub} + m\|x_i - x^*_\ell\|^2_\infty - 2\Delta_*(\sqrt{D}h_i) >^{(A.12)} \mathrm{ub}
\end{aligned}$$

where $(*)$ follows from the fact that if $\mathcal{C}_{h_i}(x_i)$ is the g-th generation cube containing x^*_ℓ, then

$$F(x^*_\ell) - \mathrm{ub} \geq F(x^*_\ell) - \mathrm{ub}_i = F(x^*_\ell) - F(x_i) \geq -\Delta_*(\sqrt{D}h_i).$$

Now for $g \geq g_0$, the condition (A.12) is not fulfilled in at most $\bar{C} = (\sqrt{\frac{2MD}{m}} + 2)^D$ cubes surrounding each minimizer, and so in total only $N\bar{C}$ cubes can survive each generation $g > g_0$. The important point is that this number is independent of ϵ. So the total number of iterations is bounded by the sum of the total number of cubes in all generations $g \leq g_0$, which is some constant indepentent of ϵ which we denote by b, and the constant $N\bar{C}$ multiplied by the remaining number of iterations $g_f - g_0$, that is

$$n_2 \leq b + (g_f - g_0)N\bar{C} \leq n_2 \leq b + g_f N\bar{C} \leq^{(A.5)} b + N\bar{C}(1/2\log\frac{C_2}{\epsilon} + 1)$$

This bound can be replaced with a bound of the form (16) with an appropriate constant. □

Theorem 3. *Let $\delta > 0, r \in \mathbb{R}^D$ and r_* be a global minimizer of F_{bi}, and assume $\|r - r_*\| \leq \delta$. Let $\sigma_\mathcal{P}, \sigma_\mathcal{Q}$ denote the Frobenius norm of the matrices whose columns are the points in $\mathcal{P}$ and $\mathcal{Q}$ respectively. Then $\Delta_*(\delta)$ is given by*

$$F_{\mathrm{bi}}(r) - F_{\mathrm{bi}}(r_*) \leq \Delta_*(\delta) \equiv \frac{2}{n}\sigma_\mathcal{P}\sigma_\mathcal{Q}\,\psi_2(\delta) \tag{18}$$

Proof. To conclude the proof of the theorem for the case $r_* = 0$ we need to show

Lemma 6. *For all $r \in \mathbb{R}^D$,*

$$\|[r]\|_{\mathrm{op}} \leq \|r\| \tag{A.13}$$

Proof. The non-zero eigenvalues λ_j of a skew-symmetric real matrix $[r]$ can be written as

$$a_1 i, -a_1 i, a_2 i, -a_2 i, \ldots$$

where $a_1 \geq a_2 \ldots > 0$. Therefore

$$\|[r]\|^2_{\mathrm{op}} = a_1^2 \leq 1/2\sum_i |\lambda_i|^2 = 1/2\|[r]\|^2_F = \|r\|^2.$$

□

For the general case $r_* \neq 0$, we define a change of variable $\tilde{p}_i = R_{r_*}p_i$ and denote by $\tilde{E}_{\mathrm{bi}}$ the energy resulting by replacing p_i with $\tilde{p}_i$ in the definition of E_{bi}. Then for all R_0, π we have

$$\tilde{E}_{\mathrm{bi}}(R_0 R^T_{r_*}, \pi) = E_{\mathrm{bi}}(R_0, \pi).$$

In particular $\tilde{r}_* = 0$ is a minimizer of $\tilde{F}_{\mathrm{bi}}$ which is defined by replacing E_{bi} with $\tilde{E}_{\mathrm{bi}}$ in the definition of F_{bi}. We claim that there exists r_1 such that

$$R_{r_1} = R_r R^T_{r_*} \text{ and } \|r_1\| \leq \|r - r_*\| \tag{A.14}$$

In the case $d = 2$ we can identify $R_r R_{r_*}^T$ with $e^{i(r-r_*)}$ and so we can simply choose $r_1 = r - r_*$. For $d = 3$ it is proven in Lemma 3.2 in [18] that the angular distance between R_r and R_{r_*} is smaller or equal to $\|r - r_*\|$. As the angular distance is invariant to multiplication by rotations this means that the angular distance between $R_r R_{r_*}^T$ and the identity is less than $\|r - r_*\|$. Since the exponential map is a radial isometry in $B_\pi(0)$ this implies the existence of r_1 satisfying (A.14).

Now for every δ such that $\|r - r_*\| \le \delta$, we using the bound from (18) for $\tilde{F}$ which is minimized at zero, and satisfies $\|r_1 - 0\| \le \delta$, to obtain

$$F_{\text{bi}}(r) - F_{\text{bi}}(r_*) = \tilde{F}_{\text{bi}}(r_1) - \tilde{F}_{\text{bi}}(0) \le \frac{2}{n}\psi_2(\delta)\sigma_{\tilde{\mathcal{P}}}\sigma_{\mathcal{Q}} = \frac{2}{n}\psi_2(\delta)\sigma_{\mathcal{P}}\sigma_{\mathcal{Q}} \quad .$$

□

Theorem 4. *Let (r_*, t_*) be a minimizer of F_{CP}, and let $(r,t) \in \mathbb{R}^s \times \mathbb{R}^d$, and $\delta_1, \delta_2 > 0$ which satisfy $\|r - r_*\| \le \delta_1 and \|t - t_*\| \le \delta_2$. Let f_* be some upper bound for the global minimum of F_{CP}. Then*

$$F_{\text{CP}}(r,t) - F_{\text{CP}}(r_*, t_*) \le \Delta_*(\delta_1, \delta_2) \tag{20}$$

where

$$\begin{aligned}\Delta_*(\delta_1,\delta_2) = \frac{1}{n}\Big[&2\psi_2(\delta_1)(\sigma_{\mathcal{P}}^2 + \sigma_{\mathcal{P}}\sqrt{nf_*}) \\ &+ 2\delta_2\psi_1(\delta_1)\sum_i \|p_i\| + n\delta_2^2\Big]\end{aligned} \tag{21}$$

Proof. The proof is very similar to the proof of Theorem 3. Let us first consider the case $(r_*, t_*) = (0,0)$, and let π_* be the corresponding mapping so that $(I_d, 0, \pi_*)$ minimizes E_{CP}. Then

$$\begin{aligned}F_{\text{CP}}(r,t) - F_{\text{CP}}(0,0) &\le E_{\text{CP}}(r,t,\pi_*) - E_{\text{CP}}(0,0,\pi_*) \\ &= \frac{1}{n}\sum_{i=1}^n \left[2\langle (I_d - R_r)p_i, q_{\pi_*(i)}\rangle + 2\langle R_r p_i, t\rangle - 2\langle t, q_{\pi_*(i)}\rangle + \|t\|^2\right] \\ &=^{(*)} \frac{1}{n}\sum_{i=1}^n \left[2\sum_{k=2}^\infty \frac{1}{k!}\langle [r]^k p_i, q_{\pi_*(i)}\rangle + 2\sum_{k=1}^\infty \frac{1}{k!}\langle [r]^k p_i, t\rangle] + \|t\|^2\right] \\ &\le \frac{1}{n}\sum_{i=1}^n \left[2\sum_{k=2}^\infty \frac{1}{k!}\|r\|^k\|p_i\|(\|p_i\| + \|q_{\pi_*(i)} - p_i\|) + 2\sum_{k=1}^\infty \frac{1}{k!}\|r\|^k\|p_i\|\,\|t\| + \|t\|^2\right] \\ &\le \frac{1}{n}\left[2\psi_2(\delta_1)(\sigma_{\mathcal{P}}^2 + \sigma_{\mathcal{P}}[\sum_i \|q_{\pi_*(i)} - p_i\|^2]^{1/2}) + 2\psi_1(\delta_1)\delta_2\sum_i\|p_i\| + n\delta_2^2\right] \\ &\le \frac{1}{n}\left[2\psi_2(\delta_1)(\sigma_{\mathcal{P}}^2 + \sigma_{\mathcal{P}}\sqrt{nf_*}) + 2\psi_1(\delta_1)\delta_2\sum_i\|p_i\| + n\delta_2^2\right]\end{aligned}$$

Here $(*)$ follows from the fact that $E_{\text{CP}}(\cdot,\cdot,\pi_*)$ is minimized at the origin and so the first order terms cancel out, and the next inequalities follow from the Cauchy-Schwarz inequality and from Lemma 6.

For general (r_*, t_*), we use a change of variables $\tilde{p}_i = R_* p_i$, $\tilde{q}_i = q_i - t_*$, and denote by $\tilde{E}_{\text{CP}}$ and $\tilde{F}_{\text{CP}}$ the functions obtained by replacing p_i, q_i by $\tilde{p}_i, \tilde{q}_i$ in the definition of these functions. For given (r,t,π) we have

$$E_{\text{CP}}(R_r, t, \pi) = \tilde{E}_{\text{CP}}(R_r R_*^T, t - t_*, \pi)$$

For r, r_*, t, t_* satisfying $\|r - r_*\| \le \delta_1$ and $\|t - t_*\| \le \delta_2$, we choose $r_1 \in \mathbb{R}^D$ satisfying (A.14), and so we can applying the theorem to $\tilde{F}_{\text{CP}}$ which is minimized at $(0,0)$ to obtain

$$\begin{aligned}F_{\text{CP}}(r,t) - F_{\text{CP}}(r_*, t_*) &= \tilde{F}_{\text{CP}}(r_1, t - t_*) - \tilde{F}_{\text{CP}}(0,0) \\ &\le \frac{1}{n}\left[2\psi_2(\delta_1)(\sigma_{\mathcal{P}}^2 + \sigma_{\mathcal{P}}\sqrt{nf_*}) + 2\psi_1(\delta_1)\delta_2\sum_i\|p_i\| + n\delta_2^2\right]\end{aligned}$$

□

B. BnB for rigid closest point

In the following we explain how we construct a quasi-BnB framework for the rigid CP problem based on the BnB architecture proposed in Go-ICP [33]. Following Go-ICP, we use a nested BnB structure: We perform an "outer" BnB search on the rotation space, wherein the upper and lower bounds are functions of the translation component t; in turn, to compute these bounds we perform an "inner" BnB search over the variable t. Namely, for given r_i, we define an upper bound for the outer BnB by

$$\bar{E}_{\mathrm{CP}}(r_i) = \min_{t \in \mathcal{C}_1(0), \pi \in \Pi_{\mathrm{CP}}} E_{\mathrm{CP}}(r_i, t, \pi). \tag{B.1}$$

To compute a quasi-lower bound for the outer BnB we note that if (r_*, t_*) minimizes F_{CP} and $r_* \in \mathcal{C}_h(r_i)$, then by using (21) where we set $r = r_i$, take δ_1 to be the maximal distance of a point in the cube from the center, $t = t_*$ and $\delta_2 = 0$ we obtain

$$F_{\mathrm{CP}}(r_*, t_*) \geq F_{\mathrm{CP}}(r_i, t_*) - \frac{2}{n}\left(1 + \sqrt{\frac{f_*}{\sigma_{\mathcal{P}}}}\right) \sigma_{\mathcal{P}} \psi_2(\sqrt{D}h), \tag{B.2}$$

and since $F_{\mathrm{CP}}(r_i, t_*) \geq \bar{E}_{\mathrm{CP}}(r_i)$ it follows that if $r_* \in \mathcal{C}_h(r_i)$ then

$$F_{\mathrm{CP}}(r_*, t_*) \geq \bar{E}_{\mathrm{CP}}(r_i) - \frac{2}{n}\left(1 + \sqrt{\frac{f_*}{\sigma_{\mathcal{P}}}}\right) \sigma_{\mathcal{P}} \psi_2(\sqrt{D}h). \tag{B.3}$$

The RHS of the equation above gives us our quasi lower bound for the rotation quasi BnB. To compute $\bar{E}_{\mathrm{CP}}(r_i)$ we compute a BnB in translation space, where throughout the translation BnB the rotation coordinate r_i is fixed. For a given translation cube $\mathcal{C}_h(t_j)$ an upper bound for the value of $\bar{E}_{\mathrm{CP}}(r_i)$ is given by evaluation of $F_{\mathrm{CP}}(r_i, t_j)$. If t_* is a minimizer of $F_{\mathrm{CP}}(r_i, \cdot)$ then a quasi-lower bound in the cube is given by

$$E_{\mathrm{CP}}(r_i, t_*) \geq E_{\mathrm{CP}}(r_i, t_j) - \frac{dh^2}{n}. \tag{B.4}$$

We note that this bound is similar to what we would get by setting $\delta_1 = 0$ and δ_2 to be the maximal distance in the cube from t_j in (21). Although the bound does not follow directly from this equation the derivation is similar, and can be obtained by studying the behavior of a minimizer of $E(r_i, \cdot, \pi_*)$, so we do not go into the details. Finally we note that when the quasi-lower bounds in the outer or inner BnB is lower than zero we replace it with zero.

The rest of the architecture of the BnB is also borrowed from Go-ICP. We use best-first-search, where the cube with the lowest lower bound is visited first. Every time the upper bound is improved, an ICP algorithm is run to improve the resolution of the solution. For more details see [33].

C. Morphological data

The morphological data for the experiment shown in Figure 5 comes from the MorphoSource dataset. The figure shows ten different second mandibular molars of spider monkeys (Ateles), which come from three different taxonomical groups. More details on the data are shown in the table below.

ID	specimen #	specimen taxonomy	ark ID
M782-661	AMNH:M:67102	Ateles belzebuth	http://n2t.net/ark:/87602/m4/M661
M783-663	AMNH:M:71787	Ateles belzebuth	http://n2t.net/ark:/87602/m4/M663
M785-665	AMNH:M:76882	Ateles belzebuth	http://n2t.net/ark:/87602/m4/M665
M787-666	USNM:241384	Ateles belzebuth	http://n2t.net/ark:/87602/m4/M666
M788-668	USNM:406674	Ateles belzebuth	http://n2t.net/ark:/87602/m4/M668
M790-669	USNM:406675	Ateles belzebuth	http://n2t.net/ark:/87602/m4/M669
M791-671	MCZ:34320	Ateles geoffroyi	http://n2t.net/ark:/87602/m4/M671
M793-673	MCZ:mamm:bom-5344	Ateles geoffroyi	http://n2t.net/ark:/87602/m4/M673
M795-675	USNM:mammals:336204	Ateles geoffroyi	http://n2t.net/ark:/87602/m4/M675
M797-677	MCZ:31759	Ateles paniscus	http://n2t.net/ark:/87602/m4/M677