

Further Discussions on Details of PCA-GM

Details on alternative cross-graph design. Our cross-graph affinity component of PCA-GM in the main paper is relatively simple. We also experimented a more complex alternative design of cross-graph module, where the matrix $\hat{\mathbf{S}}$ is updated by iterative prediction, rather than predicted from shallower GNN layer as original PCA-GM. Details of this alternative design including intra-graph embedding layers is shown in Alg. 1. As shown in the main paper, experimental result reveals the degradation in performance with this alternative design, compared to our simple but effective cross-graph module. Such phenomena may be caused by the heavy iteration adopted in this alternative design, which may affect the stability of backward gradient in training.

Algorithm 1: Iterative cross-graph embedding for graph matching

Input: CNN features $\{\mathbf{h}_{1i}^{(0)}, \mathbf{h}_{2j}^{(0)}\}_{i \in \mathcal{V}_1, j \in \mathcal{V}_2}$; number of iterations K

- 1 // first intra-graph aggregation Eq. (5, 6, 7)
- 2 $\{\mathbf{h}_{si}^{(1)}\} \leftarrow \text{GConv}_1(\mathbf{A}_s, \{\mathbf{h}_{si}^{(0)}\});$
- 3 // Initialize $\hat{\mathbf{S}}^{(0)}$ as zero matrix
- 4 $\hat{\mathbf{S}}^{(0)} \leftarrow \mathbf{0}^{N \times N};$
- 5 **for** $k \leftarrow \{0..K\}$ **do**
- 6 // cross-graph aggregation Eq. (9, 10, 11)
- 7 $\{\mathbf{h}_{1i}^{(2)}\} \leftarrow \text{CrossConv}(\hat{\mathbf{S}}^{(k-1)}\{\mathbf{h}_{1i}^{(1)}\}, \{\mathbf{h}_{2j}^{(1)}\});$
- 8 $\{\mathbf{h}_{2j}^{(2)}\} \leftarrow \text{CrossConv}(\hat{\mathbf{S}}^{(k-1)\top}, \{\mathbf{h}_{2j}^{(1)}\}, \{\mathbf{h}_{1i}^{(1)}\});$
- 9 // second intra-graph aggregation Eq. (5, 6, 7)
- 10 $\{\mathbf{h}_{si}^{(3)}\} \leftarrow \text{GConv}_2(\mathbf{A}_s, \{\mathbf{h}_{si}^{(2)}\});$
- 11 // correspondence prediction Eq. (13, 16)
- 12 build $\hat{\mathbf{M}}^{(0)}$ from $\{\mathbf{h}_{1i}^{(3)}\}, \{\mathbf{h}_{2j}^{(3)}\}$ by Eq. (13)
- 13 $\hat{\mathbf{S}}^{(k)} \leftarrow \text{Sinkhorn}(\hat{\mathbf{M}}^{(0)});$
- 14 // prepare feature for next iteration
- 15 $\{\mathbf{h}_{1i}^{(1)}\} \leftarrow \{\mathbf{h}_{1i}^{(3)}\};$
- 16 $\{\mathbf{h}_{2j}^{(1)}\} \leftarrow \{\mathbf{h}_{2j}^{(3)}\};$

Output: embedding features $\{\mathbf{h}_{1i}^{(3)}, \mathbf{h}_{2j}^{(3)}\}_{i \in \mathcal{V}_1, j \in \mathcal{V}_2}$
