

# Supplementary material for PR Product: A Substitute for Inner Product in Neural Networks

## 1. Training Curves on CIFAR10

Figures 1-4 show the training curves of some classification models on CIFAR10 used in the paper, from which we can see that the models of PR Product version get consistent lower error rates than the models of standard inner product version.

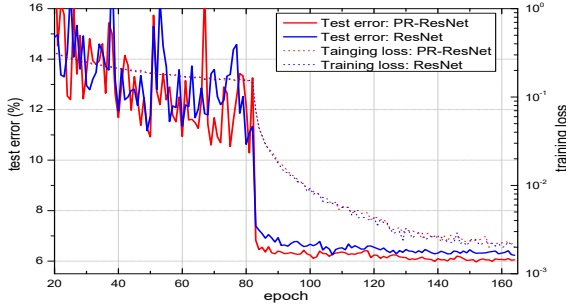


Figure 1. Training curves of the ResNet on CIFAR10.

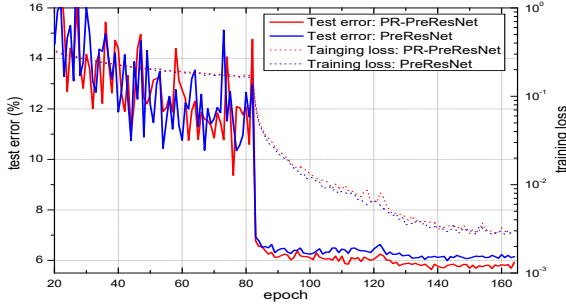


Figure 2. Training curves of the PreResNet on CIFAR10.

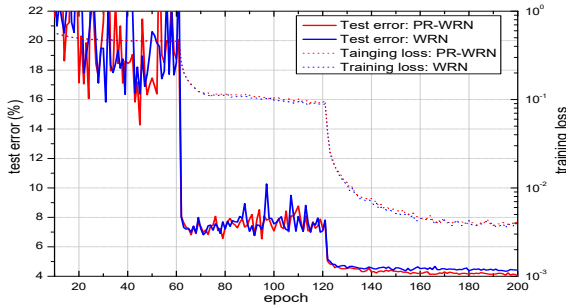


Figure 3. Training curves of the WRN on CIFAR10.

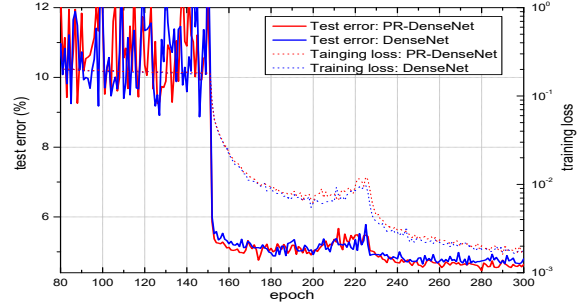


Figure 4. Training curves of the DenseNet on CIFAR10.

## 2. The Minimum of $|\sin \theta|$

We plot the minimum of  $|\sin \theta|$  in some layers of our captioning model, as shown in Figures 5-13. From these plots, we can observe that the minimum of  $|\sin \theta|$  in PR Product version is larger than the one in P Product version for most of the layers, which means the weight vector and data vector in PR Product are more orthogonal. We argue this is the reason for PR Product to take effect.

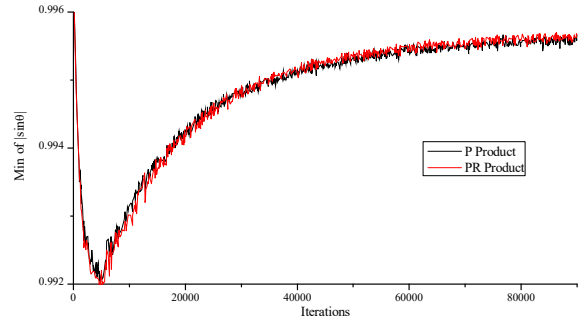


Figure 5. The minimum of  $|\sin \theta|$  of the  $\mathbf{a}_i$  to  $\mathbf{v}_i$  transfer part in the Encoder.

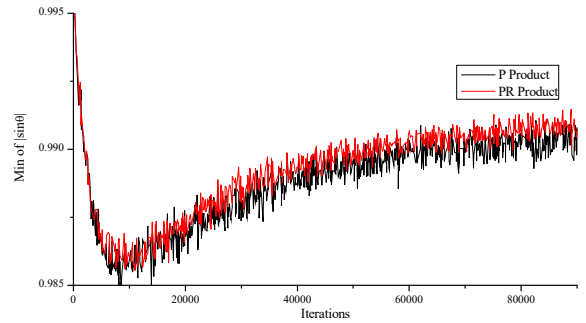


Figure 6. The minimum of  $|\sin \theta|$  of the  $\mathbf{a}_g$  to  $\mathbf{v}_g$  transfer part in the Encoder.

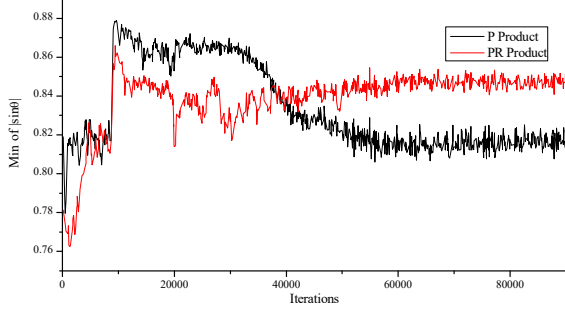


Figure 7. The minimum of  $|\sin \theta|$  of the  $W_e \Pi_t$  to hidden transfer part in the Attention LSTM.

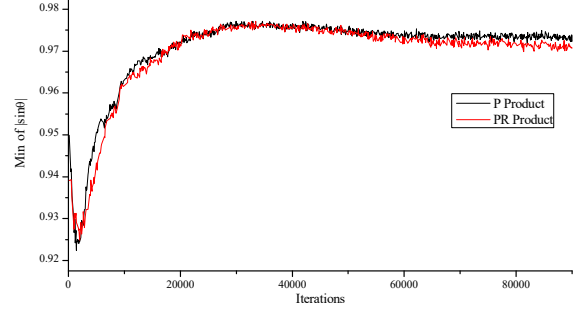


Figure 11. The minimum of  $|\sin \theta|$  of the  $h_t^1$  to hidden transfer part in the Language LSTM.

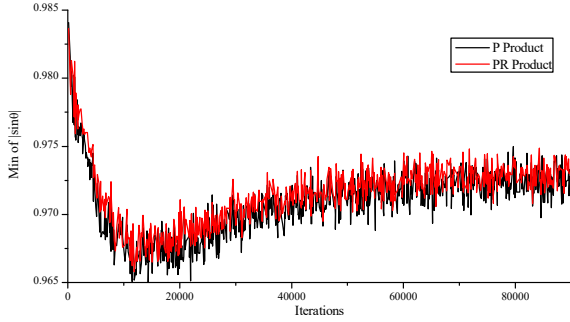


Figure 8. The minimum of  $|\sin \theta|$  of the  $v_g$  to hidden transfer part in the Attention LSTM.

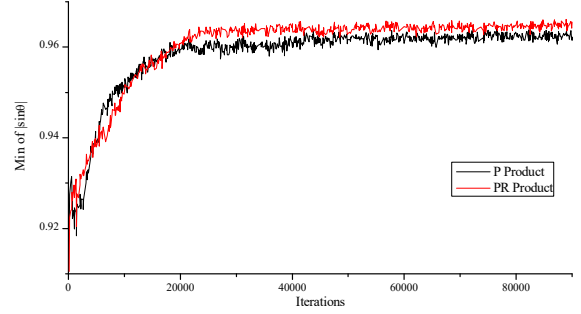


Figure 12. The minimum of  $|\sin \theta|$  of the hidden to hidden transfer part in the Language LSTM.

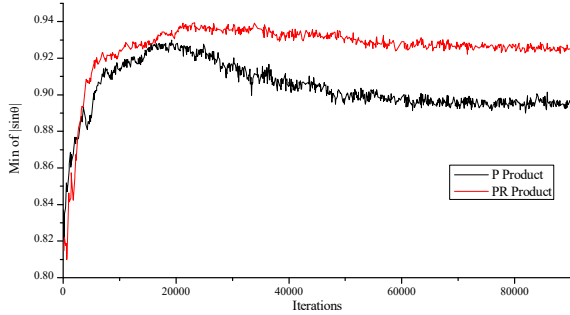


Figure 9. The minimum of  $|\sin \theta|$  of the hidden to hidden transfer part in the Attention LSTM.

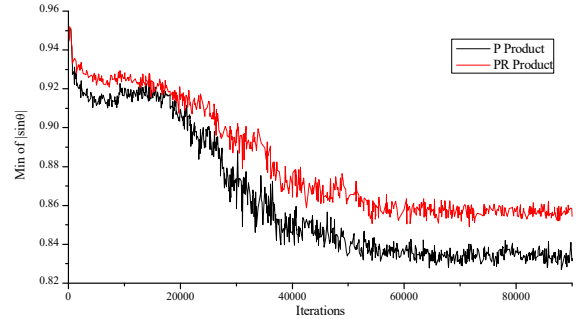


Figure 13. The minimum of  $|\sin \theta|$  of the output layer (softmax layer in the Decoder of our captioning model.)

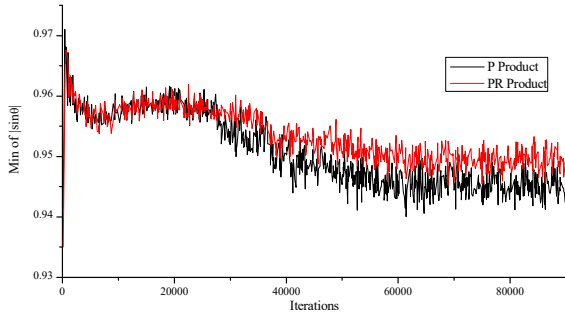


Figure 10. The minimum of  $|\sin \theta|$  of the  $\hat{v}_t$  to hidden transfer part in the Language LSTM.

### 3. Examples of Image Captioning

To intuitively illustrate the advantage of the PR Product, we show some examples of image captioning in Figure 14. The images are sampled from Karpathy’s test split of MS COCO dataset. All the three models (P version, R version, and PR version) are trained with cross-entropy loss and then fine-tuned for CIDEr optimization. The results show that PR product makes contribution to the descriptiveness of the sentences and prove that the PR Product is effective.



**GT:** several people are flying kites in an open field  
**P:** a man standing in a field flying a kite  
**R:** a man flying a kite in a field  
**PR:** a group of people flying kites in a field



**GT:** a man riding skis across a snow covered slope  
**P:** a person standing on skis in the snow  
**R:** a person standing on skis in the snow  
**PR:** a person riding skis on a snow covered slope



**GT:** a group of people in a pool with floating plates of food  
**P:** a group of people sitting at a table  
**R:** a group of people sitting around a table  
**PR:** a group of people in a pool



**GT:** a group of people gathered at the bottom of a snow mountain  
**P:** a group of people on skis on a ski lift  
**R:** a group of people riding skis on a ski lift  
**PR:** a group of people on skis on a snow covered mountain



**GT:** a large passenger jet flying through a cloudy sky  
**P:** a large airplane flying in the sky  
**R:** a large airplane flying in the sky  
**PR:** a large airplane flying in a cloudy sky



**GT:** a man holding a very big pair of scissors  
**P:** a man holding a guitar in a room  
**R:** a man standing in a room holding a bag  
**PR:** a man holding a pair of scissors in his hand



**GT:** a bowl of broccoli sits beside a lemon wedge  
**P:** a white plate of broccoli and orange slices on a table  
**R:** a white plate of broccoli and a table  
**PR:** a white plate of broccoli and a lemon wedge



**GT:** a pile of broccoli sitting next to other vegetables  
**P:** a bunch of vegetables in a market  
**R:** a bunch of vegetables on display in a market  
**PR:** a pile of broccoli and vegetables in a market



**GT:** two very worn suitcases stacked on top of each other resting on a table  
**P:** a old suitcase with stickers on top of it  
**R:** a suitcase with stickers on it on a wall  
**PR:** two suitcases stacked on top of each other



**GT:** a shelf filled with lots of different pairs of shoes  
**P:** a bunch of cats sitting in a shelf  
**R:** a group of cats sitting on top of a shelf  
**PR:** a group of shoes sitting on top of a shelf



**GT:** two teddy bears lie propped up against a wall  
**P:** a teddy bear with a ball in front of it  
**R:** a teddy bear holding a group of balloons  
**PR:** two teddy bears sitting next to each other



**GT:** a 'No Parking' sign is attached to a traffic cone on a sidewalk  
**P:** a sign on the side of a street  
**R:** a street sign on the side of a road  
**PR:** a no parking sign on the side of a street



**GT:** a table with a plate of cut pizza, two plates of salad, and a can of soda  
**P:** a plate of food on a table with a salad  
**R:** a table with plates of food on it  
**PR:** a table with plates of food and a can of soda



**GT:** a woman standing outside taking a picture with her cellphone  
**P:** a woman holding a cell phone in her hand  
**R:** a woman holding a cell phone in her hand  
**PR:** a woman taking a picture with her cell phone



**GT:** many people on a courtyard under a clock  
**P:** a man and a woman standing in front of a clock  
**R:** a man and a woman standing next to a clock on a street  
**PR:** a group of people standing in front of a clock

Figure 14. Examples of captions on MS COCO dataset. GT: human ground truth. P: sentence generated by the P Product version model. R: sentence generated by the R Product version model. PR: sentence generated by the PR Product version model. Obviously, the PR Product performs better than the P Product and the R Product.