Supplementary Material for Automatic and Robust Skull Registration Based on Discrete Uniformization

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1. Addition Registration Results for Accuracy

In order to demonstrate the effectiveness of our method, we test our method on multiple skull surfaces in addition to the data presented in the paper submitted.

As shown in Figure 1, the registration mapping from our method pushes forward the feature points on source surface onto the target surface, and the arrow lines represent the correspondence of the feature points and the registration points. The result points on the target surface is close to, if not identical to, the feature points in human sense by visualization as well as relative errors as demonstrated in Table 1. The video in the supplementary also shows the corresponding between the source surface and the target surface. If we select one point on the source surface, the corresponding point on the target surface will be drawn in red color.

Figure 2 illustrates that our registration mapping pushes forward the colored squares to corresponding squares with same color locally. And more results are shown in Figure 3,4,5. These results are all showing the correct correspondence between the source surface and the target surface. And all the results demonstrate that our method is effective and accurate.

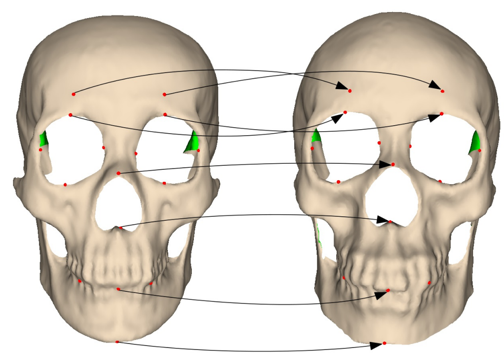

Figure 1: Our registration mapping push forward the feature points on source surface to the target surface. The arrows show the correspondence.

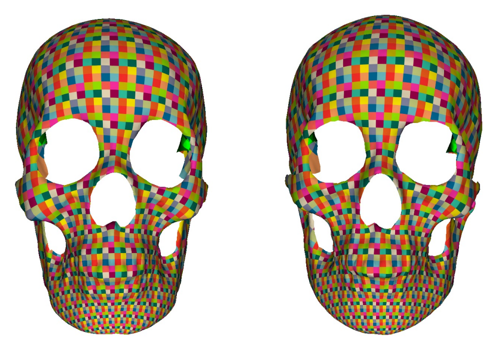

Figure 2: Our registration mapping push forward the colorful squares to corresponding color squares.

2. Our Approach vs. ICP Method

ICP (Iterative Closest Point) is a classic registration method. We have conducted the experiments with our method on 105 skulls in our database. We compared our method with classic ICP method [1], the results are shown in Table 1. The results show that the errors of our method are less than the errors of ICP method and the average improvement is 0.3323%.

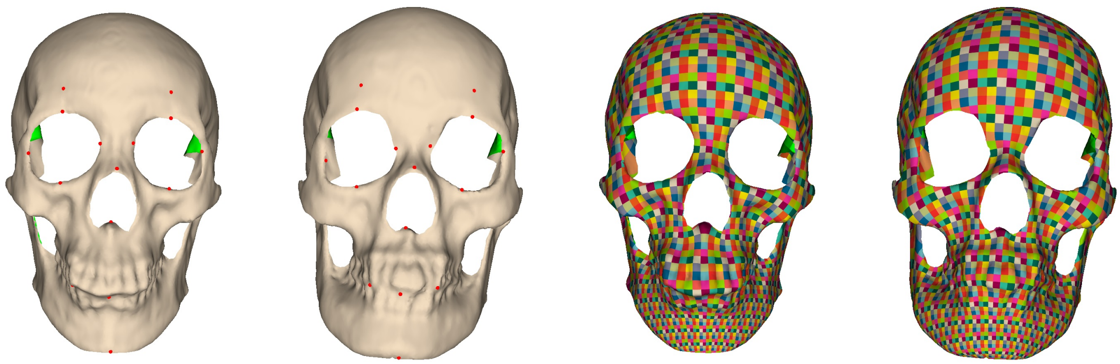

Figure 3: Registration result using data pair 0905

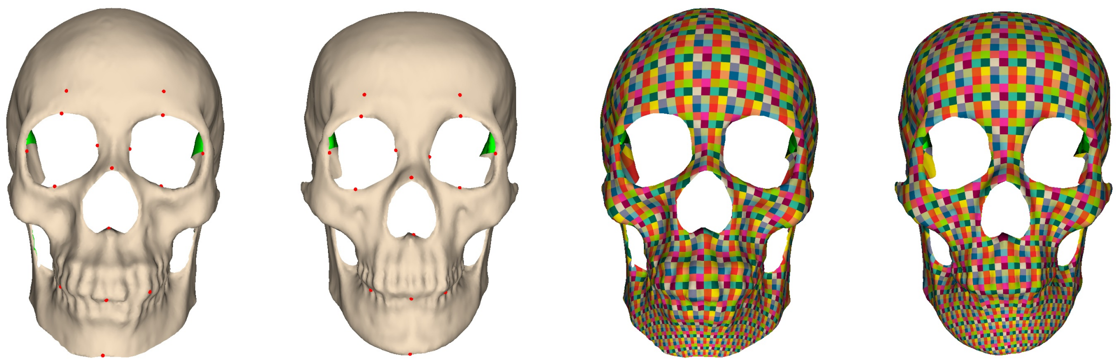

Figure 4: Registration result using data pair 1643

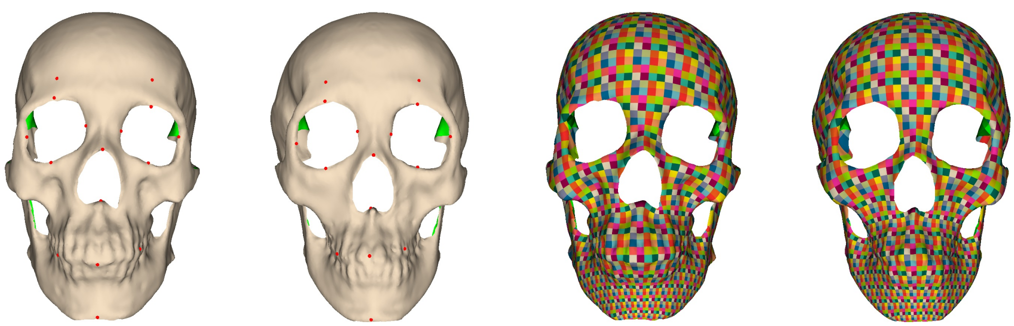

Figure 5: Registration result using data pair 3218

Table 1: Registration error comparison of our method and ICP method

No.	Our Error	ICP Error	Improvement
1	1.9682%	2.2542%	0.2860%
2	1.5481%	1.6808%	0.1327%
3	2.1935%	2.2327%	0.0392%

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No.	Our Error	ICP Error	Improvement
4	2.7229%	3.1433%	0.4204%
5	1.9749%	2.2455%	0.2706%
6	1.8778%	2.4625%	0.5847%
7	2.2242%	2.2600%	0.0358%
8	1.7474%	2.5565%	0.8091%
9	1.7320%	1.9945%	0.2625%
10	1.9184%	2.6023%	0.6839%

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No.	Our Error	ICP Error	Improvement
11	1.2427%	1.7086%	0.4659%
12	1.7013%	2.0332%	0.3319%
13	1.9915%	2.7389%	0.7474%
14	2.2114%	2.4812%	0.2698%
15	2.6469%	3.0035%	0.3566%
16	2.3150%	2.6531%	0.3381%
17	1.9443%	2.1480%	0.2037%
18	2.1358%	2.7000%	0.5642%
19	2.2479%	2.8431%	0.5952%
20	2.2764%	2.5261%	0.2497%
21	2.3663%	2.4162%	0.0499%
22	2.0729%	2.5233%	0.4504%
23	1.6448%	2.3980%	0.7532%
24	2.2018%	2.8951%	0.6933%
25	2.1035%	2.2970%	0.1935%
26	2.5958%	3.1531%	0.5573%
27	2.4228%	3.1517%	0.7289%
28	2.0890%	2.2468%	0.1578%
39	2.1801%	2.5765%	0.3964%
30	2.1515%	2.8060%	0.6545%
31	2.0274%	2.1134%	0.0860%
32	2.1691%	2.6989%	0.5298%
33	2.2185%	2.5115%	0.2930%
34	1.8979%	2.2611%	0.3632%
35	2.1589%	2.1740%	0.0151%
36	2.0827%	2.4281%	0.3454%
37	2.4766%	2.7660%	0.2894%
38	1.9430%	2.3530%	0.4100%
39	2.1815%	2.3320%	0.1505%
40	2.1880%	2.4625%	0.2745%
41	2.4646%	3.0091%	0.5445%
42	2.0819%	2.1544%	0.0725%
43	2.0240%	2.1766%	0.1526%
44	1.9694%	2.4337%	0.4643%
45	2.6712%	2.8589%	0.1877%
46	1.7941%	2.0773%	0.2832%
47	1.9851%	2.3644%	0.3793%
48	2.1668%	2.6205%	0.4537%
49	2.4121%	3.2153%	0.8032%
50	2.3501%	2.3589%	0.0088%
51	1.9240%	2.2861%	0.3621%
52	1.8868%	2.3102%	0.4234%
53	1.6170%	2.2891%	0.6721%
54	2.4413%	2.7294%	0.2881%
55	2.2334%	2.2418%	0.0084%
56	2.2969%	2.6464%	0.3495%
57	2.4740%	2.7912%	0.3172%

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No.	Our Error	ICP Error	Improvement
58	2.2834%	2.6371%	0.3537%
59	2.6587%	2.8310%	0.1723%
60	2.6744%	2.9184%	0.2440%
61	2.2622%	2.6707%	0.4085%
62	2.8569%	3.0284%	0.1715%
63	2.9625%	3.1262%	0.1637%
64	1.8879%	2.1588%	0.2709%
65	2.0160%	2.5039%	0.4879%
66	2.2824%	2.3925%	0.1101%
67	2.4341%	2.6006%	0.1665%
68	2.2785%	2.1729%	-0.1056%
69	2.0346%	2.5286%	0.4940%
70	2.0056%	2.4625%	0.4569%
71	2.1322%	3.3073%	1.1751%
72	3.0569%	3.3778%	0.3209%
73	1.9876%	2.3611%	0.3735%
74	1.8030%	2.1625%	0.3595%
75	2.9508%	2.7368%	-0.2140%
76	2.1641%	2.5646%	0.4005%
77	2.4566%	2.4783%	0.0217%
78	2.7806%	2.8569%	0.0763%
79	2.3646%	2.5279%	0.1633%
80	3.0084%	3.1219%	0.1135%
81	2.2165%	2.2241%	0.0076%
82	2.0968%	2.9752%	0.8784%
83	1.8207%	2.0395%	0.2188%
84	2.6865%	3.0064%	0.3199%
85	2.3275%	2.4319%	0.1044%
86	2.4191%	2.9482%	0.5291%
87	2.3236%	2.3920%	0.0684%
88	2.1801%	2.8973%	0.7172%
89	2.9876%	3.3357%	0.3481%
90	2.6083%	2.7044%	0.0961%
91	1.7269%	2.0636%	0.3367%
92	2.7927%	2.8161%	0.0234%
93	2.1767%	2.4954%	0.3187%
94	2.6451%	2.6538%	0.0087%
95	2.4152%	2.8029%	0.3877%
96	2.7594%	3.1823%	0.4229%
97	1.9243%	2.3784%	0.4541%
98	2.4432%	2.5865%	0.1433%
99	2.5924%	2.8927%	0.3003%
100	2.3035%	2.8337%	0.5302%
101	2.5331%	3.1045%	0.5714%
102	2.3481%	2.5078%	0.1597%
103	2.4118%	2.4961%	0.0843%
104	2.8013%	3.3270%	0.5257%

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No.	Our Error	ICP Error	Improvement
105	2.6672%	3.0093%	0.3421%
Average	2.2489%	2.5812%	0.3323%

3. Comparison with Other Conformal Mapping Method

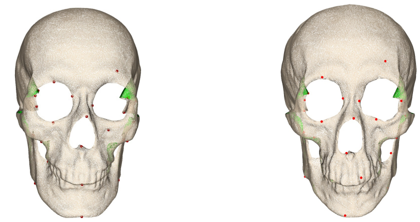
The proposed method is quite different from the existing ones. The algorithm in [4] handles surfaces with simple topologies while our proposed method is able to deal with surfaces with more complicated topologies, such as skull surface in our experiments.

The algorithm in [5] is based on discrete Yamabe flow with fixed triangulations. In theory, there is no guarantee for the existence of the solution to discrete Yamabe flow, in practice this method is vulnerable to triangulations with low quality, and the discrete Riemannian metric often becomes degenerated in the flow. Therefore, this method is impractical. The proposed method dynamically updates the triangulation, the solution exists and is unique.

The algorithm in [6] is based on holomorphic differential, which is equivalent to solve an elliptic partial differential equation using Finite Element Method. FEM requires triangulations with good qualities, otherwise the computation is unstable and the results won't converge. The proposed method has no requirements on the triangulations, it is much more robust and stable. Hence the proposed method is much more rigorous, robust and capable of handing more complicated surfaces.

Compare other conformal mapping method, the proposed method is intrinsic, independent of triangulation and robust to low quality meshes. For example, Tutte embedding method [2] is not intrinsic to the geometry of the surface, but solely depends on the triangulation. Two triangulations of the same surface will induce different Tutte embedding. Hence this method is not suitable for surface registration.

Teichmüller mapping method by Ng et.al [3] is based on holomorphic flow, which requires high quality triangulation. If the triangulation has many obtuse triangles, or the norm of the Beltrami coefficient is big, the computation becomes unstable and error-prone. Moreover, because of the lack of interior feature points as constraints, the Teichmüller mapping algorithms actually have worse registration results in our experiments. Figure 6 shows the corresponding points from Teichmüller mapping algorithms. Table 2 presents the errors of our method and Teichmüller mapping method. The results show that the error of our method is smaller than Teichmüller mapping.



(a) Feature points

(b) Corresponding points by Teichmüller mapping

Figure 6: Registration results visualized with feature points correspondence. (a) shows the feature points labeled by craniofacial experts. (b) presents the corresponding points from Teichmüller mapping results.

Table 2: Error comparison of our method and Teichmüller mapping

No.	Our Error	Teichmüller Mapping Error	Registration Improvement
1	0.0168	0.0193	0.25%

References

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[6] Wei Zeng, Yun Zeng, Yang Wang, Xiaotian Yin, Xianfeng Gu, and Dimitris Samaras. 3d non-rigid surface matching and registration based on holomorphic differentials. In *European Conference on Computer Vision*, pages 1–14. Springer, 2008.