1 Scale Invariance

Scale invariance is a desirable property of the shape descriptor, which can be achieved by one of the following four different methods:

1. Trying to detect the scale, as done in most feature descriptors (e.g. SIFT) in image analysis. However, 3D shapes are usually poorer in features and scale detection can be done reliably only at a sparse set of feature points.

2. Through the normalization of Laplace-Beltrami eigenvalues, but this method may suffer if the object has missing parts [2]. In such case, the scale invariance must be introduced locally rather than globally.

3. Using a series of transformations applied to the HKS [2] in order to avoid scale detection. This allows creating a dense descriptor. This method is considered local, thus can work with objects with missing parts.

4. The local equi-affine invariant Laplace-Beltrami operator proposed by [3].

In this work, we propose a novel local scale normalization method based on simple operations (thus belonging to the third category above). It was shown [2] that scaling a shape by a factor β results in changing $K(x,t)$ to $\beta^2\ K(x,\beta^2 t)$.

Thus, a series of transformations are applied to W-HKS as follows. At each point x, the W-HKS is sampled logarithmically in time ($t = \alpha^\tau$) and the function

$$k_\tau = K(x, \alpha^\tau) \tag{1}$$

is formed. Scaling the shape by β results in a time shift $s = 2\log_\alpha \beta$ and amplitude scaling by β^2. That is,

$$k'_\tau = \beta^2 k_{\tau+s} \tag{2}$$

[2] proposed to take the logarithmic transformation $\log\ k'_\tau = 2\log\beta + \log\ k_{\tau+s}$ which decouples the multiplicative constant from $k_{\tau+s}$. Then they proposed to take the derivative afterwards to remove the effect of the resulting additive $2\log\beta$ term and then taking the amplitude of the Fourier transform (FT) of the derivative to remove the effect of the time shift s. Since the derivative operator is sensitive to noise, their method is not robust enough.

We propose to apply the Fourier transform directly to k'_τ in (2).

$$K'(w) = \beta^2 K(w) \exp(j\ 2\pi\ w\ s). \tag{3}$$

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Then taking the amplitude of the FT,

$$\mid K'(w) \mid = \beta^2 \mid K(w) \mid \tag{4}$$

The effect of the multiplicative constant β^2 is eliminated by normalizing the $\mid K'(w) \mid$ by the sum of the amplitudes of the FT components. The amplitudes of the first significant FT components (we normally use 6) are employed to construct the scale-invariant shape descriptor. This proposed method eliminates the scale effect without having to use the noise-sensitive derivative operation or the logarithmic transformation that both were used in [2]. Thus, our method is simpler, more computational-efficient and more robust to noise. This is clearly verified in Figure 1 that shows the scale-invariant heat kernel for a HK computed at a vertex of on 3D shape and another HK computed for the same object with 3-times scale up under different noise levels. The two descriptors computed at the two different scales are virtually identical using our proposed method even at high noise levels. The method in [1] demonstrates a significant influence of the noise on the computed descriptors.

References

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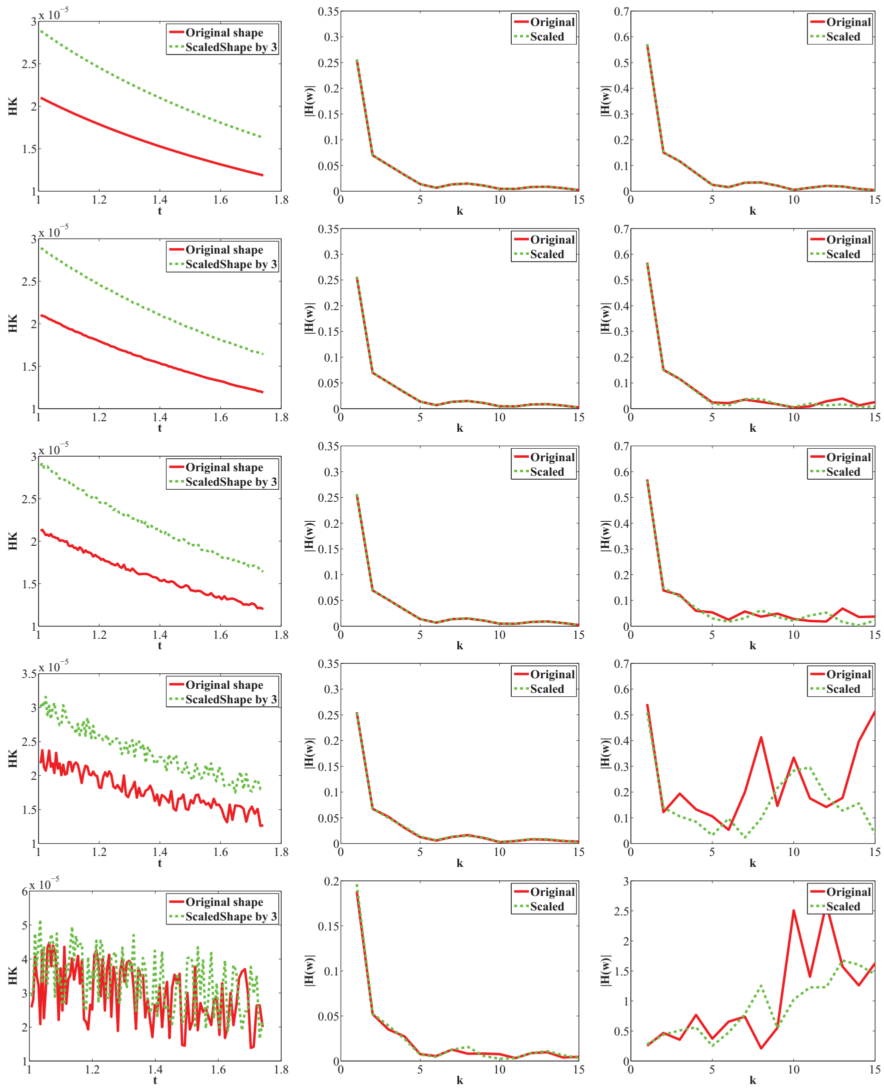

Figure 1: Construction of the scale-invariant heat kernel under several noise levels. (Left) heat kernel computed at a point on a shape at different time (red) and the scaled heat kernel (in green) computed at a corresponding point on a shape scaled up by a factor of 3. (Middle) The amplitude of the first 15 fourier transform components of $\mid K(w) \mid$ for the two heat kernels (again in red and green) using the proposed method showing complete overlapping specially in the first four noise levels. (Right) Similar but using the method in [1]. First row shows signal without noise. Then noise level is increased in the subsequent rows.