

Supplementary Material

Bayesian sparse representation
for hyperspectral image super resolution
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1 Proof of the Gibbs sampling equations

1.1 The model

Let $\mathbf{y}_i \in \mathbb{R}^L$, $\Phi \in \mathbb{R}^{L \times |\mathcal{K}|}$, $\beta_i \in \mathbb{R}^{|\mathcal{K}|}$ and $\epsilon_i \in \mathbb{R}^L$. Then $\forall i \in \{1, \dots, mn\}$ and $\forall k \in \mathcal{K} = \{1, \dots, K\}$:

$$\begin{aligned}\mathbf{y}_i &= \Phi \beta_i + \epsilon_i \\ \beta_i &= \mathbf{z}_i \odot \mathbf{s}_i \\ \varphi_k &\sim \mathcal{N}(\varphi_k | \mu_{k_o}, \Lambda_{k_o}^{-1}) \\ z_{ik} &\sim \text{Bern}(z_{ik} | \pi_{k_o}) \\ \pi_k &\sim \text{Beta}(\pi_k | a_o/K, b_o(K-1)/K) \\ s_{ik} &\sim \mathcal{N}(s_{ik} | \mu_{s_o}, \lambda_{s_o}^{-1}) \\ \epsilon_i &\sim \mathcal{N}(\epsilon_i | \mathbf{0}, \Lambda_{\epsilon_o}^{-1})\end{aligned}$$

For computational purposes, let $\mu_{s_o} = 0$, $\mu_{k_o} = \mathbf{0}$, $\Lambda_{k_o} = \lambda_{k_o} \mathbf{I}_L$ and $\Lambda_{\epsilon_o} = \lambda_{\epsilon_o} \mathbf{I}_L$, where $\mathbf{I}_L \in \mathbb{R}^{L \times L}$ is an identity matrix. We further place the following priors on the precision parameters of the normal distributions:

$$\begin{aligned}\lambda_s &\sim \Gamma(\lambda_s | c_o, d_o) \\ \lambda_\epsilon &\sim \Gamma(\lambda_\epsilon | e_o, f_o)\end{aligned}$$

We make use of the following theorem [1] in our proves:

Theorem 1 [1]: When prior distribution over \mathbf{x}_1 is given by $p(\mathbf{x}_1) = \mathcal{N}(\mathbf{x}_1 | \mu_o, \Lambda_o^{-1})$ and the likelihood function is defined as $p(\mathbf{x}_2 | \mathbf{x}_1) = \mathcal{N}(\mathbf{x}_2 | \mathbf{A}\mathbf{x}_1 + \mathbf{b}, \mathbf{L}^{-1})$, the posterior distribution over \mathbf{x}_1 can be written as $p(\mathbf{x}_1 | \mathbf{x}_2) = \mathcal{N}(\mathbf{x}_1 | \mu, \Lambda^{-1})$, where

$$\begin{aligned}\Lambda &= \Lambda_o + \mathbf{A}^T \mathbf{L} \mathbf{A} \\ \mu &= \Lambda^{-1} (\mathbf{A}^T \mathbf{L} (\mathbf{y} - \mathbf{b}) + \Lambda_o \mu_o)\end{aligned}$$

1.2 Gibbs Sampling Equations

Sample φ_k : From our model, we can write the posterior distribution over the dictionary atom $p(\varphi_k | -)$ as:

$$p(\varphi_k | -) \propto \prod_{i=1}^{mn} \mathcal{N}(\mathbf{y}_i | \Phi(\mathbf{z}_i \odot \mathbf{s}_i), \lambda_{\epsilon_o}^{-1} \mathbf{I}_L) \mathcal{N}(\varphi_k | \mathbf{0}, \lambda_{k_o}^{-1} \mathbf{I}_L)$$

In order to write the mean parameter of the likelihood function in terms of φ_k , we can write:

$$\mathbf{y}_{i\varphi_k} = \mathbf{y}_i - \Phi(\mathbf{z}_i \odot \mathbf{s}_i) + \varphi_k (z_{ik} \odot s_{ik})$$

where $\mathbf{y}_{i_{\varphi_k}}$ represents the contribution of the dictionary atom φ_k to the signal \mathbf{y}_i . This gives us the following form of the posterior distribution over φ_k :

$$p(\varphi_k | -) \propto \prod_{i=1}^{mn} \mathcal{N}(\mathbf{y}_{i_{\varphi_k}} | \varphi_k(z_{ik} \cdot s_{ik}), \lambda_{\epsilon_o}^{-1} \mathbf{I}_L) \mathcal{N}(\varphi_k | \mathbf{0}, \lambda_{k_o}^{-1} \mathbf{I}_L)$$

Using the results of Theorem 1, the posterior distribution over the dictionary atoms is given by:

$$p(\varphi_k | -) = \mathcal{N}(\varphi_k | \boldsymbol{\mu}_k, \lambda_k^{-1} \mathbf{I}_L) \quad (1)$$

where

$$\begin{aligned} \lambda_k &= \lambda_{k_o} + \lambda_{\epsilon_o} \sum_{i=1}^{mn} (z_{ik} \cdot s_{ik})^2 \\ \boldsymbol{\mu}_k &= \lambda_k^{-1} \lambda_{\epsilon_o} \sum_{i=1}^{mn} (z_{ik} \cdot s_{ik}) \mathbf{y}_{i_{\varphi_k}} \end{aligned}$$

Note that, we arrive at the above results by putting $\mathbf{A} = \sum_{i=1}^{mn} (z_{ik} \cdot s_{ik})$ and $\mathbf{b} = \mathbf{0}$ in the results of Theorem 1. The results get further simplified because of the standard multivariate Gaussian distributions with isotropic covariance/precision matrices.

Sample z_{ik} : Once φ_k has been sampled, we must sample z_{ik} (and s_{ik}) based on the updated atom. Again, using the contribution of the k^{th} dictionary atom only, the posterior over z_{ik} can be written as:

$$p(z_{ik} | -) \propto \mathcal{N}(\mathbf{y}_{i_{\varphi_k}} | \varphi_k(z_{ik} \cdot s_{ik}), \lambda_{\epsilon_o}^{-1} \mathbf{I}_L) \text{Bern}(z_{ik} | \pi_{k_o})$$

Thus,

$$\begin{aligned} p(z_{ik} = 1 | -) &\propto \pi_{k_o} \exp \left(- \frac{(\mathbf{y}_{i_{\varphi_k}} - \varphi_k s_{ik})^T \lambda_{\epsilon_o} \mathbf{I}_L (\mathbf{y}_{i_{\varphi_k}} - \varphi_k s_{ik})}{2} \right) \\ &= \pi_{k_o} \exp \left(- \frac{\lambda_{\epsilon_o}}{2} (\mathbf{y}_{i_{\varphi_k}}^T \mathbf{y}_{i_{\varphi_k}} - 2s_{ik} \mathbf{y}_{i_{\varphi_k}}^T \varphi_k + \varphi_k^T \varphi_k s_{ik}^2) \right) \\ p_1 &= \pi_{k_o} \exp \left(- \frac{\lambda_{\epsilon_o}}{2} \mathbf{y}_{i_{\varphi_k}}^T \mathbf{y}_{i_{\varphi_k}} \right) \cdot \exp \left(- \frac{\lambda_{\epsilon_o}}{2} (\varphi_k^T \varphi_k s_{ik}^2 - 2s_{ik} \mathbf{y}_{i_{\varphi_k}}^T \varphi_k) \right) \end{aligned}$$

Similarly,

$$\begin{aligned} p(z_{ik} = 0 | -) &\propto (1 - \pi_{k_o}) \exp \left(- \frac{\mathbf{y}_{i_{\varphi_k}}^T \lambda_{\epsilon_o} \mathbf{I}_L \mathbf{y}_{i_{\varphi_k}}}{2} \right) \\ &= (1 - \pi_{k_o}) \exp \left(- \frac{\lambda_{\epsilon_o}}{2} \mathbf{y}_{i_{\varphi_k}}^T \mathbf{y}_{i_{\varphi_k}} \right) \\ p_0 &= \exp \left(- \frac{\lambda_{\epsilon_o}}{2} \mathbf{y}_{i_{\varphi_k}}^T \mathbf{y}_{i_{\varphi_k}} \right) - \pi_{k_o} \exp \left(- \frac{\lambda_{\epsilon_o}}{2} \mathbf{y}_{i_{\varphi_k}}^T \mathbf{y}_{i_{\varphi_k}} \right) \end{aligned}$$

Thus, z_{ik} can be sampled from the following normalized Bernoulli distribution.

$$z_{ik} \sim \text{Bern}\left(\frac{p_1}{p_1 + p_0}\right)$$

Further simplification leads to the following formulation:

$$z_{ik} \sim \text{Bern}\left(\frac{\pi_{k_o} \xi}{1 - \pi_{k_o} + \xi \pi_{k_o}}\right) \quad (2)$$

where, $\xi = \exp\left(-\frac{\lambda_{\epsilon_o}}{2}(\boldsymbol{\varphi}_k^T \boldsymbol{\varphi}_k s_{ik}^2 - 2s_{ik} \mathbf{y}_{i\varphi_k}^T \boldsymbol{\varphi}_k)\right)$.

Sample s_{ik} : With the same reasoning as above, we can write the following about the posterior distribution over s_{ik} :

$$p(s_{ik}|-) \propto \mathcal{N}(\mathbf{y}_{i\varphi_k} | \boldsymbol{\varphi}_k(z_{ik} \cdot s_{ik}), \lambda_{\epsilon_o}^{-1} \mathbf{I}_L) \mathcal{N}(s_{ik} | 0, \lambda_{s_o}^{-1})$$

Using Theorem 1, s_{ik} can be sampled from the following distribution:

$$p(s_{ik}|-) = \mathcal{N}(s_{ik} | \mu_s, \lambda_s^{-1}) \quad (3)$$

where,

$$\begin{aligned} \lambda_s &= \lambda_{s_o} + (\boldsymbol{\varphi}_k z_{ik})^T \lambda_{\epsilon_o} \mathbf{I}_L (\boldsymbol{\varphi}_k z_{ik}) \\ &= \lambda_{s_o} + \lambda_{\epsilon_o} z_{ik}^2 \boldsymbol{\varphi}_k^T \boldsymbol{\varphi}_k \\ \mu_s &= \lambda_s^{-1} \left((\boldsymbol{\varphi}_k z_{ik})^T \lambda_{\epsilon_o} \mathbf{I}_L \mathbf{y}_{i\varphi_k} \right) \\ &= \lambda_s^{-1} \lambda_{\epsilon_o} z_{ik} \boldsymbol{\varphi}_k^T \mathbf{y}_{i\varphi_k} \end{aligned}$$

Sample π_k : Based on the model, the posterior distribution over π_k can be written as:

$$\begin{aligned} p(\pi_k|-) &\propto \prod_{i=1}^{mn} \text{Bern}(z_{ik} | \pi_{k_o}) \text{Beta}(\pi_{k_o} | a_o/K, b_o(K-1)/K) \\ &= \pi_{k_o}^{\sum_{i=1}^{mn} z_{ik}} (1 - \pi_{k_o})^{mn - \sum_{i=1}^{mn} z_{ik}} \times \pi_{k_o}^{\frac{a_o}{K} - 1} (1 - \pi_{k_o})^{\frac{b_o(K-1)}{K} - 1} \\ &= \pi_{k_o}^{\frac{a_o}{K} + \sum_{i=1}^{mn} z_{ik} - 1} (1 - \pi_{k_o})^{\frac{b_o(K-1)}{K} + mn - \sum_{i=1}^{mn} z_{ik} - 1} \\ &= \text{Beta}\left(\frac{a_o}{K} + \sum_{i=1}^{mn} z_{ik}, \frac{b_o(K-1)}{K} + mn - \sum_{i=1}^{mn} z_{ik}\right) \end{aligned}$$

Thus,

$$\pi_k \sim \text{Beta}\left(\frac{a_o}{K} + \sum_{i=1}^{mn} z_{ik}, \frac{b_o(K-1)}{K} + mn - \sum_{i=1}^{mn} z_{ik}\right) \quad (4)$$

Sample λ_s : In the model, we have assumed that $\mu_{s_k} = 0$ and $\lambda_{s_k} = \lambda_s$, $\forall k \in \mathcal{K}$. This simplification allows us to write the likelihood function for λ_s in terms of standard multivariate Gaussian with isotropic covariance matrix $\lambda_{s_o}^{-1} \mathbf{I}_K$. Thus, we can write the posterior over λ_s as following:

$$\begin{aligned} p(\lambda_s | -) &\propto \prod_{i=1}^{mn} \mathcal{N}(\mathbf{s}_i | \mathbf{0}, \lambda_{s_o}^{-1} \mathbf{I}_K) \Gamma(\lambda_{s_o} | c_o, d_o) \\ &= \frac{1}{(2\pi)^{\frac{mnK}{2}} |\lambda_{s_o}^{-1} \mathbf{I}_K|^{\frac{mn}{2}}} \exp\left(-\frac{\lambda_{s_o}}{2} \sum_{i=1}^{mn} \mathbf{s}_i^T \mathbf{s}_i\right) \frac{1}{\Gamma(c_o)} d_o^{c_o} \lambda_{s_o}^{c_o-1} \exp(-d_o \lambda_{s_o}) \end{aligned}$$

where $\Gamma(\cdot)$ is the gamma function and $|\cdot|$ denotes the determinant of the matrix. Neglecting the constants and making use of the property that $|\lambda \mathbf{I}_K| = \lambda^K$, we arrive at the following:

$$\begin{aligned} p(\lambda_s | -) &\propto \lambda_{s_o}^{\frac{Kmn}{2}} \exp\left(-\frac{\lambda_{s_o}}{2} \sum_{i=1}^{mn} \mathbf{s}_i^T \mathbf{s}_i\right) \lambda_{s_o}^{c_o-1} \exp(-d_o \lambda_{s_o}) \\ &= \lambda_{s_o}^{\frac{Kmn}{2} + c_o - 1} \exp\left(-\lambda_{s_o} \left(\frac{1}{2} \sum_{i=1}^{mn} \mathbf{s}_i^T \mathbf{s}_i + d_o\right)\right) \\ &\propto \Gamma(\lambda_{s_o} | \frac{Kmn}{2} + c_o, \frac{1}{2} \sum_{i=1}^{mn} \mathbf{s}_i^T \mathbf{s}_i + d_o) \end{aligned}$$

Therefore,

$$\lambda_s \sim \Gamma\left(\frac{Kmn}{2} + c_o, \frac{1}{2} \sum_{i=1}^{mn} \|\mathbf{s}_i\|_2^2 + d_o\right) \quad (5)$$

Sample λ_ϵ : Based on our model, we can write the posterior over λ_ϵ as:

$$p(\lambda_\epsilon | -) \propto \prod_{i=1}^{mn} \mathcal{N}(\mathbf{y}_i | \Phi(\mathbf{z}_i \odot \mathbf{s}_i), \lambda_{\epsilon_o}^{-1} \mathbf{I}_L) \Gamma(\lambda_{\epsilon_o} | e_o, f_o)$$

Similar to λ_s we can arrive at the following for sampling λ_ϵ :

$$\lambda_\epsilon \sim \Gamma\left(\frac{Lmn}{2} + e_o, \frac{1}{2} \sum_{i=1}^{mn} \|\mathbf{y}_i - \Phi(\mathbf{z}_i \odot \mathbf{s}_i)\|_2^2 + f_o\right) \quad (6)$$

2 MSE analysis for Lemma 1 in the paper:

We can write MSE as below:

$$MSE = \mathbb{E}[\|\tilde{\alpha} - \alpha\|_2^2] \quad (7)$$

In our settings, we can re-write the above definition as:

$$MSE = \sum_{\mathbf{z} \in \mathcal{Z}} P(\mathbf{z}) \mathbb{E}[\|\tilde{\alpha} - \alpha\|_2^2 | \mathbf{z}] \quad (8)$$

Below, we further analyze the expression for MSE , starting from expanding the second term in the above equation.

$$\mathbb{E}[\|\tilde{\alpha} - \alpha\|_2^2 | \mathbf{z}] = \mathbb{E}[\|\tilde{\alpha}\|_2^2 - 2\tilde{\alpha}^T \alpha + \|\alpha\|_2^2 | \mathbf{z}] \quad (9)$$

$$= \|\tilde{\alpha}\|_2^2 - 2\tilde{\alpha}^T \mathbb{E}[\alpha | \mathbf{z}] + \mathbb{E}[\|\alpha\|_2^2 | \mathbf{z}] \quad (10)$$

We can further write $\mathbb{E}[\|\alpha\|_2^2 | \mathbf{z}]$ as:

$$\begin{aligned} \mathbb{E}[\|\alpha\|_2^2 | \mathbf{z}] &= \mathbb{E}[\|\mathbb{E}[\alpha | \mathbf{z}] + \alpha - \mathbb{E}[\alpha | \mathbf{z}]\|_2^2 | \mathbf{z}] \\ &= \mathbb{E}[\|(\mathbb{E}[\alpha | \mathbf{z}]) + (\alpha - \mathbb{E}[\alpha | \mathbf{z}])\|_2^2 | \mathbf{z}] \\ &= \mathbb{E}[\|\mathbb{E}[\alpha | \mathbf{z}]\|_2^2 + 2\mathbb{E}[\alpha | \mathbf{z}](\alpha - \mathbb{E}[\alpha | \mathbf{z}]) + \|\alpha - \mathbb{E}[\alpha | \mathbf{z}]\|_2^2 | \mathbf{z}] \\ &= \mathbb{E}[\|\mathbb{E}[\alpha | \mathbf{z}]\|_2^2 + 2\alpha\mathbb{E}[\alpha | \mathbf{z}] - 2\|\mathbb{E}[\alpha | \mathbf{z}]\|_2^2 + \|\alpha - \mathbb{E}[\alpha | \mathbf{z}]\|_2^2 | \mathbf{z}] \\ &= \|\mathbb{E}[\alpha | \mathbf{z}]\|_2^2 + 2\|\mathbb{E}[\alpha | \mathbf{z}]\|_2^2 - 2\|\mathbb{E}[\alpha | \mathbf{z}]\|_2^2 + \mathbb{E}[\|\alpha - \mathbb{E}[\alpha | \mathbf{z}]\|_2^2 | \mathbf{z}] \\ &= \|\mathbb{E}[\alpha | \mathbf{z}]\|_2^2 + \mathbb{E}[\|\alpha - \mathbb{E}[\alpha | \mathbf{z}]\|_2^2 | \mathbf{z}] \end{aligned}$$

Denoting the second term in the above equation as $\mathbb{V}_{\mathbf{z}}$ (i.e. conditional variance) and putting the values in (10) gives:

$$\mathbb{E}[\|\tilde{\alpha} - \alpha\|_2^2 | \mathbf{z}] = \|\tilde{\alpha}\|_2^2 - 2\tilde{\alpha}^T \mathbb{E}[\alpha | \mathbf{z}] + \|\mathbb{E}[\alpha | \mathbf{z}]\|_2^2 + \mathbb{V}_{\mathbf{z}} \quad (11)$$

$$= \|\tilde{\alpha} - \mathbb{E}[\alpha | \mathbf{z}]\|_2^2 + \mathbb{V}_{\mathbf{z}} \quad (12)$$

Putting the values back in (8).

$$MSE = \sum_{\mathbf{z} \in \mathcal{Z}} P(\mathbf{z}) \|\tilde{\alpha} - \mathbb{E}[\alpha | \mathbf{z}]\|_2^2 + \sum_{\mathbf{z} \in \mathcal{Z}} P(\mathbf{z}) \mathbb{V}_{\mathbf{z}} \quad (13)$$

$$= \mathbb{E}[\|\tilde{\alpha} - \mathbb{E}[\alpha | \mathbf{z}]\|_2^2] + \mathbb{E}[\mathbb{V}_{\mathbf{z}}] \quad (14)$$

To find the minimizer, we derivate MSE w.r.t $\tilde{\alpha}$ and equate it to 0.

$$0 = \mathbb{E}\left[\frac{\partial}{\partial \tilde{\alpha}} \|\tilde{\alpha} - \mathbb{E}[\alpha | \mathbf{z}]\|_2^2\right] + \mathbb{E}\left[\frac{\partial}{\partial \tilde{\alpha}} \mathbb{V}_{\mathbf{z}}\right] \quad (15)$$

$$= \mathbb{E}\left[2\|\tilde{\alpha} - \mathbb{E}[\alpha | \mathbf{z}]\|_2 \cdot \frac{\tilde{\alpha} - \mathbb{E}[\alpha | \mathbf{z}]}{\|\tilde{\alpha} - \mathbb{E}[\alpha | \mathbf{z}]\|_2}\right] \quad (16)$$

$$= \tilde{\alpha} - \mathbb{E}[\mu_{\mathbf{z}}(\alpha)] \quad (17)$$

where, we have denoted $\mathbb{E}[\alpha | \mathbf{z}]$ as $\mu_{\mathbf{z}}(\alpha)$. It is clear from (17) that $\tilde{\alpha}_{\text{opt}} = \mathbb{E}[\mu_{\mathbf{z}}(\alpha)]$.

3 Further results

We have used the following definition of RMSE here and in the paper:

$$RMSE = \sqrt{\frac{\|\mathbf{T} - \tilde{\mathbf{T}}\|_2^2}{MNL}} \quad (18)$$

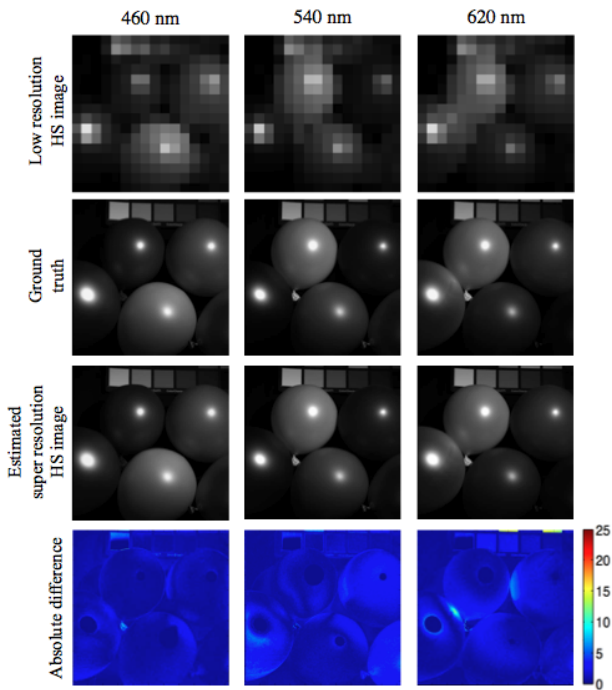
where $\tilde{\mathbf{T}} \in \mathbb{R}^{M \times N \times L}$ is the computed estimate of the the ground truth $\mathbf{T} \in \mathbb{R}^{M \times N \times L}$

3.1 Results on CAVE database

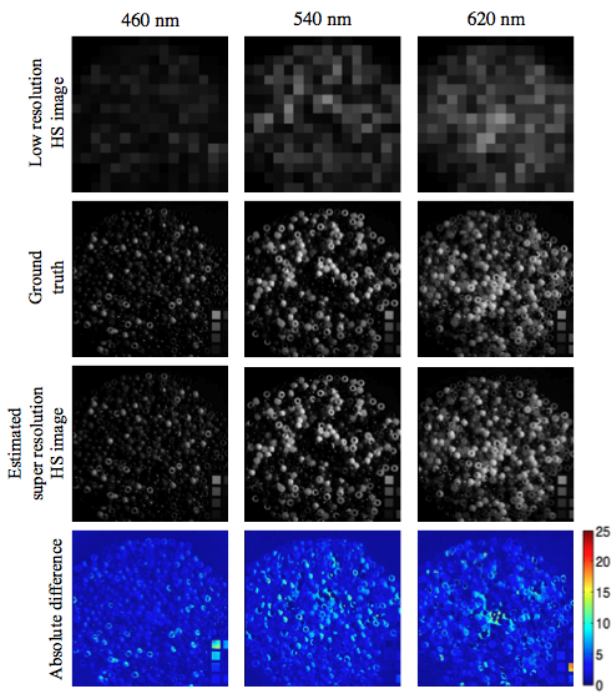
Table 1: Results on CAVE database [2]: The RMSE values are reported for all the available images. The values are in the range of 8 bit images. The images are named according to [2]. For the rows highlighted in blue, we also provide the spectral images below. We have also included the corresponding results of G-SOMP[3] for comparison.

	Image	Pro- posed	GSOMP [3]		Image	Pro- posed	GSOMP [3]
1	Balloons	2.1	2.3	16	Faces	1.9	2.2
2	Beads	5.4	6.1	17	Photo and face	1.6	2.2
3	CD	5.3	7.5	18	Hairs	2.2	2.1
4	Cloth	4.0	4.0	19	Oil painting	1.9	4.0
5	Clay	2.7	1.9	20	Paints	3.2	6.9
6	Statue	1.4	2.1	21	Water color	2.5	4.0
7	Feathers	4.0	4.1	22	Beers	2.1	2.2
8	Flowers	4.6	4.7	23	Jelly beans	4.8	5.8
9	Glass tiles	2.6	4.1	24	Lemon slices	2.0	2.9
10	Chart & toys	3.1	3.7	25	Lemons	2.4	2.7
11	Pompoms	4.0	4.4	26	Peppers	2.7	3.0
12	Sponges	4.0	2.3	27	Strawberries	2.6	3.7
13	Spools	4.6	5.0	28	Sushi	2.9	3.2
14	Stuffed toys	4.0	5.1	29	Tomatoes	2.4	2.4
15	Super balls	2.6	3.2	30	Yellow peppers	2.1	2.3

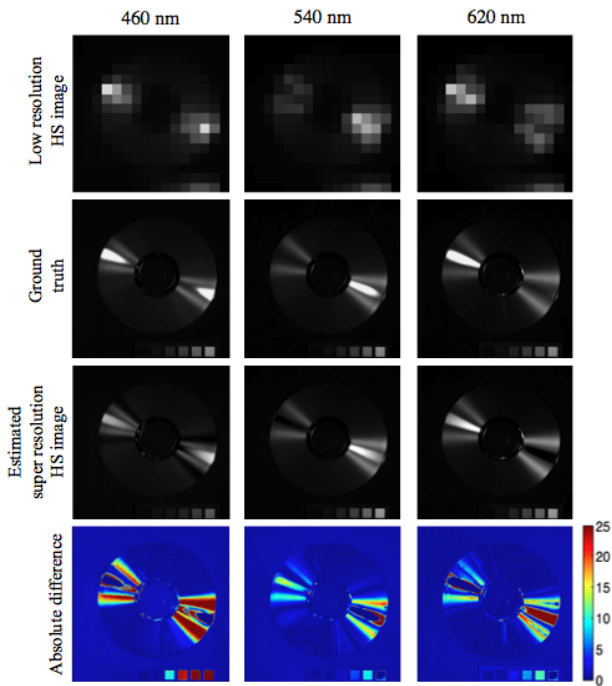
Fig. 1a - 1g show the spectral images from the CAVE database. The figures show the spectral images at 460, 540 and 620 nm. First row of each figure corresponds to the low resolution hyperspectral image. The second row shows the ground truth image for the constructed super resolution hyperspectral image, shown in the third row. The last row shows the absolute difference between the ground truth and the constructed image, where the scale is in the range of 8 bit images.



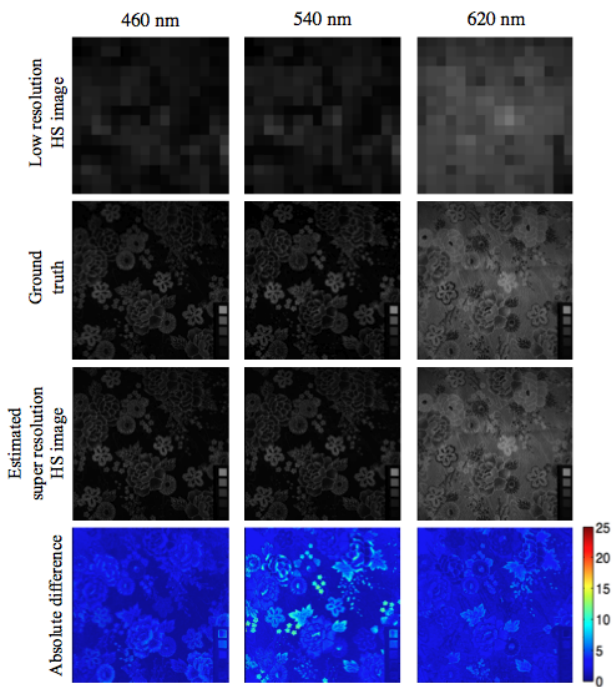
(a) Balloons (Sr. # 1 in Table 1)



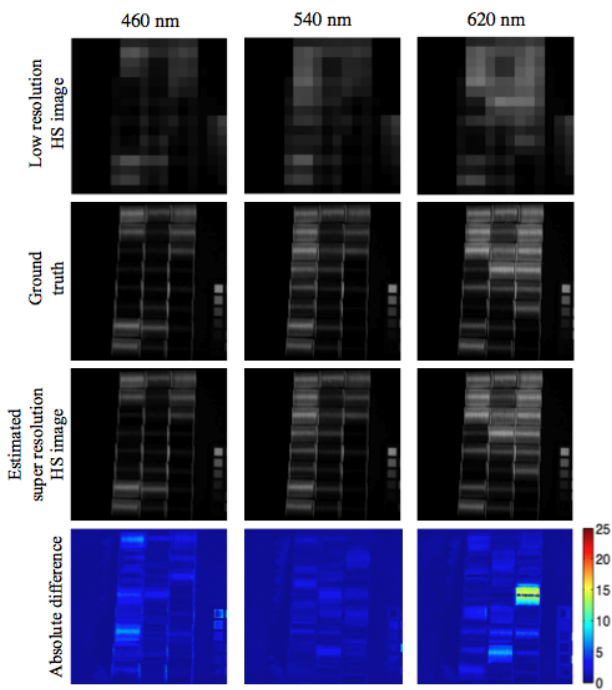
(b) Beads (Sr. # 2 in Table 1)



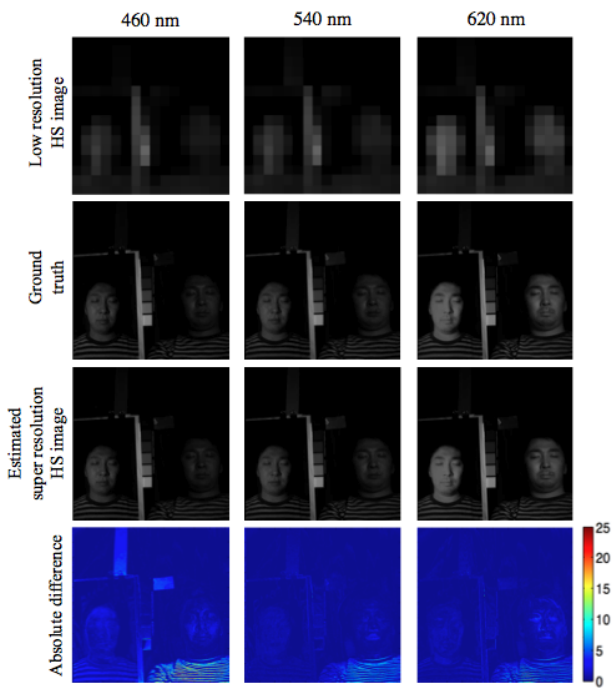
(c) CD (Sr. # 3 in Table 1)



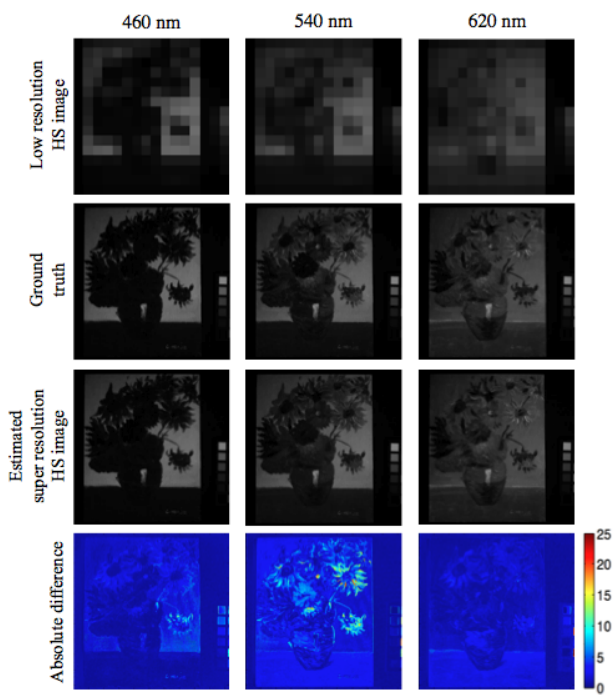
(d) Cloth (Sr. # 4 in Table 1)



(e) Thread (Sr. # 13 in Table 1)



(f) Photo (Sr. # 17 in Table 1)



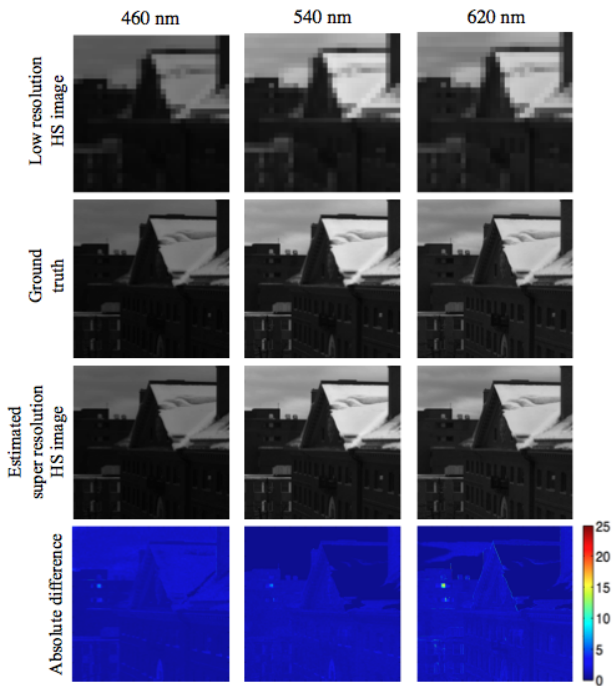
(g) Painting (Sr. # 19 in Table 1)

3.2 Results on Harvard database

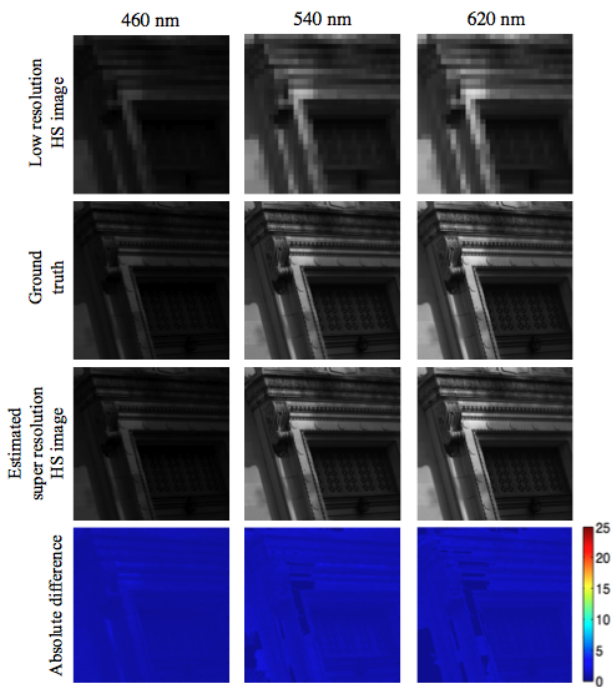
Table 2: Results on Harvard database [4]: The RMSE values are reported for all the available images. The values are in the range of 8 bit images. The images are named according to [4]. For the rows highlighted in blue, we also provide the spectral images below. Again, notice that the images selected for the paper are not based on the best achieved results.

	Image	Proposed	GSOMP [3]		Image	Proposed	GSOMP [3]
1	Image 1	1.1	1.2	26	Image d3	1.3	1.7
2	Image 2	2.6	3.4	27	Image d4	0.5	2.4
3	Image a1	1.3	1.7	28	Image d7	0.8	2.1
4	Image a2	1.0	1.1	29	Image d8	2.2	2.5
5	Image a5	0.3	0.6	30	Image d9	1.3	8.4
6	Image a6	2.1	3.2	31	Image e0	0.7	1.6
7	Image a7	0.7	3.0	32	Image e1	2.8	6.1
8	Image b0	1.5	2.0	33	Image e2	3.1	3.1
9	Image b1	0.9	1.9	34	Image e3	1.2	10.3
10	Image b2	1.1	1.6	35	Image e4	0.8	0.7
11	Image b3	3.2	4.1	36	Image e5	1.2	1.2
12	Image b4	5.2	5.7	37	Image e6	1.3	2.5
13	Image b5	0.9	0.9	38	Image e7	0.7	1.4
14	Image b6	3.2	3.7	39	Image f1	4.1	4.5
15	Image b7	8.2	9.9	40	Image f2	1.2	1.7
16	Image b8	4.3	5.9	41	Image f3	1.6	2.2
17	Image b9	0.9	2.3	42	Image f4	0.6	1.2
18	Image c1	1.3	1.9	43	Image f5	1.1	0.7
19	Image c2	2.6	2.8	44	Image f6	1.6	1.3
20	Image c4	1.0	1.0	45	Image f7	1.0	2.0
21	Image c5	0.7	6.1	46	Image f8	1.2	1.5
22	Image c7	1.1	1.7	47	Image h0	2.4	3.3
23	Image c8	5.0	4.2	48	Image h1	0.5	1.3
24	Image c9	1.7	4.3	49	Image h2	0.7	1.0
25	Image d2	0.8	2.7	50	Image h3	0.5	0.5

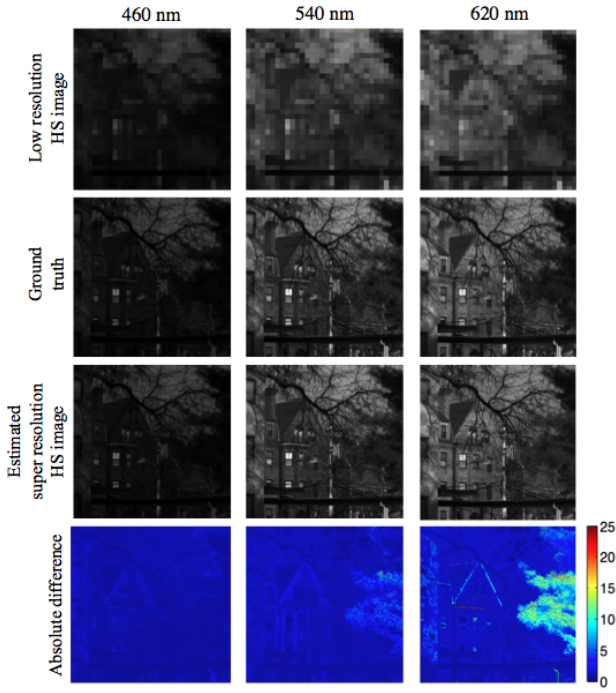
Fig. 1h - 1n show the spectral images for the Harvard database.



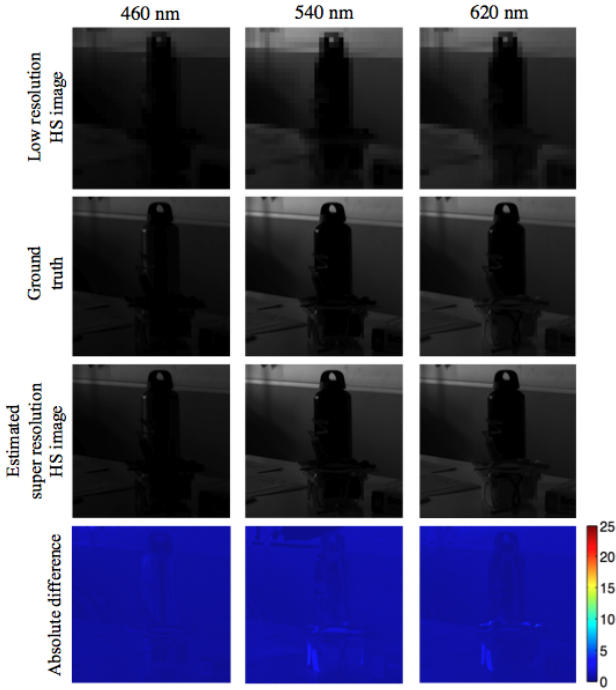
(h) Image 1 (Sr. # 1 in Table 2)



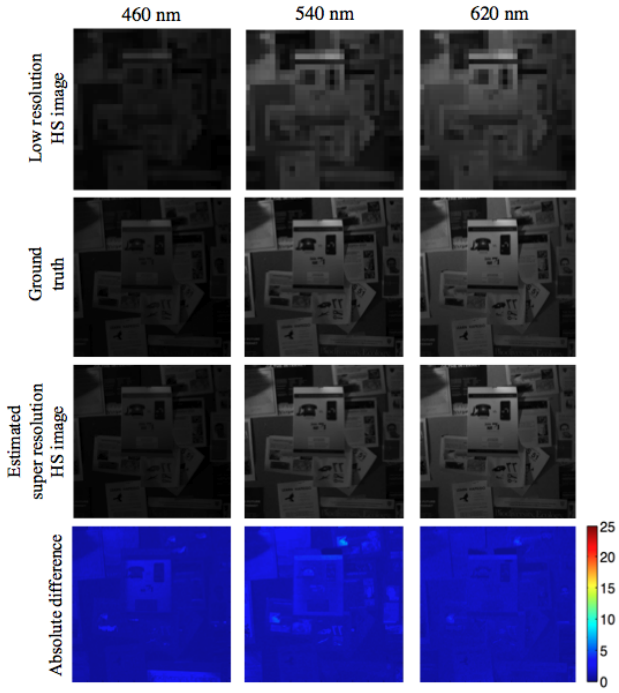
(i) Image b5 (Sr. # 13 in Table 2)



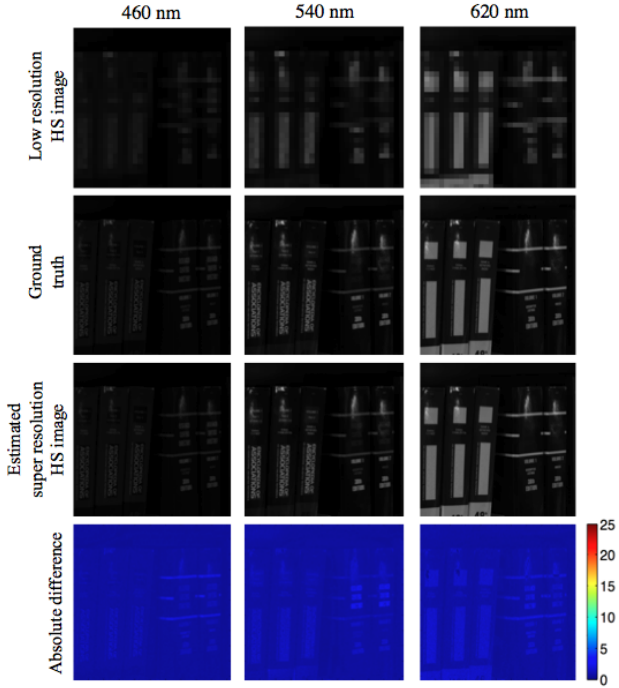
(j) Image b8 (Sr. # 16 in Table 2)



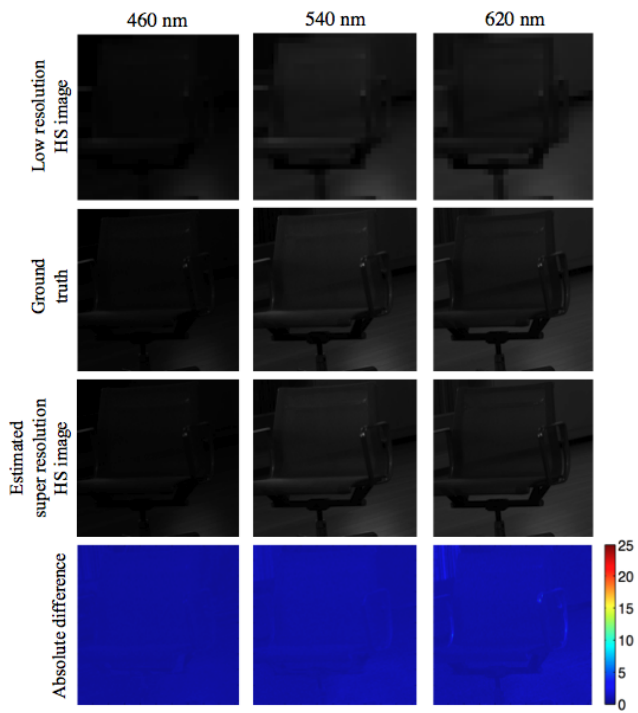
(k) Image d4 (Sr. # 27 in Table 2)



(l) Image d7 (Sr. # 28 in Table 2)



(m) Image h2 (Sr. # 49 in Table 2)



(n) Image h3 (Sr. # 50 in Table 2)

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