KL Divergence based Agglomerative Clustering for Automated Vitiligo Grading

Supplementary Material

1. Introduction

In the supplementary add on to the main paper, we present proof for Lemma 1., and comparisons against MRF and SWA in detail.

Lemma 1. *For a strictly convex function f on a closed interval $[a, b]$, let $c = w_1 a + w_2 b$ be an interior point on the interval, where $w_1 + w_2 = 1$ and $w_1, w_2 \in \mathbb{R}^+$, then*

$$(b - a)(f'(b) - f'(a)) \quad \geq \quad w_1 f(a) + w_2 f(b) - f(c).$$

Proof. As noted earlier in the main paper, the lemma can be proved based on the monotonic non-decreasing slope property of convex functions on a closed interval. Pictorially, the right hand side is the length d of the blue arrow shown in Fig. 1. The lemma claims that the maximum distance of the slope line from the curve is lesser than the slope difference at the end points multiplied by the interval.

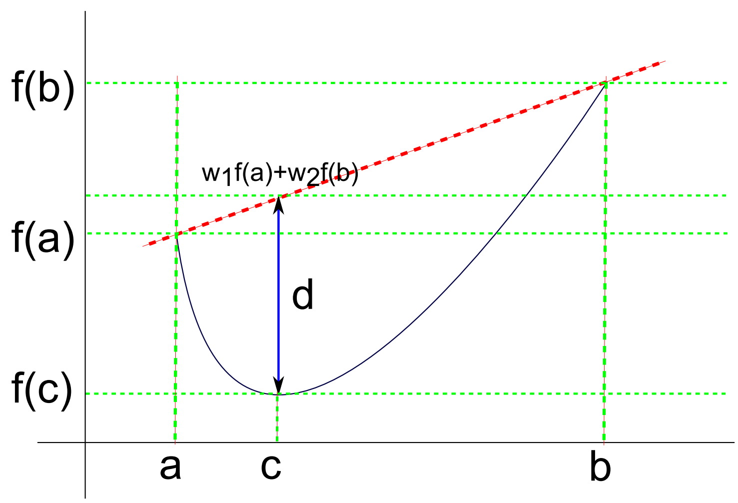

Figure 1. Strictly convex function. The maximum distance of the slope line from the curve is lesser than the slope difference at the end points multiplied by the interval.

For a strictly convex function in a bounded interval $[a, b]$, due to the monotonically non-decreasing property of the slope, we can write

$$f(a) + (b - a)f'(a) \leq f(b) \leq f(a) + (b - a)f'(b) \tag{1}$$

Let us have any internal point on the interval $a \leq c \leq b$. Then the logic can be extended to c as

$$f(a) + (c - a)f'(a) \leq f(c) \leq f(a) + (c - a)f'(c) \tag{2}$$

From Eq. 1, we can write

$$\begin{aligned}
(b-a)[f'(b)-f'(a)] &\geq f(b)-f(a)-(b-a)f'(a) && (3)\\
&\geq f(b)-f(a)-(b-c+c-a)f'(a) && (4)\\
&\geq [f(b)-(b-c)f'(a)]-[f(a)+(c-a)f'(a)] && (5)\\
&\geq [f(b)-(b-c)f'(a)]-f(c) \quad \text{Using Eq. 2} && (6)
\end{aligned}$$

substituting

$$\begin{aligned}
c &= w_1 a + w_2 b\\
b &= w_1 b + w_2 b\\
b-c &= w_1(b-a)
\end{aligned}$$

in Eq. 6, where $w_1 + w_2 = 1$, we get

$$\begin{aligned}
(b-a)[f'(b)-f'(a)] &\geq [w_1 f(b) + w_2 f(b) - w_1(b-a)f'(a)] - f(c) && (7)\\
&= w_2 f(b) + w_1[f(b)-(b-a)f'(a)] - f(c) && (8)\\
&\geq w_2 f(b) + w_1 f(a) - f(c) \quad \text{Using Eq. 1} && (9)
\end{aligned}$$

□

2. Comparison against methods

For MRF method [1], we use the MRF filter implemented in the ITK imaging toolkit
http://www.itk.org/Doxygen/html/group__MRFFilters.html.
The comparison for Dice's coefficient for the partially depigmented region is shown in Fig. 2. Comparisons against the Hausdorff distance criteria are shown in Fig. 3.

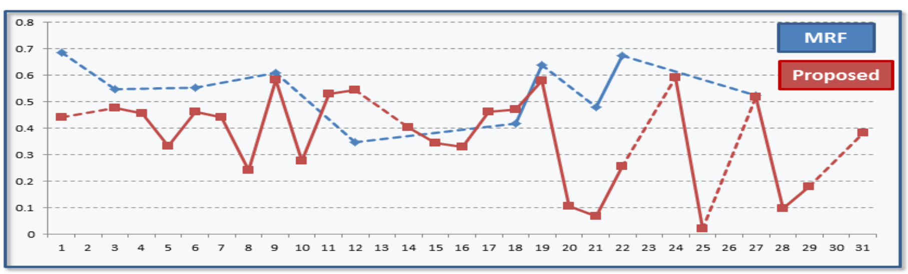

Figure 2. Dice's similarity coefficients for the partially depigmented regions. Dotted lines represent missed cases.

For SWA method [2], we use the implementation provided at
http://www.cse.buffalo.edu/∼jcorso/r/supervoxels/.
The comparison for Dice's coefficient for the partially depigmented region is shown in Fig. 4. Comparisons against the Hausdorff distance criteria are shown in Fig. 5.

We also enclose a small video along with this material (testVideo.mp4). The video shows the working of our implementation of the proposed method. To demonstrate the live mode of the software, we use a photograph of a patient in an office environment as a target. Note that this is just a demonstration, the actual images reported in the main paper were acquired directly from the patients.

References

[1] R. Kindermann, J. L. Snell, et al. *Markov random fields and their applications*, volume 1. American Math. Soc., Providence, RI, 1980. 2

[2] E. Sharon, M. Galun, D. Sharon, R. Basri, and A. Brandt. Hierarchy and adaptivity in segmenting visual scenes. *Nature*, 442(7104):810–813, 2006. 3

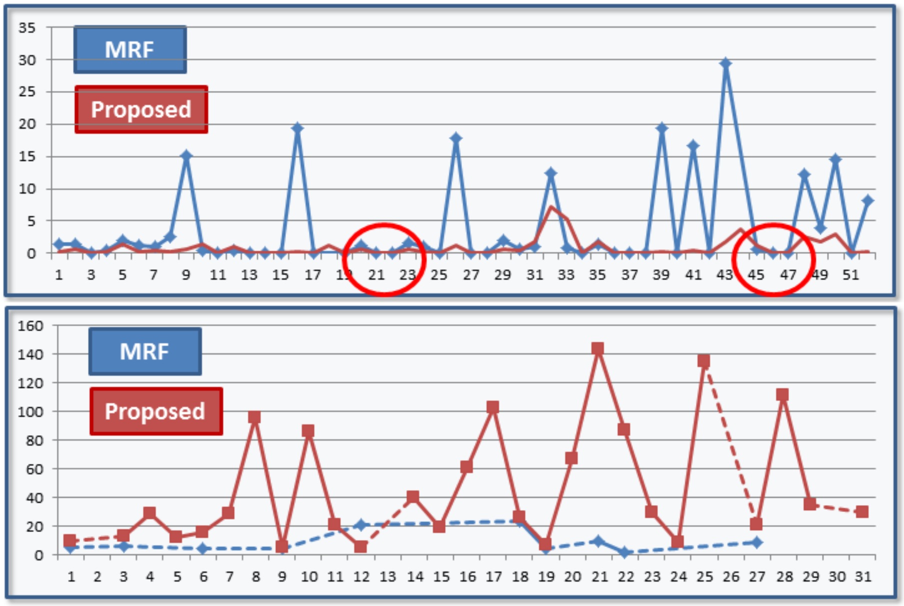

Figure 3. Hasudorff Distance comparison (lower is better). Top: complete depigmentation, bottom: partial depigmentation. Dotted lines represent missed cases. The circles denote the locations of the missed cases.

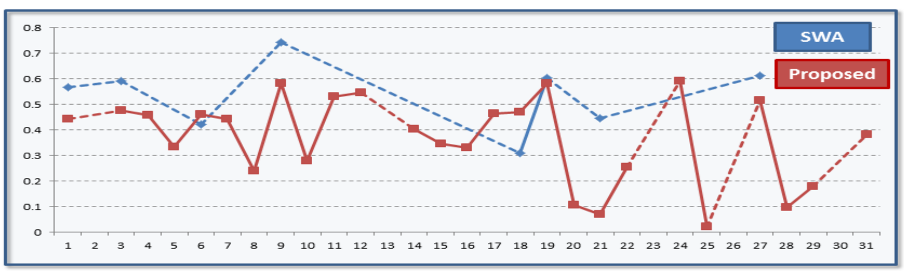

Figure 4. Dice's similarity coefficients for the partially depigmented regions. Dotted lines represent missed cases.

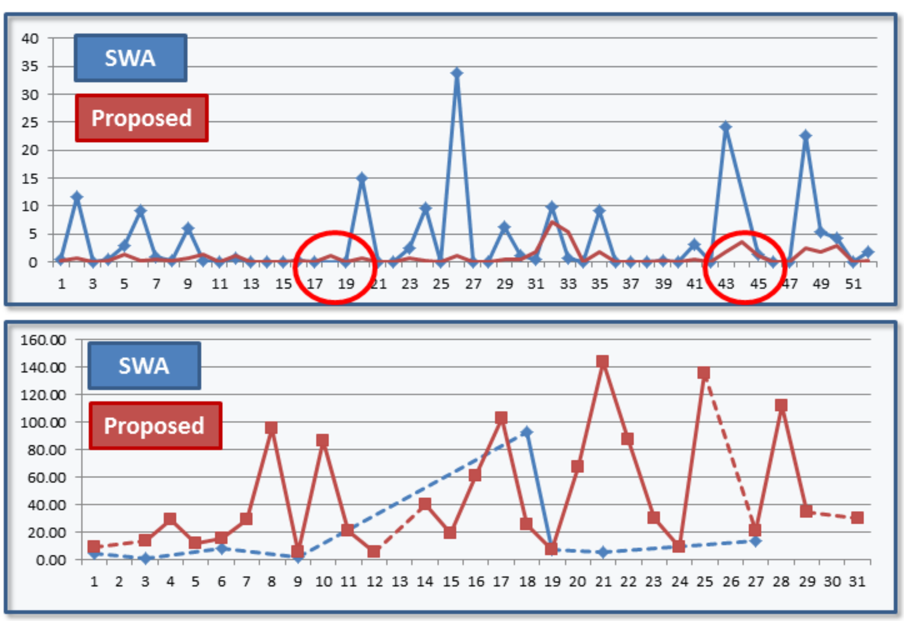

Figure 5. Hasudorff Distance comparison (lower is better). Top: complete depigmentation, bottom: partial depigmentation. Dotted lines represent missed cases. The circles denote the locations of the missed cases.