# Supplementary Material: Generalized Deformable Spatial Pyramid: Geometry-Preserving Dense Correspondence Estimation

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## 1. The message passing for the pairwise term $V_{ij}^2$

This section explains how to calculate a message to pass for the pairwise term $V_{ij}^2$ as we stated in Section 3.3 in the main paper. We successfully show how to simplify the message passing term for the $V_{ij}^1$ by reordering the term according to the dependencies on each variable, as in Eq. (7) in the main paper. Simplifying the message passing term for $V_{ij}^2$ follows the similar way of that for $V_{ij}^1$, but details in the reorganized term become different as the variables are not identically combined as in $V_{ij}^1$.

The pairwise term $V_{ij}^2$ calculates the state discrepancy between two nodes where $i$ is a child node and $j$ is its parent node:

$$V_{ij}^2 = \alpha \left\| \mathbf{t}_i - ((s_j R(r_j) \cdot \overrightarrow{\mathbf{o}_j \mathbf{o}_i} - \overrightarrow{\mathbf{o}_j \mathbf{o}_i}) + \mathbf{t}_j) \right\|_1 + \beta \left\| r_j - r_i \right\|_1 + \gamma \left\| s_j - s_i \right\|_1 .$$

If we let $\Delta_u(r_j, s_j) = (s_j R(r_j) \cdot \overrightarrow{\mathbf{o}_j \mathbf{o}_i} - \overrightarrow{\mathbf{o}_j \mathbf{o}_i}) \cdot \hat{u}$ and $\Delta_u(r_j, s_j) = (s_j R(r_j) \cdot \overrightarrow{\mathbf{o}_j \mathbf{o}_i} - \overrightarrow{\mathbf{o}_j \mathbf{o}_i}) \cdot \hat{v}$ in the above equation, we can derive a message from the node $i$ to the node $j$ in the similar way in Eq. (7) in our paper:

$$\begin{aligned}
& h(u_j, v_j, r_j, s_j) \\
& \quad = \min_{u_i, v_i, r_i, s_i} (\alpha f_1 + \alpha f_2 + \beta f_3 + \gamma f_4 + h) \\
& \quad = \min_{u_i} \{ \alpha f_1 + \min_{v_i} \{ \alpha f_2 + \min_{r_i} [\beta f_3 + \min_{s_i} (\gamma f_4 + h)]\}\} \\
& \qquad \text{where } \ f_1 = \left\| u_i - (\Delta_u(r_j, s_j) + u_j) \right\|_1 , \\
& \qquad \qquad f_2 = \left\| v_i - (\Delta_v(r_j, s_j) + v_j) \right\|_1 , \\
& \qquad \qquad f_3 = \left\| r_j - r_i \right\|_1 , f_4 = \left\| s_j - s_i \right\|_1 , \\
& \qquad \qquad \text{and } \ h = h(u_i, v_i, r_i, s_i).
\end{aligned}$$

The message from the node $i$ to the node $j$ can be generally represented as the second line in the above equation [1, 2], and we can reorder the equation according to the dependencies of each variable as in the third line of the equation.

**Algorithm 1** The Distance Transform (DT) for computing messages with $V_{ij}^2$ in the four dimensional case ($u_i$, $v_i$, $r_i$, $s_i$)

**Require:** A message from a child node $i$ to a parent node $j$ before updating, The center coordinate $\mathbf{o}_i$, $\mathbf{o}_j$ of node $i$ and $j$ respectively.
**Ensure:** A message from a child $i$ to a parent $j$ after updating

**procedure** MessageUpdating
  **for** $u_i$, $v_i$, $r_i$ **do** // *calculating* $h(u_i, v_i, r_i, s_j)$
    1D DT for $s_i$
  **end for**
  **for** $u_i$, $v_i$, $s_j$ **do** // *calculating* $h(u_i, v_i, r_j, s_j)$
    1D DT for $r_i$
  **end for**
  **for** $u_i$, $r_j$, $s_j$ **do** // *calculating* $h(u_i, v_j, r_j, s_j)$
    1D DT for $v_i$ with offset -$\Delta_v(r_j, s_j)$
  **end for**
  **for** $v_j$, $r_j$, $s_j$ **do** // *calculating* $h(u_j, v_j, r_j, s_j)$
    1D DT for $u_i$ with offset -$\Delta_u(r_j, s_j)$
  **end for**
**end procedure**

Then, the message can be easily calculated via a series of four distance transforms [1]. Note that minimizing for $s_i$ and $r_i$ is interchangeable, and also for $u_i$ and $v_i$ as well.

The Algorithm 1 above solves the series of four minimization problems using the 1D distance transform. The Fig. 1 represents how to conduct the 1D distance transform with the offset in the Algorithm 1.

## 2. Qualitative analysis of DAISY Filter Flow's results [3] on the non-rigid deformation pairs

As we stated in the experiment section in our main paper, we display the qualitative analysis of DAISY Filter Flow's results on the non-rigid deformation pairs in Fig. 2. The results indicate that the geometry of foreground objects is not



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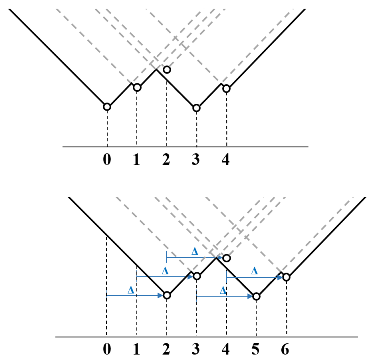

Figure 1: The visualization of 1D distance transform with the offset $\Delta$.

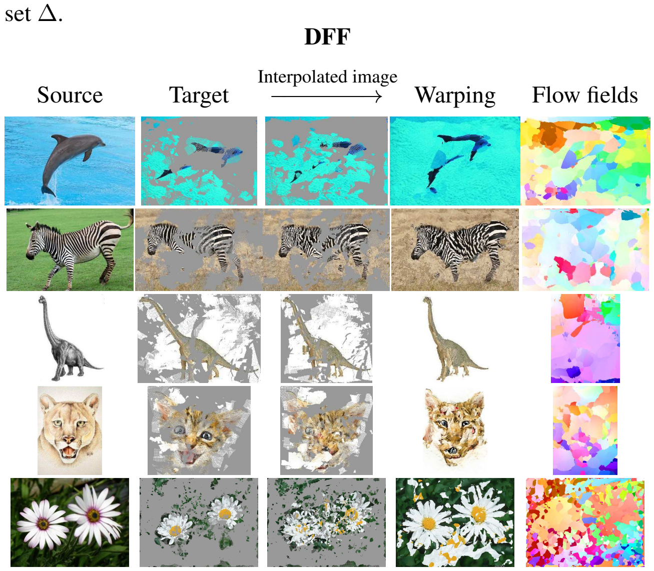


Figure 2: Qualitative analysis on the dense correspondence search results of DAISY Filter Flow [3].

preserved and the number of pixels which are transferred to the warping results is significantly reduced. The outcome implies that the majority of correspondences are not correctly estimated, even if the warping results look similar to their source images. The flow fields also validate these results; the flow fields of DFF [3] demonstrate significant irregularities overall, comparing to those of our model which show piecewise-smoothness.

## 3. The demonstration Video Link

We uploaded a demonstration video online. The video can be found on our project page: http://sites.google.com/site/hurjunhwa/research/gdsp. The video contains seven scenarios of matching images under non-rigid deformation. The video demonstrates the intermediate results of estimated dense correspondence fields both in the grid-cell levels and in the pixel levels. The video helps to comprehend our matching algorithm more effectively.

## References

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