Real-time Joint Estimation of Camera Orientation and Vanishing Points: Supplementary Material

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1. Introduction

The proposed method has been developed based on nonlinear Bayesian filtering, and, in the implementation, we utilized the extended Kalman filter (EKF) [1]. In this supplementary material, we provide the detailed derivations of the proposed method, including the derivations of the jacobians of the proposed nonlinear system models.

2. State

In the proposed system, the state vector $\mathbf{x}$ is defined by

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} \mathbf{x}_v^{\mathrm{T}} & \mathbf{y}_1^{\mathrm{T}} & \mathbf{y}_2^{\mathrm{T}} & \cdots \end{bmatrix}^{\mathrm{T}} \\ &= \begin{bmatrix} \mathbf{q}_{WC}^{\mathrm{T}} & \omega_C^{\mathrm{T}} & \mathbf{y}_1^{\mathrm{T}} & \mathbf{y}_2^{\mathrm{T}} & \cdots \end{bmatrix}^{\mathrm{T}}, \end{aligned} \tag{1}$$

where $\mathbf{q}_{WC} = [q_1, q_2, q_3, q_4]^{\mathrm{T}}$ denotes a quaternion for the current camera orientation, ω_C an angular velocity, and $\mathbf{y}_i = [\theta_i, \phi_i]^{\mathrm{T}}$ a vanishing direction (VD).

3. Camera Motion Model

Since we aim at estimating the camera orientation in an image sequence, we assume a constant angular velocity model as the camera motion model. The model function is defined as

$$\mathbf{f}_v = \begin{bmatrix} \mathbf{q}_{WC}^{new} \\ \omega_C^{new} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_{WC}^{old} \times \mathbf{q}((\omega_C^{old} + \mathbf{\Omega})\Delta t) \\ \omega_C^{old} + \mathbf{\Omega} \end{bmatrix}, \tag{2}$$

where the quaternion for the rotation $(\omega_C^{old} + \mathbf{\Omega})\Delta t$ in the axis-angle representation, $\mathbf{q}((\omega_C^{old} + \mathbf{\Omega})\Delta t)$, is as follows.

$$\mathbf{q}((\omega_C^{old} + \mathbf{\Omega})\Delta t) = \begin{bmatrix} \cos(\|\omega_C^{old} + \mathbf{\Omega}\| \frac{\Delta t}{2}) \\ \frac{\omega_C^{old} + \mathbf{\Omega}}{\|\omega_C^{old} + \mathbf{\Omega}\|} \sin(\|\omega_C^{old} + \mathbf{\Omega}\| \frac{\Delta t}{2}) \end{bmatrix} \tag{3}$$

The multiplication of the quaternions $\mathbf{q}$ and $\mathbf{p}$ is represented by

$$\mathbf{q} \times \mathbf{p} = \mathbf{Q}(\mathbf{q})\mathbf{p} = \mathbf{P}(\mathbf{p})\mathbf{q}, \tag{4}$$

where the matrix $\mathbf{Q}(\mathbf{q})$ and $\mathbf{P}(\mathbf{p})$ are defined by

$$\mathbf{Q}(\mathbf{q}) = \begin{bmatrix} q_1 & -q_2 & -q_3 & -q_4 \\ q_2 & q_1 & -q_4 & q_3 \\ q_3 & q_4 & q_1 & -q_2 \\ q_4 & -q_3 & q_2 & q_1 \end{bmatrix}, \tag{5}$$

$$\mathbf{P}(\mathbf{p}) = \begin{bmatrix} p_1 & -p_2 & -p_3 & -p_4 \\ p_2 & p_1 & p_4 & -p_3 \\ p_3 & -p_4 & p_1 & p_2 \\ p_4 & p_3 & -p_2 & p_1 \end{bmatrix}. \tag{6}$$

4. Measurement Model

The measurement model h_{ij} for the i-th VD of the state vector and the j-th line features is defined by

$$h_{ij} = \mathbf{d}_i^{\mathrm{T}} \mathbf{R}(\mathbf{q}_{WC}) \mathbf{n}_{ij}. \tag{7}$$

The i-th unit VD vector $\mathbf{d}_i$ in Cartesian coordinates is relevant to the i-th VD vector $\mathbf{y}_i$ in spherical coordinates as

$$\mathbf{d}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} \cos\phi_i \cos\theta_i \\ \cos\phi_i \sin\theta_i \\ \sin\phi_i \end{bmatrix}, \tag{8}$$

$$\mathbf{y}_i = \begin{bmatrix} \theta_i \\ \phi_i \end{bmatrix} = \begin{bmatrix} \mathrm{atan2}(y_i, x_i) \\ \mathrm{atan2}(z_i, \sqrt{x_i^2 + y_i^2}) \end{bmatrix}, \tag{9}$$

where $\mathrm{atan2}$ is the arctangent function. The rotation matrix $\mathbf{R}$ for a quaternion $\mathbf{q}$ is calculated by

$$\mathbf{R}(\mathbf{q}) = \begin{bmatrix} q_1^2 + q_2^2 - q_3^2 - q_4^2 & 2(q_2 q_3 - q_1 q_4) & 2(q_2 q_4 + q_1 q_3) \\ 2(q_2 q_3 + q_1 q_4) & q_1^2 - q_2^2 + q_3^2 - q_4^2 & 2(q_3 q_4 - q_1 q_2) \\ 2(q_2 q_4 - q_1 q_3) & 2(q_3 q_4 + q_1 q_2) & q_1^2 - q_2^2 - q_3^2 + q_4^2 \end{bmatrix}. \tag{10}$$

5. Initialization

Let $\mathbf{d}_{C,i} = [d_1, d_2, d_3]^{\mathrm{T}}$ denote a VD vector in the camera coordinates and $\mathbf{q}_{WR} = [q_1, q_2, q_3, q_4]^{\mathrm{T}}$ the quaternion of the current camera orientation. Then, using Eq. (10), the VD $\mathbf{d}_i$ is

$$\begin{aligned} \mathbf{d}_i &= \mathbf{R}(\mathbf{q}_{WC})\, \mathbf{d}_{C,i} \\ &= \begin{bmatrix} q_1^2 + q_2^2 - q_3^2 - q_4^2 & 2(q_2 q_3 - q_1 q_4) & 2(q_2 q_4 + q_1 q_3) \\ 2(q_2 q_3 + q_1 q_4) & q_1^2 - q_2^2 + q_3^2 - q_4^2 & 2(q_3 q_4 - q_1 q_2) \\ 2(q_2 q_4 - q_1 q_3) & 2(q_3 q_4 + q_1 q_2) & q_1^2 - q_2^2 - q_3^2 + q_4^2 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}. \end{aligned} \tag{11}$$

Eq. (11) is used to compute the covariance of a new VD, $\mathbf{P}_{\mathbf{y}_{new}}$. The covariance $\mathbf{P}_{\mathbf{y}_{new}}$ is defined as

$$\mathbf{P}_{\mathbf{y}_{new}} = \frac{\partial \mathbf{y}_{new}}{\partial \mathbf{x}_v} \mathbf{P}_{\mathbf{x}_v} \frac{\partial \mathbf{y}_{new}}{\partial \mathbf{x}_v}^{\mathrm{T}} + \widetilde{\mathbf{P}}_{\mathbf{y}_{new}}. \tag{12}$$

In Eq. (12), the covariances $\mathbf{P}_{\mathbf{x}_v}$ and $\widetilde{\mathbf{P}}_{\mathbf{y}_{new}}$ are already known. Therefore, one needs to compute just the jacobian $\frac{\partial \mathbf{y}_{new}}{\partial \mathbf{x}_v}$. From Eq. (8), (9), and (11), the jacobian is derived as

$$\frac{\partial \mathbf{y}_{new}}{\partial \mathbf{x}_v} = \frac{\partial \mathbf{y}_{new}}{\partial \mathbf{d}_i} \frac{\partial \mathbf{d}_i}{\partial \mathbf{x}_v} \tag{13}$$

The left jacobian of the right term of Eq. (13) is derived from Eq. (9), as follows.

$$\frac{\partial \mathbf{y}_{new}}{\partial \mathbf{d}_i} = \begin{bmatrix} -\frac{y_i}{x_i^2 + y_i^2} & \frac{x_i}{x_i^2 + y_i^2} & 0 \\ -\frac{z_i}{x_i^2 + y_i^2 + z_i^2} \frac{x_i}{\sqrt{x_i^2 + y_i^2}} & -\frac{z_i}{x_i^2 + y_i^2 + z_i^2} \frac{y_i}{\sqrt{x_i^2 + y_i^2}} & \frac{\sqrt{x_i^2 + y_i^2}}{x_i^2 + y_i^2 + z_i^2} \end{bmatrix} \tag{14}$$

The right jacobian of the right term of Eq. (13) is derived from Eq. (11), as

$$\frac{\partial \mathbf{d}_i}{\partial \mathbf{x}_v} = \begin{bmatrix} \frac{\partial \mathbf{d}_i}{\partial \mathbf{q}_{WC}} & \frac{\partial \mathbf{d}_i}{\partial \omega_C} \end{bmatrix}, \tag{15}$$

where the right jacobian of the right term of Eq. (15) is set to a 3×3 zero matrix and the left jacobian of the right term of Eq. (15) can be derived as follows.

$$\frac{\partial \mathbf{d}_i}{\partial \mathbf{q}_{WC}} = \begin{bmatrix} 2q_1 d_1 - 2q_4 d_2 + 2q_3 d_3 & 2q_2 d_1 + 2q_3 d_2 + 2q_4 d_3 & 2q_3 d_1 + 2q_2 d_2 + 2q_1 d_3 & 2q_4 d_1 - 2q_1 d_2 + 2q_2 d_3 \\ 2q_4 d_1 + 2q_1 d_2 - 2q_2 d_3 & 2q_3 d_1 - 2q_2 d_2 - 2q_1 d_3 & 2q_2 d_1 + 2q_3 d_2 + 2q_4 d_3 & 2q_1 d_1 - 2q_4 d_2 + 2q_3 d_3 \\ -2q_3 d_1 + 2q_2 d_2 + 2q_4 d_3 & 2q_4 d_1 + 2q_3 d_2 - 2q_2 d_3 & -2q_1 d_1 + 2q_4 d_2 - 2q_3 d_3 & 2q_1 d_1 + 2q_3 d_2 + 2q_4 d_3 \end{bmatrix} \tag{16}$$

6. Prediction Step

In the prediction step of the EKF system, an estimate of the current state is produced from Eq. (2), and the current covariance $\hat{\mathbf{P}}$ is estimated by

$$\hat{\mathbf{P}} = \begin{bmatrix} \frac{\partial \mathbf{f}_v}{\partial \mathbf{x}_v} \mathbf{P}_{\mathbf{x}_v} \frac{\partial \mathbf{f}_v}{\partial \mathbf{x}_v}^{\mathrm{T}} + \frac{\partial \mathbf{f}_v}{\partial \mathbf{\Omega}} \mathbf{P}_{\Omega} \frac{\partial \mathbf{f}_v}{\partial \mathbf{\Omega}}^{\mathrm{T}} & \frac{\partial \mathbf{f}_v}{\partial \mathbf{x}_v} \mathbf{P}_{\mathbf{x}_v \mathbf{y}} \\ \mathbf{P}_{\mathbf{y}\mathbf{x}_v} \frac{\partial \mathbf{f}_v}{\partial \mathbf{x}_v}^{\mathrm{T}} & \mathbf{P}_{\mathbf{y}} \end{bmatrix}, \tag{17}$$

where $\mathbf{P}_{\Omega}$ is a covariance of the noise Ω of the motion model, $\mathbf{P}_{\mathbf{x}_v \mathbf{y}}$ and $\mathbf{P}_{\mathbf{y}\mathbf{x}_v}$ are covariances for the camera orientation and the VDs, and $\mathbf{P}_{\mathbf{y}}$ is the covariance of the VDs. Unlike that the covariances are already known, the two jacobians $\frac{\partial \mathbf{f}_v}{\partial \mathbf{x}_v}$ and $\frac{\partial \mathbf{f}_v}{\partial \mathbf{\Omega}}$ should be computed. First, the jacobian of the camera motion model for the camera state, $\frac{\partial \mathbf{f}_v}{\partial \mathbf{x}_v}$, is derived from Eq. (2) as

$$\frac{\partial \mathbf{f}_v}{\partial \mathbf{x}_v} = \begin{bmatrix} \frac{\partial \mathbf{q}_{WC}^{new}}{\partial \mathbf{q}_{WC}^{old}} & \frac{\partial \mathbf{q}_{WC}^{new}}{\partial \omega_C^{old}} \\ \frac{\partial \omega_C^{new}}{\partial \mathbf{q}_{WC}^{old}} & \frac{\partial \omega_C^{new}}{\partial \omega_C^{old}} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{q}_{WC}^{new}}{\partial \mathbf{q}_{WC}^{old}} & \frac{\partial \mathbf{q}_{WC}^{new}}{\partial \omega_C^{old}} \\ \mathbf{0}_{3\times 4} & \mathbf{I}_3 \end{bmatrix}, \tag{18}$$

where $\mathbf{0}_{3\times 4}$ is a 3×4 zero matrix, and $\mathbf{I}_3$ is a 3×3 identity matrix. Here, the jacobian of the new camera orientation for the old camera orientation, $\frac{\partial \mathbf{q}_{WC}^{new}}{\partial \mathbf{q}_{WC}^{old}}$, is easily derived using Eq. (6), as follows.

$$\frac{\partial \mathbf{q}_{WC}^{new}}{\partial \mathbf{q}_{WC}^{old}} = \mathbf{P}\left(\mathbf{q}\left(\left(\omega_C^{old} + \mathbf{\Omega}\right)\Delta t\right)\right)\mathbf{I}_4 \tag{19}$$

Let $\omega_C^{old} = [w_1, w_2, w_3]^{\mathrm{T}}$. Then, the jacobian $\frac{\partial \mathbf{q}_{WC}^{new}}{\partial \omega_C^{old}}$ is derived as

$$\frac{\partial \mathbf{q}_{WC}^{new}}{\partial \omega_C^{old}} = \mathbf{Q}\left(\mathbf{q}_{WC}^{old}\right) \begin{bmatrix} m(\omega_C^{old}, \Delta t, 1) & m(\omega_C^{old}, \Delta t, 2) & m(\omega_C^{old}, \Delta t, 3) \\ n(\omega_C^{old}, \Delta t, 1) & o(\omega_C^{old}, \Delta t, 1, 2) & o(\omega_C^{old}, \Delta t, 1, 3) \\ o(\omega_C^{old}, \Delta t, 2, 1) & n(\omega_C^{old}, \Delta t, 2) & o(\omega_C^{old}, \Delta t, 2, 3) \\ o(\omega_C^{old}, \Delta t, 3, 1) & o(\omega_C^{old}, \Delta t, 3, 2) & n(\omega_C^{old}, \Delta t, 3) \end{bmatrix}, \tag{20}$$

where

$$m(\omega_C^{old}, \Delta t, i) = -\frac{w_i}{\|\omega_C^{old}\|} \sin\left(\frac{\|\omega_C^{old}\|\Delta t}{2}\right)\frac{\Delta t}{2}, \tag{21}$$

$$n(\omega_C^{old}, \Delta t, i) = \frac{\|\omega_C^{old}\| - w_i^2/\|\omega_C^{old}\|}{\|\omega_C^{old}\|^2} \sin\left(\frac{\|\omega_C^{old}\|\Delta t}{2}\right) + \frac{w_i^2}{\|\omega_C^{old}\|^2}\cos\left(\frac{\|\omega_C^{old}\|\Delta t}{2}\right)\frac{\Delta t}{2}, \tag{22}$$

$$o(\omega_C^{old}, \Delta t, i, j) = -\frac{w_i w_j}{\|\omega_C^{old}\|^3} \sin\left(\frac{\|\omega_C^{old}\|\Delta t}{2}\right) + \frac{w_i w_j}{\|\omega_C^{old}\|^2}\cos\left(\frac{\|\omega_C^{old}\|\Delta t}{2}\right)\frac{\Delta t}{2}. \tag{23}$$

In addition, the jacobian $\frac{\partial \mathbf{f}_v}{\partial \mathbf{\Omega}}$ can be derived in a manner similar to the derivation of Eq. (18).

7. Update Step

In the update step of the EKF system, the computation of the jacobian of the measurement model for the camera state, $\frac{\partial h_{ij}}{\partial \mathbf{x}_v}$, is required for correcting the estimate. The jacobian is represented as

$$\frac{\partial h_{ij}}{\partial \mathbf{x}_v} = \left[\frac{\partial h_{ij}}{\partial \mathbf{q}_{WC}} \quad \frac{\partial h_{ij}}{\partial \omega_C} \quad \cdots \quad \frac{\partial h_{ij}}{\partial \mathbf{y}_i} \quad \cdots\right]. \tag{24}$$

The jacobian of the measurement model for the camera orientation, $\frac{\partial h_{ij}}{\partial \mathbf{q}_{WC}}$, is derived using Eq. (7) and (10) as

$$\frac{\partial h_{ij}}{\partial \mathbf{q}_{WC}} = \left[\frac{\partial h_{ij}}{\partial q_1} \quad \frac{\partial h_{ij}}{\partial q_2} \quad \frac{\partial h_{ij}}{\partial q_3} \quad \frac{\partial h_{ij}}{\partial q_4}\right], \tag{25}$$

where

$$\frac{\partial h_{ij}}{\partial q_1} = 2\mathbf{d}_i^{\mathrm{T}} \begin{bmatrix} q_1 & -q_4 & q_3 \\ q_4 & q_1 & -q_2 \\ -q_3 & -q_2 & q_1 \end{bmatrix} \mathbf{n}_{ij}, \tag{26}$$

$$\frac{\partial h_{ij}}{\partial q_2} = 2\mathbf{d}_i^{\mathrm{T}} \begin{bmatrix} q_2 & q_3 & q_4 \\ q_3 & -q_2 & -q_1 \\ q_4 & -q_1 & -q_2 \end{bmatrix} \mathbf{n}_{ij}, \tag{27}$$

$$\frac{\partial h_{ij}}{\partial q_3} = 2\mathbf{d}_i^{\mathrm{T}} \begin{bmatrix} -q_3 & q_2 & q_1 \\ q_2 & q_3 & q_4 \\ -q_1 & q_4 & -q_3 \end{bmatrix} \mathbf{n}_{ij}, \tag{28}$$

$$\frac{\partial h_{ij}}{\partial q_4} = 2\mathbf{d}_i^{\mathrm{T}} \begin{bmatrix} -q_4 & -q_1 & q_2 \\ q_1 & -q_4 & q_3 \\ q_2 & q_3 & q_4 \end{bmatrix} \mathbf{n}_{ij}. \tag{29}$$

The jacobian of the measurement model for the angular velocity, $\frac{\partial h_{ij}}{\partial \omega_C}$, is a 3-dimensional row vector with all elements equal to zero, since the measurement model does not involve the variables of the angular velocity. The jacobian of the measurement model for the i-th VD, $\frac{\partial h_{ij}}{\partial \mathbf{y}_i}$, is derived as

$$\frac{\partial h_{ij}}{\partial \mathbf{y}_i} = \frac{\partial h_{ij}}{\partial \mathbf{d}_i} \frac{\partial \mathbf{d}_i}{\partial \mathbf{y}_i}, \tag{30}$$

where the left jacobian of the right term, $\frac{\partial h_{ij}}{\partial \mathbf{d}_i}$, is computed using Eq. (7), as

$$\frac{\partial h_{ij}}{\partial \mathbf{d}_i} = \left(\mathbf{R}(\mathbf{q}_{WC})\mathbf{n}_{ij}\right)^{\mathrm{T}}, \tag{31}$$

and the right jacobian of the right term, $\frac{\partial \mathbf{d}_i}{\partial \mathbf{y}_i}$, is computed using Eq. (8), as follows.

$$\frac{\partial \mathbf{d}_i}{\partial \mathbf{y}_i} = \begin{bmatrix} -\cos\phi_i \sin\theta_i & -\sin\phi_i \cos\theta_i \\ \cos\phi_i \cos\theta_i & -\sin\phi_i \sin\theta_i \\ 0 & \cos\phi_i \end{bmatrix} \tag{32}$$

When computing the jacobian $\frac{\partial h_{ij}}{\partial \mathbf{x}_v}$, all the jacobians $\frac{\partial h_{kl}}{\partial \mathbf{y}_k}$ except for $k = i$ and $l = j$ are 2-dimensional row vectors with all elements equal to zero.

References

[1] D. Simon. *Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches*. Wiley-Interscience, Hoboken (N.J.), 2006.