Supplementary material for: Active Sample Selection and Correction Propagation on a Gradually-Augmented Graph

Anonymous CVPR submission

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1. Derivation of the Correction Propagation Algorithm in Eq. (33)

Solution:
By substituting Eq. (32) into Eq. (29) in the paper, we obtain

$$\begin{aligned}\mathbf{Y}_u^+ &= \boldsymbol{\Gamma}_{uu}^+(\mathbf{W}_{ul}\mathbf{Y}_l + w\mathbf{Z}_{us}\mathbf{Y}_s) \\ &= \big(\boldsymbol{\Gamma}_{uu} - \boldsymbol{\Gamma}_{uk}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\boldsymbol{\Gamma}_{ku}\big)\mathbf{W}_{ul}\mathbf{Y}_l + w\big(\boldsymbol{\Gamma}_{uu} - \boldsymbol{\Gamma}_{uk}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\boldsymbol{\Gamma}_{ku}\big)\mathbf{Z}_{us}\mathbf{Y}_s\end{aligned} \tag{1}$$

Since $w \to +\infty$, the first term can be computed as

$$\begin{aligned}&\big(\boldsymbol{\Gamma}_{uu} - \boldsymbol{\Gamma}_{uk}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\boldsymbol{\Gamma}_{ku}\big)\mathbf{W}_{ul}\mathbf{Y}_l \\ =\;& \boldsymbol{\Gamma}_{uu}\mathbf{W}_{ul}\mathbf{Y}_l - \boldsymbol{\Gamma}_{uk}\boldsymbol{\Gamma}_{kk}^{-1}\boldsymbol{\Gamma}_{ku}\mathbf{W}_{ul}\mathbf{Y}_l \\ =\;& \boldsymbol{\Gamma}_{uu}\mathbf{W}_{ul}\mathbf{Y}_l - \boldsymbol{\Gamma}_{uk}\boldsymbol{\Gamma}_{kk}^{-1}\mathbf{Z}_{su}\boldsymbol{\Gamma}_{uu}\mathbf{W}_{ul}\mathbf{Y}_l \;(\because \boldsymbol{\Gamma}_{ku} = \mathbf{Z}_{su}\boldsymbol{\Gamma}_{uu}) \\ =\;& \mathbf{Y}_u - \boldsymbol{\Gamma}_{uk}\boldsymbol{\Gamma}_{kk}^{-1}\mathbf{Z}_{su}\mathbf{Y}_u \;(\because \boldsymbol{\Gamma}_{uu}\mathbf{W}_{ul}\mathbf{Y}_l = \mathbf{Y}_u) \\ =\;& \mathbf{Y}_u - \boldsymbol{\Gamma}_{uk}\boldsymbol{\Gamma}_{kk}^{-1}\mathbf{Y}_k.\end{aligned} \tag{2}$$

The second term can be calculated as

$$\begin{aligned}&w\big(\boldsymbol{\Gamma}_{uu} - \boldsymbol{\Gamma}_{uk}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\boldsymbol{\Gamma}_{ku}\big)\mathbf{Z}_{us}\mathbf{Y}_s \\ =\;& w\boldsymbol{\Gamma}_{uu}\mathbf{Z}_{us}\mathbf{Y}_s - w\boldsymbol{\Gamma}_{uk}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\boldsymbol{\Gamma}_{ku}\mathbf{Z}_{us}\mathbf{Y}_s \\ =\;& w\boldsymbol{\Gamma}_{uk}\mathbf{Y}_s - w\boldsymbol{\Gamma}_{uk}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\boldsymbol{\Gamma}_{kk}\mathbf{Y}_s \end{aligned} \tag{3}$$

$$= w\boldsymbol{\Gamma}_{uk}\big(\mathbf{I}_{N_s} - (\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\boldsymbol{\Gamma}_{kk}\big)\mathbf{Y}_s. \tag{4}$$

Therefore,

$$\begin{aligned}&w\big(\boldsymbol{\Gamma}_{uu} - \boldsymbol{\Gamma}_{uk}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\boldsymbol{\Gamma}_{ku}\big)\mathbf{Z}_{us}\mathbf{Y}_s \\ =\;& w\boldsymbol{\Gamma}_{uk}\big((\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk}) - (\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\boldsymbol{\Gamma}_{kk}\big)\mathbf{Y}_s \\ =\;& w\boldsymbol{\Gamma}_{uk}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk} - \boldsymbol{\Gamma}_{kk})\mathbf{Y}_s \\ =\;& \boldsymbol{\Gamma}_{uk}(\mathbf{I}_{N_s}/w + \boldsymbol{\Gamma}_{kk})^{-1}\mathbf{Y}_s \\ =\;& \boldsymbol{\Gamma}_{uk}\boldsymbol{\Gamma}_{kk}^{-1}\mathbf{Y}_s \qquad (\because w \to +\infty).\end{aligned}$$

Substituting Eq. (2) into Eq. (3) yields

$$\begin{aligned}\mathbf{Y}_u^+ &= \mathbf{Y}_u - \boldsymbol{\Gamma}_{uk}\boldsymbol{\Gamma}_{kk}^{-1}\mathbf{Y}_k + \boldsymbol{\Gamma}_{uk}\boldsymbol{\Gamma}_{kk}^{-1}\mathbf{Y}_s \\ &= \mathbf{Y}_u + \boldsymbol{\Gamma}_{uk}\boldsymbol{\Gamma}_{kk}^{-1}(\mathbf{Y}_s - \mathbf{Y}_k).\end{aligned} \tag{5}$$

Hereby, we finish the solution.

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