Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry Supplementary Material

Yuchao Dai[1], Hongdong Li[1,2] and Laurent Kneip[1,2]

[1] Research School of Engineering, Australian National University

[2]ARC Centre of Excellence for Robotic Vision (ACRV)

Abstract

In this supplementary material, we provide detailed derivations of other types of rolling shutter essential matrices as well as their linear algorithms.

1. Deriving the 7×7 rolling shutter essential matrix for uniform RS camera

Under the uniform rolling shutter camera model, the scanline coplanarity constraint can be expressed as:

$$[u_i^{'}, v_i^{'}, 1][\mathbf{t}+u_i^{'}\mathbf{d}_2 - u_i\mathbf{R}_{u_iu_i^{'}}\mathbf{d}_1]_\times \mathbf{R}_{u_iu_i^{'}}[u_i, v_i, 1]^T = 0, \quad (1)$$

where $\mathbf{R}_{u_i,u_i^{'}}$ defines the relative rotation while $\mathbf{t}_{u_i,u_i^{'}} = \mathbf{t}+u_i^{'}\mathbf{d}_2 - u_i\mathbf{R}_{u_iu_i^{'}}\mathbf{d}_1$ defines the relative translation. where $\mathbf{R}$ defines the rotation between the central row of the second frame to the central row of the first row.

$$\begin{aligned} &[u_i', \ v_i', 1]([\mathbf{t}]_\times \mathbf{R}_{u_i,u_i'} & (2)\\ &\quad - u_i\mathbf{R}_{u_i,u_i'}[\mathbf{d}_1]_\times & (3)\\ &\quad - u_i'[\mathbf{d}_2]_\times \mathbf{R}_{u_i,u_i'})[u_i, v_i, 1]^T = 0, & (4)\end{aligned}$$

Expanding this equation with the aid of the small rotation approximation results in

$$\mathbf{R}_{u_i,u_i'} = (\mathbf{I}+u_i'[\mathbf{w}_2]_\times)\mathbf{R}_0(\mathbf{I}-u_i[\mathbf{w}_1]_\times), \quad (5)$$

By defining the following auxiliary variables,

$$\begin{aligned} &\mathbf{E}_0 = [\mathbf{t}]_\times\mathbf{R},\\ &\mathbf{E}_1 = \mathbf{R}[\mathbf{d}_1]_\times + [\mathbf{t}]_\times\mathbf{R}[\mathbf{w}_1]_\times,\\ &\mathbf{E}_2 = [\mathbf{d}_2]_\times\mathbf{R} + [\mathbf{t}]_\times[\mathbf{w}_2]_\times\mathbf{R},\\ &\mathbf{E}_3 = \mathbf{R}[\mathbf{w}_1]_\times[\mathbf{d}_1]_\times,\\ &\mathbf{E}_4 = [\mathbf{d}_2]_\times\mathbf{R}[\mathbf{w}_1]_\times + [\mathbf{w}_2]_\times\mathbf{R}[\mathbf{d}_1]_\times + [\mathbf{t}]_\times[\mathbf{w}_2]_\times\mathbf{R}[\mathbf{w}_1]_\times,\\ &\mathbf{E}_5 = [\mathbf{d}_2]_\times[\mathbf{w}_2]_\times\mathbf{R},\\ &\mathbf{E}_6 = [\mathbf{w}_2]_\times\mathbf{R}[\mathbf{w}_1]_\times[\mathbf{d}_1]_\times,\\ &\mathbf{E}_7 = [\mathbf{d}_2]_\times[\mathbf{w}_2]_\times\mathbf{R}[\mathbf{w}_1]_\times,\end{aligned} \quad (6)$$

we arrive that,

$$\begin{aligned} &[u_i', \ v_i', 1](\mathbf{E}_0 - u_i\mathbf{E}_1 + u_i'\mathbf{E}_2 + u_i^2\mathbf{E}_3 & (7)\\ &- u_iu_i'\mathbf{E}_4 + u_i'\mathbf{E}_5 & (8)\\ &+ u_i^2u_i'\mathbf{E}_6 - u_iu_i^{'2}\mathbf{E}_7)[u_i, v_i, 1]^T = 0. & (9)\end{aligned}$$

Finally we obtain:

$$\left[u_i^{'3}, u_i^{'2}v_i', u_i^{'2}, u_i'v_i', u_i', v_i', 1\right] \mathbf{F} \left[u_i^3, u_i^2v_i, u_i^2, u_iv_i, u_i, v_i, 1\right]^T = 0, \quad (10)$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & f_{13} & f_{14} & f_{15} & f_{16} & f_{17}\\ 0 & 0 & f_{23} & f_{24} & f_{25} & f_{26} & f_{27}\\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} & f_{36} & f_{37}\\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} & f_{46} & f_{47}\\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} & f_{56} & f_{57}\\ f_{61} & f_{62} & f_{63} & f_{64} & f_{65} & f_{66} & f_{67}\\ f_{71} & f_{72} & f_{73} & f_{74} & f_{75} & f_{76} & f_{77} \end{bmatrix}.$$

This gives a 7×7 uniform RS essential matrix $\mathbf{F}$, whose elements are functions of the 18 unknowns (*i.e.* $\{\mathbf{R}, \mathbf{t}, \mathbf{w}_1, \mathbf{w}_2, \mathbf{d}_1, \mathbf{d}_2\}$). Also note the induced epipolar curves are *cubic*.

In total there are 45 homogeneous variables, thus minimum 44 points in general configuration are sufficient to solve this 7×7 RS essential matrix.

1.1. Detail of the linear 44-point solver

For the uniform rolling shutter relative pose problem, we first solve for the uniform rolling shutter essential matrix $\mathbf{F} \in \mathbb{R}^{7\times 7}$. Then from the 45 elements in $\mathbf{F}$, recover the eight matrices $\mathbf{E}_i, i = 0, \cdots, 7$. Finally, the relative pose $(\mathbf{R}, \mathbf{t})$, rotational velocities $\mathbf{w}_1, \mathbf{w}_2$ and translational velocities $\mathbf{d}_1, \mathbf{d}_2$ are extracted from the eight matrices.

Due to its special structure, the uniform RS essential matrix $\mathbf{F}$ consists of 45 homogeneous variables, *i.e.*, 44 DoF. According to the uniform RS essential matrix Eq.-(10), by collecting 44 correspondences, we can solve for the uniform RS essential matrix $\mathbf{M}$ linearly through the singular value decomposition (SVD).

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & E_6^{11} & E_6^{21} & E_6^{31} + E_3^{11} & E_3^{21} & E_3^{31} \\ 0 & 0 & E_6^{12} & E_6^{22} & E_6^{32} + E_3^{12} & E_3^{22} & E_3^{32} \\ -E_7^{11} & -E_7^{21} & E_6^{13} - E_4^{11} - E_7^{31} & E_6^{23} - E_4^{21} & E_3^{13} - E_1^{11} - E_4^{31} + E_6^{33} & E_3^{23} - E_1^{21} & E_3^{33} - E_1^{31} \\ -E_7^{12} & -E_7^{22} & -E_4^{12} - E_7^{32} & -E_4^{22} & -E_1^{12} - E_4^{32} & -E_1^{22} & -E_1^{32} \\ E_5^{11} - E_7^{13} & E_5^{21} - E_7^{23} & E_2^{11} - E_4^{13} + E_5^{31} - E_7^{33} & E_2^{21} - E_4^{23} & E_0^{11} - E_1^{13} + E_2^{31} - E_4^{33} & E_0^{21} - E_1^{23} & E_0^{31} - E_1^{33} \\ E_5^{12} & E_5^{22} & E_2^{12} + E_5^{32} & E_2^{22} & E_0^{12} + E_2^{32} & E_0^{22} & E_0^{32} \\ E_5^{13} & E_5^{23} & E_2^{13} + E_5^{33} & E_2^{23} & E_0^{13} + E_2^{33} & E_0^{23} & E_0^{33} \end{bmatrix} \tag{11}$$

1.2. Normalization

In solving the linear rolling shutter essential matrix $\mathbf{F}$ through linear 20 point algorithm, it is important to implement a proper normalization.

Below we describe two approaches for performing such a normalization: 1) Normalizing the image coordinates data (u_i, v_i) and $(u_i^{'}, v_i^{'})$ in the way as described in [1]. 2) Under the linear rolling shutter relative pose formulation, the inputs are monomials $(u_i^2, u_i v_i, u_i, v_i, 1)$ and $(u_i^{'2}, u_i^{'}v_i^{'}, u_i^{'}, v_i^{'}, 1)$, a better normalization should be defined on $(u_i^2, u_i v_i, u_i, v_i, 1)$ and $(u_i^{'2}, u_i^{'}v_i^{'}, u_i^{'}, v_i^{'}, 1)$ rather than (u_i, v_i) and $(u_i^{'}, v_i^{'})$. Therefore, in this paper, we propose to normalize $(u_i^2, u_i v_i, u_i, v_i, 1)$ and $(u_i^{'2}, u_i' v_i', u_i', v_i', 1)$ in the way as in [1].

2. Details about recovering the atomic essential matrices from a 5×5 linear RS essential matrix

Once a 5×5 linear RS essential matrix $\mathbf{F}$ is found, our next step is to recover the individual atomic essential matrices $\mathbf{E}_0, \mathbf{E}_1$ and $\mathbf{E}_2$. In the main paper we derived 21 linear equations defined on the three essential matrices. Because these three essential matrices contain 27 elements, we need six extra constraints to solve for $\mathbf{E}_0, \mathbf{E}_1$ and $\mathbf{E}_2$. To this end, we resort to the inherent constraints on the standard 3×3 essential matrices, *e.g.* $\det(\mathbf{E}) = 0$ and $2\mathbf{E}\mathbf{E}^T\mathbf{E} - \mathbf{Tr}(\mathbf{E}\mathbf{E}^T)\mathbf{E} = 0$, since $\mathbf{E}_0, \mathbf{E}_1$ and $\mathbf{E}_2$ are standard 3×3 essential matrices. Note that these non-linear constraints generally give rise to cubic (3-order) equations. Next we will show how to reduce them to quadratic ones.

2.1. Enforcing inherent constraints on the atomic essential matrices

Theorem 1. *A real nonzero 3×3 matrix $\mathbf{E}$ is a fundamental matrix if and only id it satisfy the equation:*

$$\det(\mathbf{E}) = 0. \tag{12}$$

Theorem 2. *A real nonzero 3×3 matrix E is an essential matrix is and only if it satisfies the equation:*

$$\mathbf{E}\mathbf{E}^T\mathbf{E} - \frac{1}{2}\mathrm{trace}(\mathbf{E}\mathbf{E}^T)\mathbf{E} = 0. \tag{13}$$

Theorem 3. *If three essential matrices $\mathbf{E}_0, \mathbf{E}_1, \mathbf{E}_2$ consists of a common rotation,* i.e., $\mathbf{E}_0 = [\mathbf{t}]_\times \mathbf{R}$, $\mathbf{E}_1 = [\mathbf{t}_1]_\times \mathbf{R}$, $\mathbf{E}_2 = [\mathbf{t}_2]_\times \mathbf{R}$, *the* column reorganized matrices $\mathbf{F}_1 = [\mathbf{E}_0^1, \mathbf{E}_1^1, \mathbf{E}_2^1]$, $\mathbf{F}_2 = [\mathbf{E}_0^2, \mathbf{E}_1^2, \mathbf{E}_2^2]$, $\mathbf{F}_3 = [\mathbf{E}_0^3, \mathbf{E}_1^3, \mathbf{E}_2^3]$ *are rank deficient.*

$$\det(\mathbf{F}_1) = 0, \det(\mathbf{F}_2) = 0, \det(\mathbf{F}_3) = 0. \tag{14}$$

Proof. According to the definition, $\mathbf{F}_1 = [\mathbf{E}_0^1, \mathbf{E}_1^1, \mathbf{E}_2^1] = [\mathbf{t} \times \mathbf{R}^1, \mathbf{t}_1 \times \mathbf{R}^1, \mathbf{t}_2 \times \mathbf{R}^1]$. Therefore, all the rows of $\mathbf{F}_1$ are orthogonal to $\mathbf{R}^1$, we must have $\mathrm{rank}(\mathbf{F}_1) = 2$ or $\det(\mathbf{F}_1) = 0$. Similarly, we have $\det(\mathbf{F}_2) = 0$ and $\det(\mathbf{F}_3) = 0$. □

Note that $\mathbf{F}_1, \mathbf{F}_2$ and $\mathbf{F}_3$ are not necessarily an essential matrix.

By collecting the rank deficient constraints on essential matrices $\mathbf{E}_0$, $\mathbf{E}_1$, $\mathbf{E}_2$ and column reorganized matrices $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$. In total, we have the following 6 rank constraints:

$$\begin{aligned} \mathrm{rank}(\mathbf{E}_0) &= \mathrm{rank}([\mathbf{t}]_\times \mathbf{R}) = 2, \\ \mathrm{rank}(\mathbf{E}_1) &= \mathrm{rank}([\mathbf{v}_1]_\times \mathbf{R}) = 2, \\ \mathrm{rank}(\mathbf{E}_2) &= \mathrm{rank}([\mathbf{v}_2]_\times \mathbf{R}) = 2, \\ \mathrm{rank}(\mathbf{F}_1) &= \mathrm{rank}([\mathbf{E}_0^1, \mathbf{E}_1^1, \mathbf{E}_2^1]) = 2, \\ \mathrm{rank}(\mathbf{F}_2) &= \mathrm{rank}([\mathbf{E}_0^2, \mathbf{E}_1^2, \mathbf{E}_2^2]) = 2, \\ \mathrm{rank}(\mathbf{F}_3) &= \mathrm{rank}([\mathbf{E}_0^3, \mathbf{E}_1^3, \mathbf{E}_2^3]) = 2. \end{aligned} \tag{15}$$

By enforcing the above six constraints together with the 21 linear equations, the atomic essential matrices $\mathbf{E}_0, \mathbf{E}_1$ and $\mathbf{E}_2$ can be recovered. Besides the rank constraints, the cubic equations defined on the essential matrix also constrain $\mathbf{E}_0, \mathbf{E}_1$ and $\mathbf{E}_2$. By exploiting the special structure of these essential matrices, we could reach the following method which involves quadratic equations only.

References

[1] R. Hartley. In defense of the eight-point algorithm. *IEEE Trans. Pattern Anal. Mach. Intell.*, 19(6):580–593, Jun 1997. 2