Fast Temporal Activity Proposals for Efficient Detection of Human Actions in Untrimmed Videos (Supplementary Material)

Fabian Caba Heilbron[1] fabian.caba@kaust.edu.sa

Juan Carlos Niebles[2,3] jniebles@cs.stanford.edu

Bernard Ghanem[1] bernard.ghanem@kaust.edu.sa

[1]King Abdullah University of Science and Technology (KAUST), Saudi Arabia
[2]Department of Computer Science, Stanford University [3]Universidad del Norte, Colombia
http://www.cabaf.net/temporalproposals

Abstract

In this supplementary material, we complement our paper submission by providing additional analysis of the optimization problems (Section 1) and qualitative results (Section 2).

1. Optimization details

In this section, we elaborate on the details required to solve the optimization problems proposed in our paper, specifically, the Class-Independent Proposal Learning and Class-Induced Proposal Learning problems. These details will help reproduce the results we achieve in the experimental section. For completeness, we reiterate both problems in Eqs (1) and (2).

$$(\mathbf{D}_U, \mathbf{A}^*) = \underset{\mathbf{D},\mathbf{A}}{\arg\min}\ \frac{1}{n}\|\mathbf{X} - \mathbf{D}\mathbf{A}\|_F^2 + \lambda\|\mathbf{A}\|_{2,1} \tag{1}$$

$$(\mathbf{D}_S, \mathbf{A}^*, \mathbf{W}^*) = \underset{\mathbf{D},\mathbf{A},\mathbf{W}}{\arg\min}\ \frac{1}{n}\|\mathbf{X} - \mathbf{D}\mathbf{A}\|_F^2 + \lambda_1\|\mathbf{A}\|_{2,1} + \lambda_2\|\mathbf{W}^T\mathbf{A} - \mathbf{Y}\|_F^2 + \lambda_3\|\mathbf{W}\|_F^2, \tag{2}$$

1.1. Solving Eq (1) Using Alternating Optimization

To solve Eq (1), we follow a conventional strategy of fixed point optimization, which iteratively updates each of the variables $\mathbf{D}_U$ and $\mathbf{A}$ separately by fixing one of them at a time.

1.1.1 Fix $\mathbf{D}$ and update $\mathbf{A}$ (Step 1)

It requires the solution of Eq (3):

$$\mathbf{A}^{k+1} = \underset{\mathbf{A}}{\arg\min}\ \frac{1}{n}\|\mathbf{X} - \mathbf{D}\mathbf{A}\|_F^2 + \lambda\|\mathbf{A}\|_{2,1} \tag{3}$$

The problem in Eq (3) is a classical extension of the Lasso problem with $L_{2,1}$ matrix norm. We can solve it efficiently using ADMM by introducing a slack variable $\mathbf{Z}$, which separates the two terms in the optimization.

$$\begin{aligned} &\underset{\mathbf{A},\mathbf{Z}}{\arg\min}\ \frac{1}{n}\|\mathbf{X} - \mathbf{D}\mathbf{A}\|_F^2 + \lambda\|\mathbf{Z}\|_{2,1} \\ &\text{subject to } \mathbf{A} = \mathbf{Z} \end{aligned} \tag{4}$$

The augmented Lagrangian is:

$$\mathbf{L} = \frac{1}{n}\|\mathbf{X} - \mathbf{DA}\|_F^2 + \lambda\|\mathbf{Z}\|_{2,1} + \mathrm{Tr}(\mathbf{\Lambda}^T(\mathbf{A} - \mathbf{Z})) + \frac{\rho}{2}\|\mathbf{A} - \mathbf{Z}\|_F^2 \tag{5}$$

Solving the above problem requires an iterative process, where (A, Z) are updated separately by minimizing the augmented Lagrangian function while $\mathbf{\Lambda}$ is updated by performing gradient ascent on the dual problem.

Update A:

$$\arg\min_{\mathbf{A}} \frac{1}{n}\|\mathbf{X} - \mathbf{DA}\|_F^2 + \mathrm{Tr}(\mathbf{\Lambda}^T\mathbf{A}) + \frac{\rho}{2}\|\mathbf{A} - \mathbf{Z}\|_F^2 \tag{6}$$

This is a strongly convex problem that is solved by setting the gradient to 0. To do this, we need to solve the following linear system:

$$(\mathbf{D}^T\mathbf{D} + \rho\mathbf{I})\mathbf{A} = \mathbf{D}^T\mathbf{X} - \mathbf{\Lambda} + \rho\mathbf{Z} \tag{7}$$

If $(\mathbf{D}^T\mathbf{D} + \rho\mathbf{I})^{-1}$ can be efficiently computed offline, we can solve this subproblem as:

$$\mathbf{A} = (\mathbf{D}^T\mathbf{D} + \rho\mathbf{I})^{-1}(\mathbf{D}^T\mathbf{X} - \mathbf{\Lambda} + \rho\mathbf{Z}) \tag{8}$$

Update Z:

$$\arg\min_{\mathbf{Z}} \frac{\rho}{2}\|\mathbf{A} - \mathbf{Z}\|_F^2 - \mathrm{Tr}(\mathbf{\Lambda}^T\mathbf{Z}) + \lambda\|\mathbf{Z}\|_{2,1} \tag{9}$$

$$\arg\min_{\mathbf{Z}} \frac{1}{2}\|\mathbf{A} - \mathbf{Z}\|_F^2 - \frac{1}{\rho}\mathrm{Tr}(\mathbf{\Lambda}^T\mathbf{Z}) + \frac{\lambda}{\rho}\|\mathbf{Z}\|_{2,1} \tag{10}$$

$$\arg\min_{\mathbf{Z}} \frac{1}{2}\|\mathbf{Z} - (\mathbf{A} + \frac{\mathbf{\Lambda}}{\rho})\|_F^2 + \frac{\lambda}{\rho}\|\mathbf{Z}\|_{2,1} \tag{11}$$

$$\arg\min_{\{\hat{\mathbf{z}}_i\}_{i=1}^d} \sum_{i=1}^{d}(\frac{1}{2}\|\hat{\mathbf{z}}_i - (\hat{\mathbf{A}}_i + \frac{\hat{\mathbf{\Lambda}}_i}{\rho}))\|_2^2 + \frac{\lambda}{\rho}\|\hat{\mathbf{z}}_i\|_2 \tag{12}$$

where d is the number of rows of $\mathbf{A}$ and $\hat{\mathbf{z}}_i$ denotes the i^{th} row of $\mathbf{Z}$. We can solve for each row of $\mathbf{Z}$ independently using the identity:

$$\begin{aligned}&\arg\min_{\hat{\mathbf{z}}_i} \sum_{i=1}^{d}(\frac{1}{2}\|\hat{\mathbf{z}}_i - (\hat{\mathbf{A}}_i + \frac{\hat{\mathbf{\Lambda}}_i}{\rho}))\|_2^2 + \frac{\lambda}{\rho}\|\hat{\mathbf{z}}_i\|_2 \\ &= \max(0, 1 - \frac{\lambda}{\rho\|\hat{\mathbf{A}}_i + \frac{\hat{\mathbf{\Lambda}}_i}{\rho}\|_2})(\hat{\mathbf{A}}_i + \frac{\hat{\mathbf{\Lambda}}_i}{\rho})\end{aligned} \tag{13}$$

Update Λ:

$$\mathbf{\Lambda} \Longleftarrow \mathbf{\Lambda} + \rho(\mathbf{A} - \mathbf{Z}) \tag{14}$$

1.1.2 Fix A and update D (Step2)

This update requires the solution to Eq (15), which is a linear least squares problem in matrix form.

$$\mathbf{D^{k+1}} = \arg\min_{\mathbf{D}} \frac{1}{n}\|\mathbf{X} - \mathbf{DA}\|_F^2$$

$$\mathbf{D^{k+1}} = \arg\min_{\mathbf{D}} \|\mathbf{X}^T - \mathbf{A}^T\mathbf{D}^T\|_F^2 \tag{15}$$

We can solve the problem using Eq (16):

$$\mathbf{D^{k+1}} = ((\mathbf{AA}^T)^{-1}\mathbf{AX}^T)^T = \mathbf{xA}^T(\mathbf{AA}^T)^{-1} \tag{16}$$

Notice that we initialize $\mathbf{D}$ using K-Means.

1.2. Solving Eq (2) Using Alternating Optimization

We solve this problem using alternating optimization.

1.2.1 Fix D and update A (Step 1)

In this step, we solve the problem in Eq (17)

$$\mathbf{A}^{k+1} = \arg\min_{\mathbf{A}} \frac{1}{n}\|\mathbf{X} - \mathbf{DA}\|_F^2 + \lambda_1\|\mathbf{A}\|_{2,1} + \lambda_2\|\mathbf{W}^T\mathbf{A} - \mathbf{Y}\|_F^2$$

$$\mathbf{A}^{k+1} = \arg\min_{\mathbf{A}} \|\mathbf{U} - \mathbf{VA}\|_F^2 + \lambda_1\|\mathbf{A}\|_F^2 \tag{17}$$

where, $\mathbf{U} = [\frac{1}{\sqrt{n}}\mathbf{X}^T\sqrt{\lambda_2}\mathbf{Y}^T]^T$ and $\mathbf{V} = [\frac{1}{\sqrt{n}}\mathbf{D}\sqrt{\lambda_2}\mathbf{W}^T]^T$. Note that problem in Eq (17) can be solved in the same way we solve Eq (3).

1.2.2 Fix all variables except D (Step 2)

Notice that we get the same optimization problem described in Eq (15).

1.2.3 Fix all the variables except W (step 3)

We get a least squares problem in matrix form:

$$\mathbf{W}^{k+1} = \arg\min_{\mathbf{W}} \lambda_2\|\mathbf{W}^T\mathbf{A} - \mathbf{Y}\|_F^2 + \lambda_3\|\mathbf{W}\|_F^2 \tag{18}$$

$$\mathbf{W}^{k+1} = \arg\min_{\mathbf{W}} \|\mathbf{A}^T\mathbf{W} - \mathbf{Y}^T\|_F^2 + \frac{\lambda_3}{\lambda_2}\|\mathbf{W}\|_F^2 \tag{19}$$

Eq (19) can be solved by handling the following linear system:

$$2(\mathbf{AA}^T)\mathbf{W} - 2\mathbf{AY}^T + \frac{2\lambda_3}{\lambda_2}\mathbf{W} = 0$$

$$(\mathbf{AA}^T + \frac{\lambda_3}{\lambda_2}\mathbf{I})\mathbf{W} = \mathbf{AY}^T$$

$$\mathbf{W}^{k+1} = (\mathbf{AA}^T + \frac{\lambda_3}{\lambda_2}\mathbf{I})^{-1}\mathbf{AY}^T \tag{20}$$

2. Complementary Qualitative Results

Here, we provide complementary qualitative results of the proposal segments generated by our method. In Figure 1, we show the top-5 best ranked proposal segments from the entire Thumos14 testing set. We observe that the most confident predictions produced by our method are strongly correlated with a known action category.

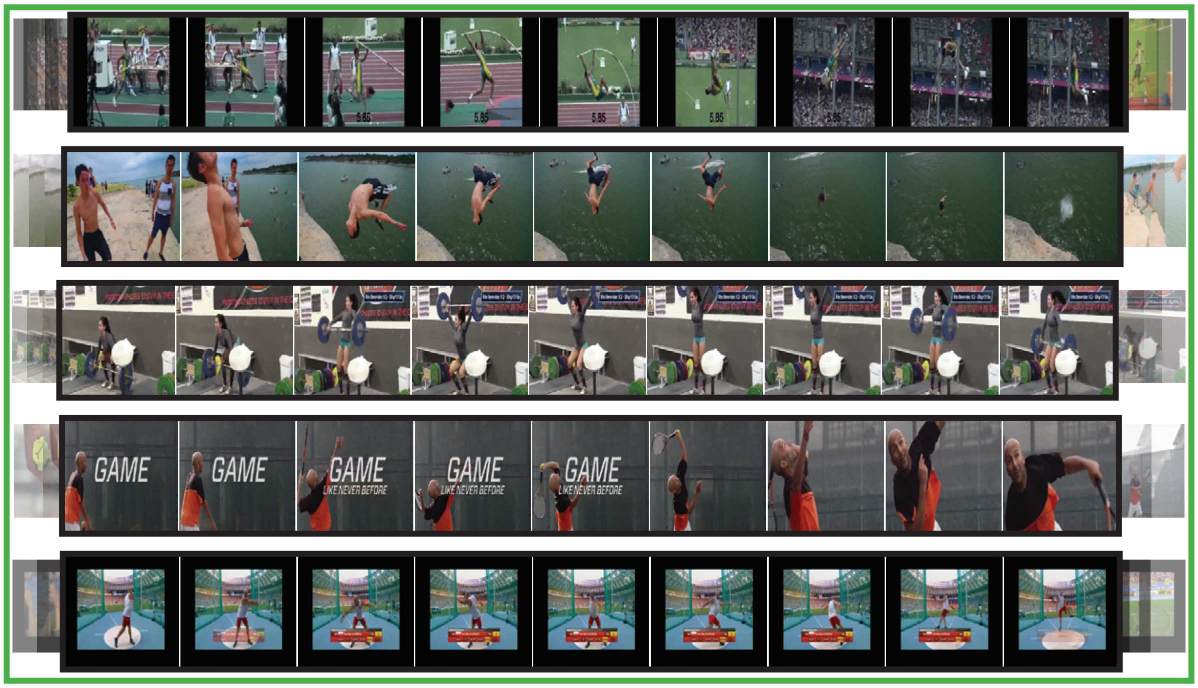

Figure 1. Top-5 best ranked proposals from entire Thumos14 testing set.

Figure 2. Bottom-5 worst ranked proposals from entire Thumos14 testing set.

We also investigate the bottom-5 worst ranked proposals in Figure 2. Our proposal method is able to discard (or assign a low score) the proposal candidates that are not related with human actions. Interestingly, we also discard the candidate proposals that contains unknown actions. For example, the third row in Figure 2 shows a proposal that confines a penalty kick foul action which it is not annotated in Thumos14.

Finally, we add additional examples of retrieved proposals in Figure 3. We provide a live example of these proposals in the following link: http://www.cabaf.net/temporalproposals/demo.

Figure 3. Additional examples of retrieved proposals. Watch the proposal segments in the following link: http://www.cabaf.net/temporalproposals/demo.