Moral Lineage Tracing (Supplement)

Florian Jug[1,*], Evgeny Levinkov[2,*], Corinna Blasse[1], Eugene W. Myers[1], Bjoern Andres[2]

[1]*Max Planck Institute of Molecular Cell Biology and Genetics, Dresden*
[2]*Max Planck Institute for Informatics, Saarbrücken*

1. Proof of Lemma 2

PROOF If Condition 1 in Def. 2 holds for a set $C \subseteq E$, then all inequalities (1) are satisfied by the $x \in \{0,1\}^E$ such that $x^{-1}(1) = C$. Otherwise, there would exist a $t \in \mathbb{N}$, a cycle Y of G_t and an $e \in Y$ such that $x_e = 1$ and $\forall e' \in Y \setminus \{e\}$: $x_{e'} = 0$. This implies $|Y \cap C| = 1$, in contradiction to the assumption that $C \cap E_t$ is a multicut of G_t. Conversely, if all inequalities (1) are satisfied by an $x \in \{0,1\}^E$, then $C := x^{-1}(1)$ satisfies Condition 1 in Def. 2. Otherwise, there would exist a $t \in \mathbb{N}$ for which $C \cap E_t$ is not a multicut of G_t. Thus, there would exist a cycle Y of G_t and an $e \in Y$ such that $Y \cap C = \{e\}$, by definition of a multicut. Hence, the inequality (1) for that cycle Y and that edge e of Y would be violated by x. The sufficiency of chordless cycles follows from (1) and is established, *e.g.*, in [**?**].

If Condition 2 in Def. 2 holds for a set $C \subseteq E$, then all inequalities (2) are satisfied by the $x \in \{0,1\}^E$ such that $x^{-1}(1) = C$. Otherwise, there would exist $t \in \mathbb{N}$, $\{v,w\} \in E_{t,t+1}$ and a path $P \in vw$-paths(G_t^+) such that $x_{vw} = 1$ and $x_P = 0$. From $x_{vw} = 1$ follows $\{v,w\} \in E_{t,t+1} \cap C$. From $x_P = 0$ follows that v and w are connected by P in $(V_t^+, E_t^+ \cap \bar{C})$. Both statements together contradict the assumption. Conversely, if all inequalities (2) are satisfied by an $x \in \{0,1\}^E$, then $C := x^{-1}(1)$ satisfies Condition 2 in Def. 2. Otherwise, there would exist $t \in \mathbb{N}$, $\{v,w\} \in E_{t,t+1} \cap C$ and a path $P \in vw$-paths$(V_t^+, E_t^+ \cap \bar{C})$. From this follows $x_{vw} = 1$ and $x_P = 0$, in contradiction to the assumption that (2) is satisfied.

If Condition 3 in Def. 2 holds for a set $C \subseteq E$, then all inequalities (3) are satisfied by the $x \in \{0,1\}^E$ such that $x^{-1}(1) = C$. Otherwise, there would exist $t \in \mathbb{N}$, $v_t, w_t \in V_t$, $v_{t+1}, w_{t+1} \in V_{t+1}$, a path $P \in v_{t+1}, w_{t+1}$-paths(G_{t+1}), and a cut $T \in v_t w_t$-cuts(G_t) such that $x_{v_t, v_{t+1}} = 0$ and $x_{w_t, w_{t+1}} = 0$ and $x_P = 0$ and $x_T = 1$. P witnesses the existence of a $v_{t+1} w_{t+1}$-path in $(V, E_{t+1} \cap \bar{C})$. The existence of T certifies the non-existence of a $v_t w_t$-path in $(V, E_t \cap \bar{C})$. Both statements together contradict the assumption. Conversely, if all inequalities (3) are satisfied by an $x \in \{0,1\}^E$, then $C := x^{-1}(1)$ satisfies Condition 3 in Def. 2. Otherwise, there would exist $t \in \mathbb{N}$, $v_t, w_t \in V_t$ and $v_{t+1}, w_{t+1} \in V_{t+1}$ such that $\{v,w\} \in E_{t,t+1} \cap \bar{C}$ and $\{v_{t+1}, w_{t+1}\} \in E_{t,t+1} \cap \bar{C}$ and such that there exist $P \in v_{t+1} w_{t+1}$-paths$(V_{t+1}, E_{t+1} \cap \bar{C})$ and $T \in v_t w_t$-cuts$(V_t, E_t \cap \bar{C})$. Hence, $x_{v_t, v_{t+1}} = 0$ and $x_{w_t, w_{t+1}} = 0$ and $x_P = 0$ and $x_T = 1$, in contradiction to the assumption that (3) is satisfied. □