Supplementary Material:
Do It Yourself Hyperspectral Imaging with Everyday Digital Cameras

Seoung Wug Oh[1] Michael S. Brown[2] Marc Pollefeys[3] Seon Joo Kim[1]
[1]Yonsei University [2]National University of Singapore [3]ETH Zurich

1. Alternating Least Squares Optimization

In this supplementary material, we present the details of the alternating least squares optimization used in the paper. The objective function can be expressed in matrix form as follows:

$$\hat{\mathbf{R}}, \hat{\mathbf{a}} = \arg\min_{\mathbf{R},\mathbf{a}} \left\{ \sum_{m=1}^{N_c} \sum_{k=1}^{3} |\mathbf{p}_{m,k} - \mathbf{R}^T \mathbf{A}_{m,k}\mathbf{a}|_2^2 + \alpha \|\mathbf{WBR}\|_F^2 + \beta |\mathbf{WEa}|_2^2 \right\}, \quad s.t. \quad \mathbf{BR}, \mathbf{Ea} \geq 0, \tag{1}$$

where $\mathbf{W}$ is the second-order difference matrix defined as:

$$\mathbf{W} = \begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 \end{bmatrix}, \tag{2}$$

$\mathbf{B}_{v,i} = b_i(v)$ with i is from 1 to N_r, $\mathbf{E}_{v,j} = a_j(v)$ with j is from 1 to N_a, and v is from 1 to 31. v represents 31 bands from 400nm to 700nm with the intervals of 10nm.

A least squares solution for this system of bilinear equations can be found by iteratively solving the two linear subproblems [1, 2]. To minimize Eq. (1), we adopt the alternating least squares method in [1] and alternate between solving for the illumination $\mathbf{a}$ by fixing the surface reflectance $\mathbf{R}$ and then solving for $\mathbf{R}$ with fixed $\mathbf{a}$.

- **Step 1: Fix $\mathbf{R}$ and solve for $\mathbf{a}$.**
  By fixing $\mathbf{R}$, Equation (1) becomes a linear system with respect to $\mathbf{a}$ with the smoothness term and the positivity constraint:

$$\hat{\mathbf{a}} = \arg\min_{\mathbf{a}} \{ |\mathbf{g} - \mathbf{Fa}|_2^2 + \beta |\mathbf{WEa}|_2^2 \}, \quad s.t. \quad \mathbf{Ea} \geq 0, \tag{3}$$

  where $\mathbf{g} = [\mathbf{p}_{1,1}^T, \mathbf{p}_{1,2}^T, \mathbf{p}_{1,3}^T, \cdots, \mathbf{p}_{N_c,1}^T, \mathbf{p}_{N_c,2}^T, \mathbf{p}_{N_c,3}^T]^T$ and $\mathbf{F} = [(\mathbf{R}^T\mathbf{A}_{1,1})^T, (\mathbf{R}^T\mathbf{A}_{1,2})^T, (\mathbf{R}^T\mathbf{A}_{1,3})^T, \cdots, (\mathbf{R}^T\mathbf{A}_{N_c,1})^T, (\mathbf{R}^T\mathbf{A}_{N_c,2})^T, (\mathbf{R}^T\mathbf{A}_{N_c,3})^T]^T$.

  Eq. (3) can be further simplified by concatenating two problems in Eq. (3). A solution for Eq. (3) can be obtained by solving the following least squares problem with the positivity constraint:

$$\hat{\mathbf{a}} = \arg\min_{\mathbf{a}} |\tilde{\mathbf{g}} - \tilde{\mathbf{F}}\mathbf{a}|_2^2 \quad s.t. \quad \mathbf{Ea} \geq 0, \tag{4}$$

  where $\tilde{\mathbf{g}} = [\mathbf{g}^T, \mathbf{0}^T]^T$ with the zero vector $\mathbf{0}$ and $\tilde{\mathbf{F}} = [\mathbf{F}^T, \beta\mathbf{W}_\mathbf{a}^\mathbf{T}]^T$ with $\mathbf{W_a} = \mathbf{WE}$.

- **Step 2: Fix $\mathbf{a}$ and solve for $\mathbf{R}$.**
  By fixing $\mathbf{a}$, Eq. (1) also becomes a linear system with respect to $\mathbf{R}$:

$$\hat{\mathbf{R}} = \arg\min_{\mathbf{R}} \{ \|\mathbf{G} - \mathbf{R}^T\mathbf{T}\|_F^2 + \alpha \|\mathbf{WBR}\|_F^2 \}, \quad s.t. \quad \mathbf{BR} \geq 0, \tag{5}$$

  where $\mathbf{G} = [\mathbf{p}_{1,1}, \mathbf{p}_{1,2}, \mathbf{p}_{1,3}, \cdots, \mathbf{p}_{N_c,1}, \mathbf{p}_{N_c,2}, \mathbf{p}_{N_c,3}]$ and $\mathbf{T} = [\mathbf{A}_{1,1}\mathbf{a}, \mathbf{A}_{1,2}\mathbf{a}, \mathbf{A}_{1,3}\mathbf{a}, \cdots, \mathbf{A}_{N_c,1}\mathbf{a}, \mathbf{A}_{N_c,2}\mathbf{a}, \mathbf{A}_{N_c,3}\mathbf{a}]$.

  Similar to Eq. (4), Eq. (5) also can be simplified. A solution for Eq. (5) can be obtained by solving the following least squares problem:

$$\hat{\mathbf{R}} = \arg\min_{\mathbf{R}} \|\tilde{\mathbf{G}} - \mathbf{R}\tilde{\mathbf{T}}\|_F^2 \quad s.t. \quad \mathbf{BR} \geq 0, \tag{6}$$

  where $\tilde{\mathbf{G}} = [\mathbf{G}, \mathbf{O}]$ with zero matrix $\mathbf{O}$ and $\tilde{\mathbf{T}} = [\mathbf{T}, \alpha\mathbf{W_r}]$ with $\mathbf{W_r} = \mathbf{WB}$.

We found out that the initialization of $\mathbf{R}$ does not significantly affect the results empirically, and we initialize every spectral reflectance the same as the first reflectance basis. Iteratively solving step 1 and 2 (alternating Eq. (4) and Eq. (6)), we successfully minimize the objective function in Eq. (1) within 100 iterations. An optimization for 24 surfaces of Macbeth color chart less than 10 seconds on our system with a Intel 3.40 GHz CPU.

References

[1] E.-W. Bai and Y. Liu. Least squares solutions of bilinear equations. *Systems & control letters*, 55(6):466–472, 2006. 1

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