Supplemental Document:
Detecting Vanishing Points using Global Image Context in a Non-Manhattan World

Menghua Zhai   Scott Workman   Nathan Jacobs

ted@cs.uky.edu   scott@cs.uky.edu   jacobs@cs.uky.edu

Computer Science, University of Kentucky

1. Proof of Perpendicularity

In the pinhole camera model (Fig. 1), define $O = [0,0,0]^{\mathsf{T}}$ as the camera focal point, $o = [0,0,f]^{\mathsf{T}}$ as the principal point. Given a zenith vanishing point, p_z, and two points on the horizon line, p_1, p_2, we show that the zenith direction is perpendicular to the horizon line:

Theorem 1. *Zenith direction, l_{op_z} is perpendicular to the horizon line, $l_{p_1 p_2}$.*
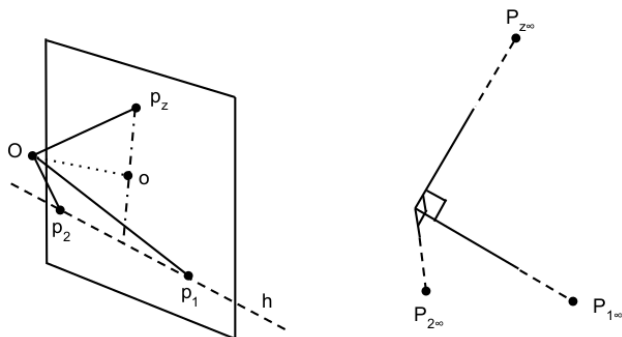

Figure 1: $P_{z\infty}$ is the infinite point corresponding to p_z; $P_{1\infty}$ and $P_{2\infty}$ are infinite points corresponding to p_1 and p_2. Since $P_{z\infty}$ is perpendicular to $P_{1\infty}$ and $P_{2\infty}$, p_z is perpendicular to p_1 and p_2.

Proof.

$$
\begin{aligned}
(p_z - o)^{\mathsf{T}}(p_1 - p_2) &= [x_{p_z}, y_{p_z}, 0] \begin{bmatrix} x_{p_1} - x_{p_2} \\ y_{p_1} - y_{p_2} \\ 0 \end{bmatrix} \\
&= p_z^{\mathsf{T}} p_1 - p_z^{\mathsf{T}} p_2 = 0
\end{aligned}
$$

$\square$

2. Choosing the Number of Horizon Samples

The horizon accuracy of the algorithm scales linearly with the number of horizon samples. For the evaluation in the paper (Sec. 5), we used 300 samples. Intuitively, the more candidates, the longer time it takes to process, but usually leads to more accurate predictions. Fig. 2 shows the sensitivity of the number of horizon samples on the horizon accuracy using our method.

As observed in Fig. 2, our algorithm requires fewer horizon line candidates to converge on YUD than on ECD.

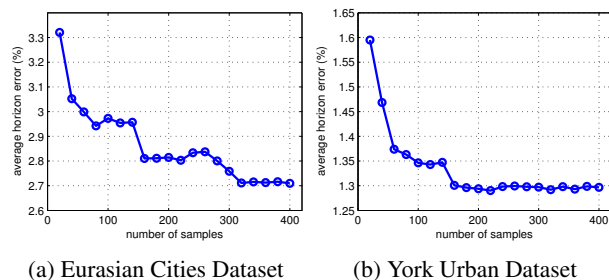(a) Eurasian Cities Dataset   (b) York Urban Dataset

Figure 2: As the the number of horizon candidates (x-axis) increases, the average horizon error (y-axis) across the entire dataset (validation + test) decreases. Horizon error is defined as the maximum distance from the detected horizon line to the ground-truth horizon line, normalized by the image height.

3. Results

We present our results on both the Eurasian Cities Dataset and the York Urban Dataset. These results cover both the validation set (ID: 1~25) and the test set (ID: 26~103 in ECD; ID: 26~102 in YUD).
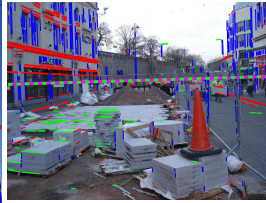
For each query image, we present the horizon ground truth (green dash), the predicted horizon line (magenta) and the line segments associated with each vanishing point. For clarity, only line segments of the zenith VP and the top two horizontal VPs are shown. The results are listed in ascending order by horizon error.
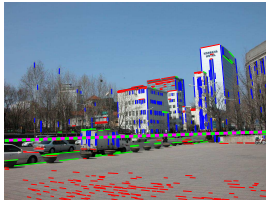
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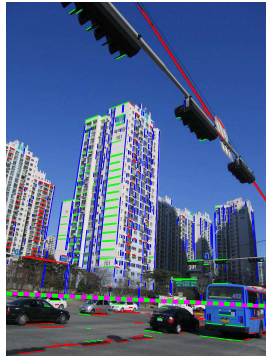
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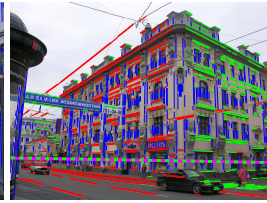
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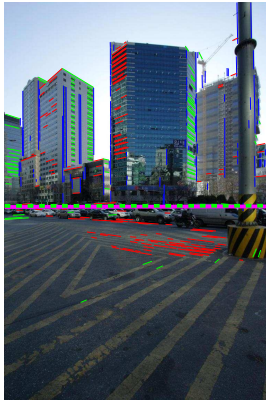
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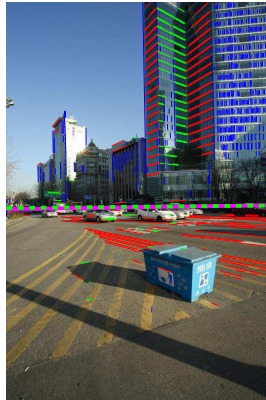
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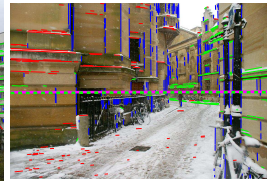
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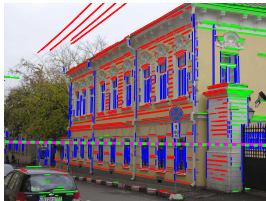
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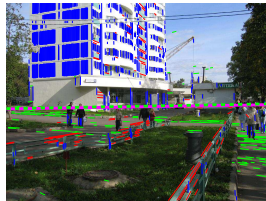
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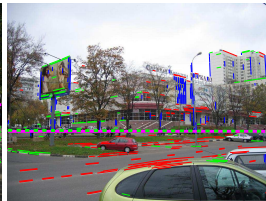
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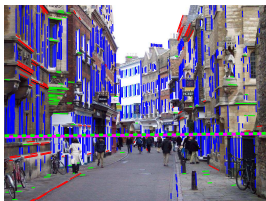
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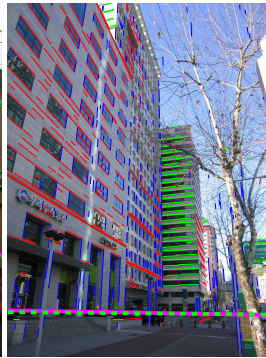
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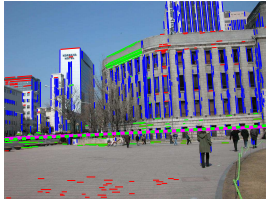
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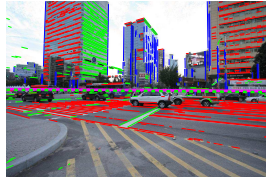
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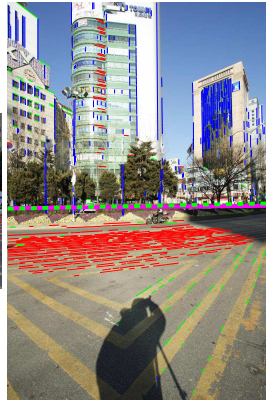
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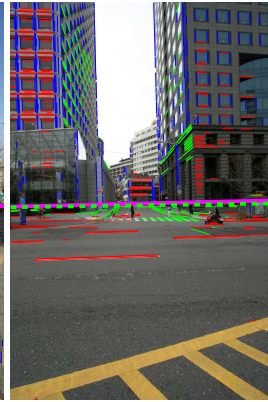
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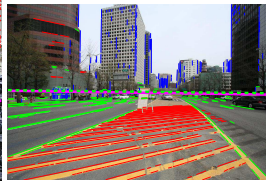
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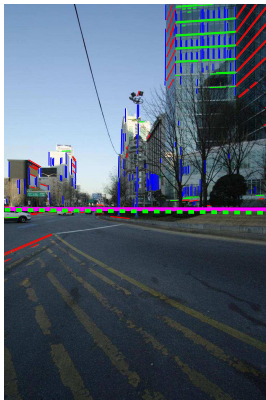
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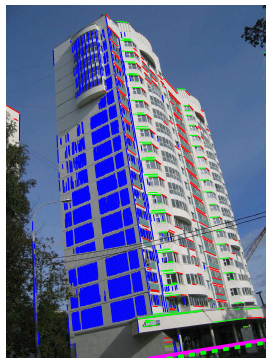
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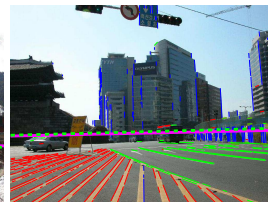
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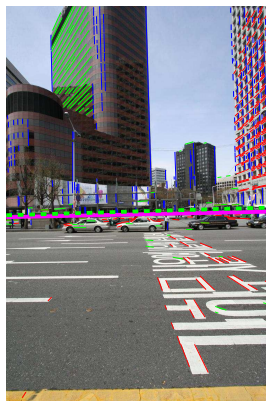
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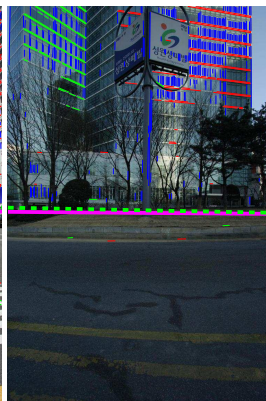
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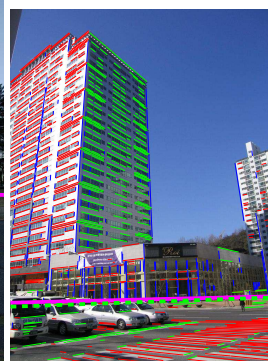
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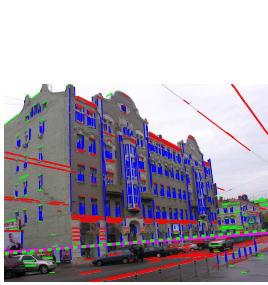
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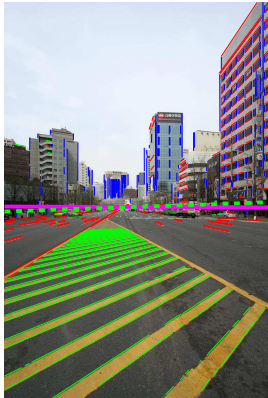
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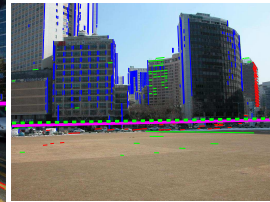
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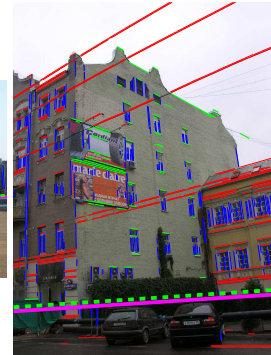
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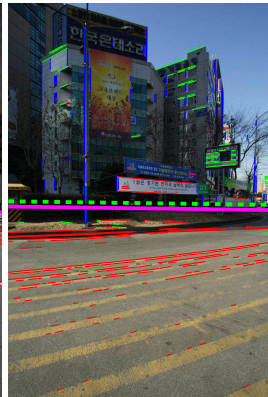
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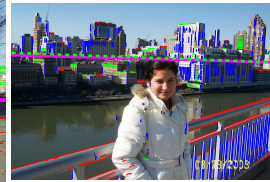
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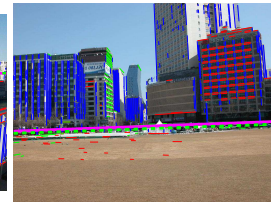
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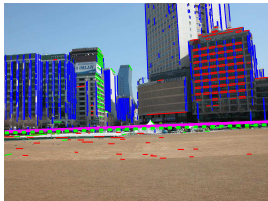
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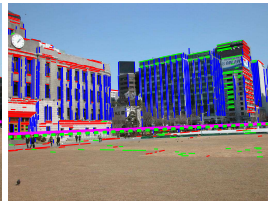
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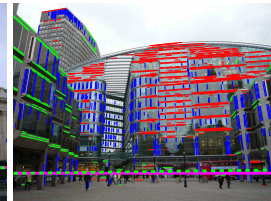
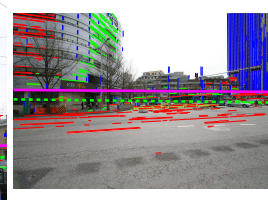ECD: 050, Error: 1.56%
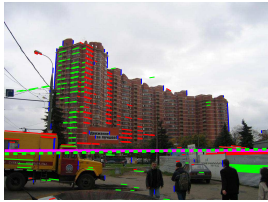
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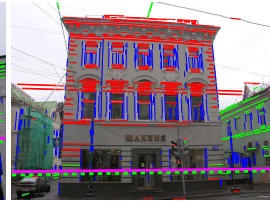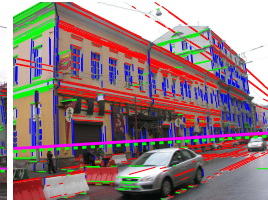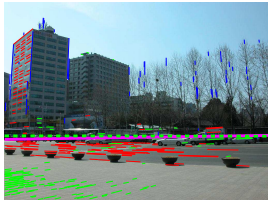
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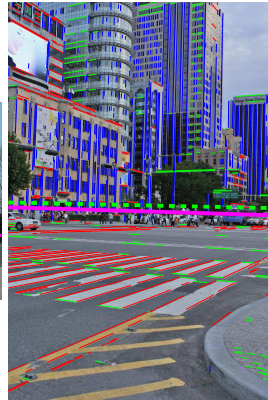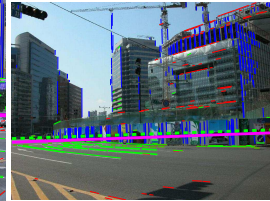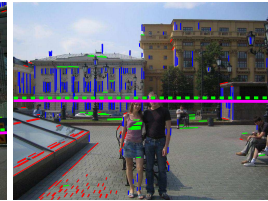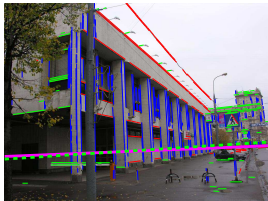
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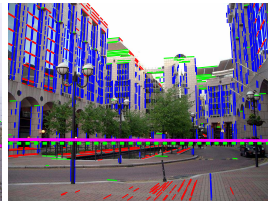
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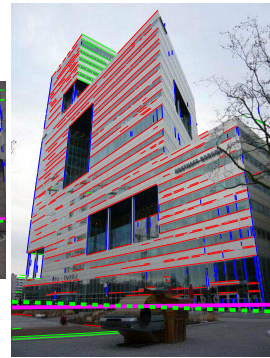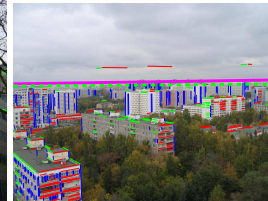
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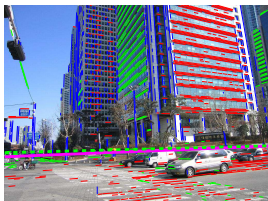
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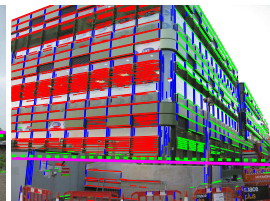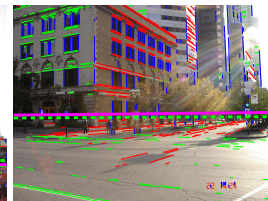
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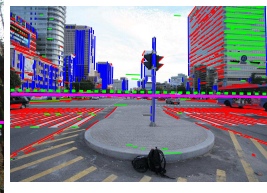
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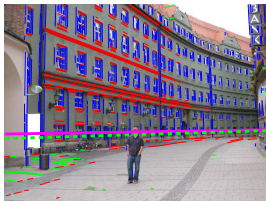
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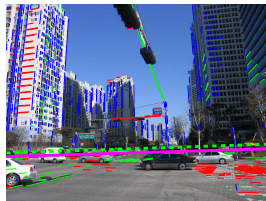
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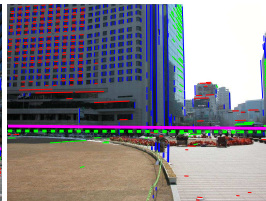
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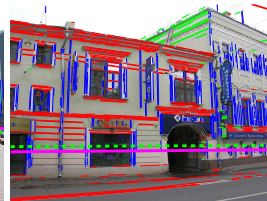
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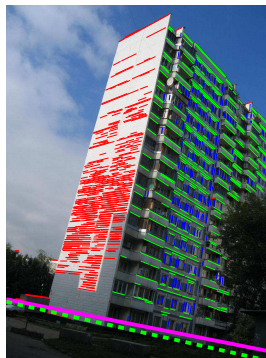
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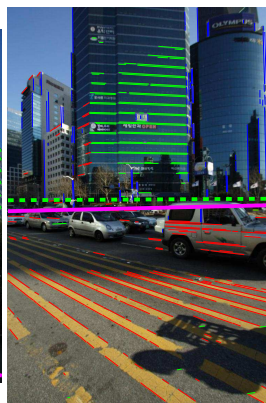
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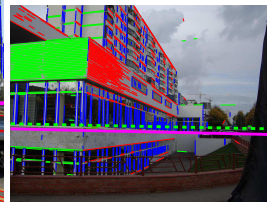
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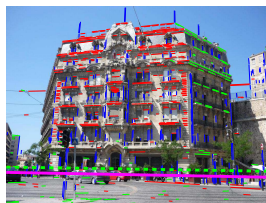
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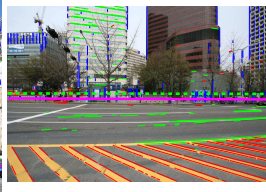
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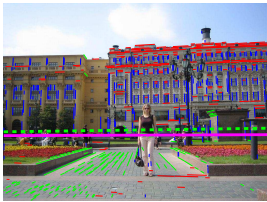
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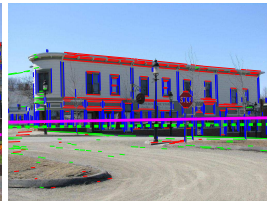
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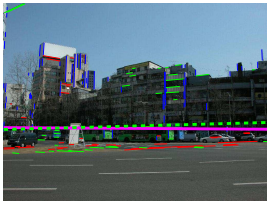
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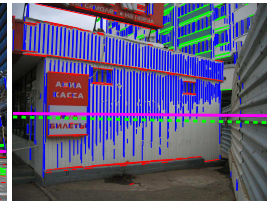
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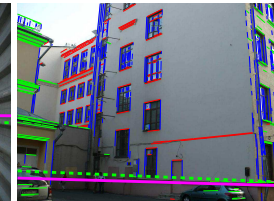
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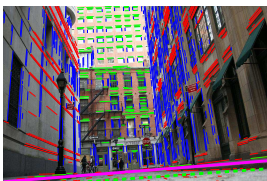
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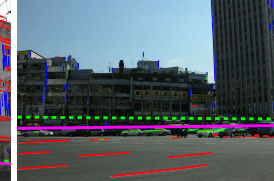
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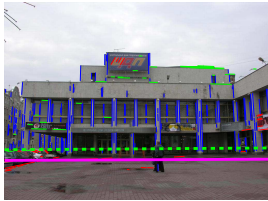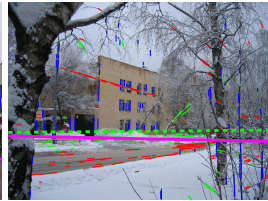
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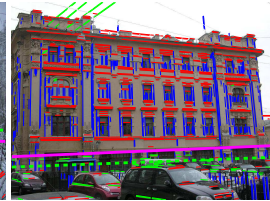
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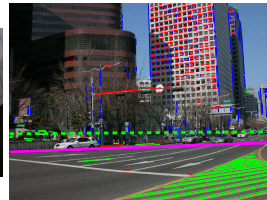
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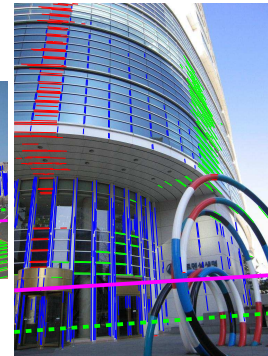
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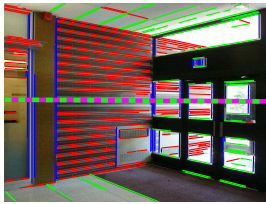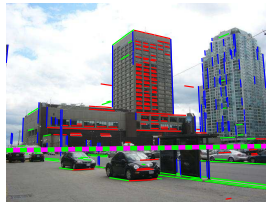
YUD: 014, Error: 0.07%   YUD: 098, Error: 0.07%   YUD: 043, Error: 0.08%   YUD: 016, Error: 0.10%
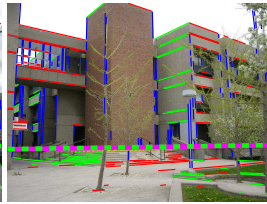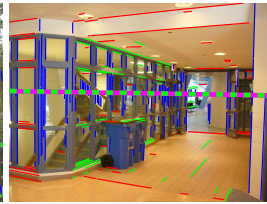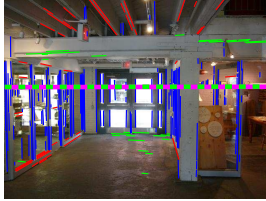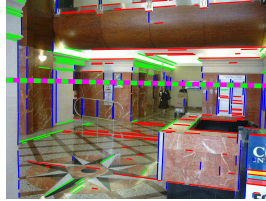
YUD: 085, Error: 0.13%   YUD: 033, Error: 0.17%   YUD: 012, Error: 0.19%   YUD: 092, Error: 0.21%
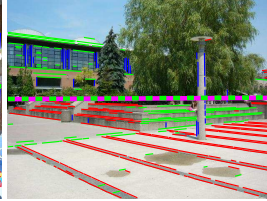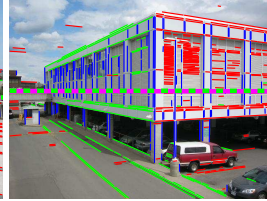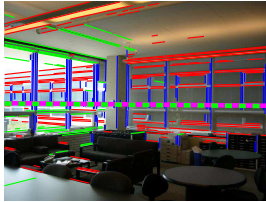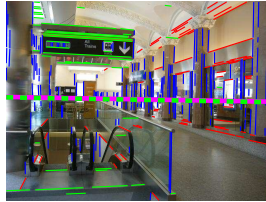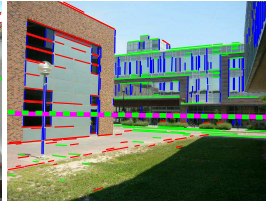
YUD: 052, Error: 0.22%   YUD: 058, Error: 0.22%   YUD: 021, Error: 0.23%   YUD: 051, Error: 0.26%
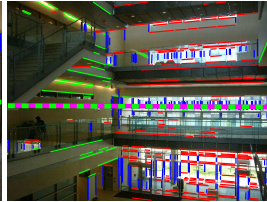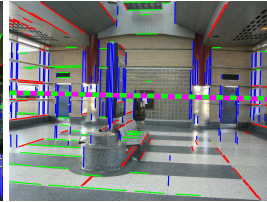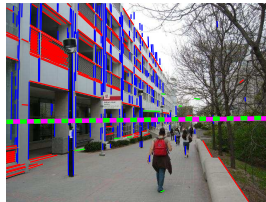
YUD: 077, Error: 0.27%   YUD: 040, Error: 0.28%   YUD: 096, Error: 0.28%   YUD: 006, Error: 0.30%
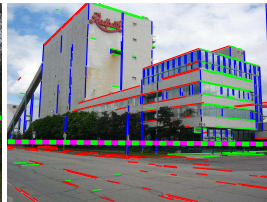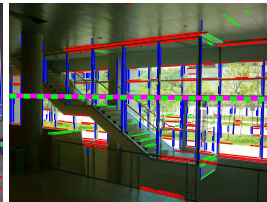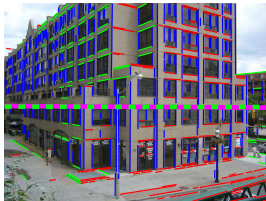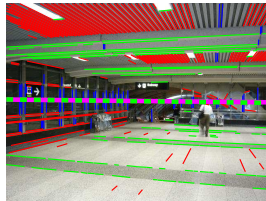
YUD: 090, Error: 0.31%   YUD: 050, Error: 0.31%   YUD: 047, Error: 0.33%   YUD: 053, Error: 0.33%
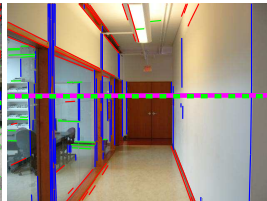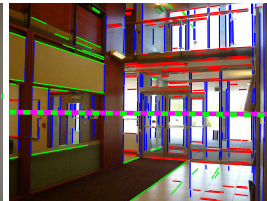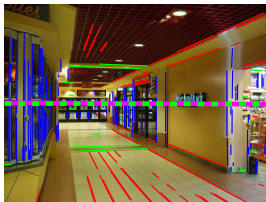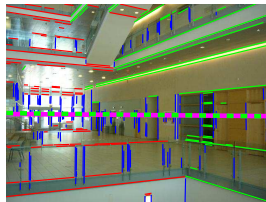
YUD: 060, Error: 0.35%   YUD: 007, Error: 0.40%   YUD: 056, Error: 0.44%   YUD: 036, Error: 0.45%
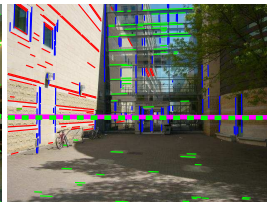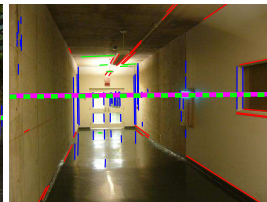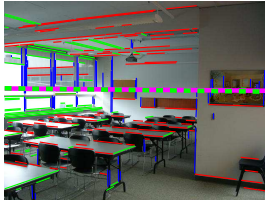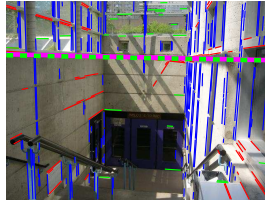
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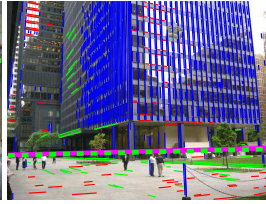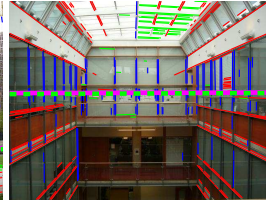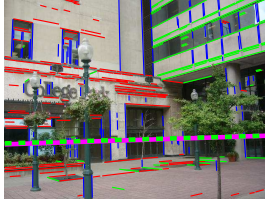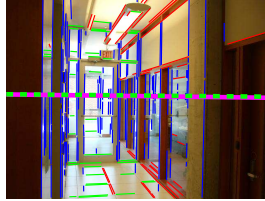
YUD: 081, Error: 0.49% YUD: 054, Error: 0.50% YUD: 089, Error: 0.50% YUD: 004, Error: 0.52%
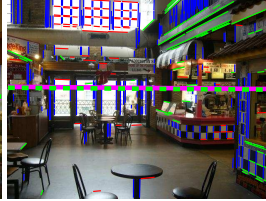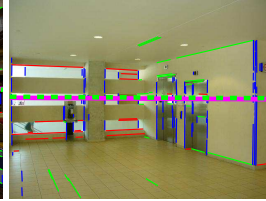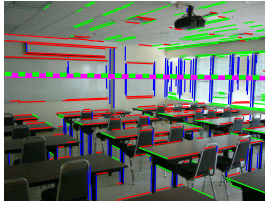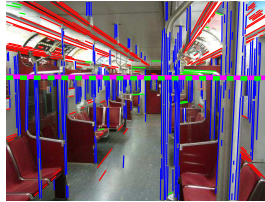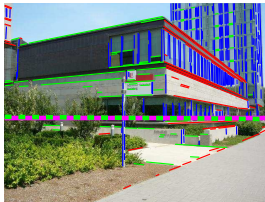
YUD: 010, Error: 0.54% YUD: 035, Error: 0.54% YUD: 011, Error: 0.55% YUD: 020, Error: 0.56%

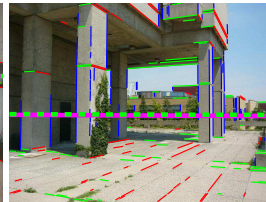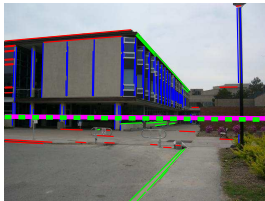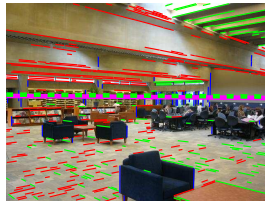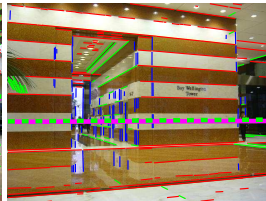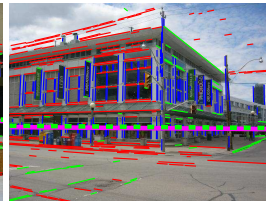YUD: 023, Error: 0.56% YUD: 093, Error: 0.58% YUD: 032, Error: 0.58% YUD: 062, Error: 0.59%
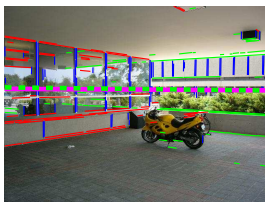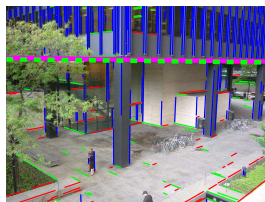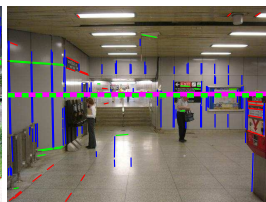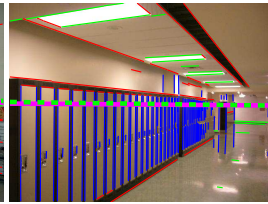
YUD: 046, Error: 0.64% YUD: 038, Error: 0.66% YUD: 079, Error: 0.69% YUD: 094, Error: 0.69%

YUD: 026, Error: 0.74% YUD: 078, Error: 0.74% YUD: 034, Error: 0.76% YUD: 076, Error: 0.78%
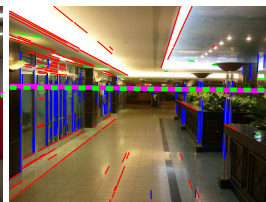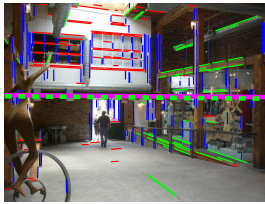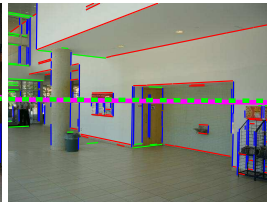
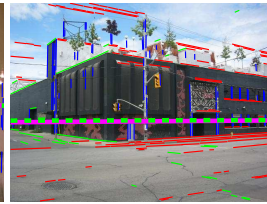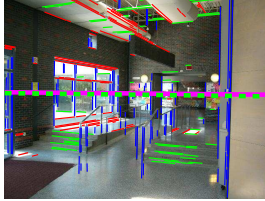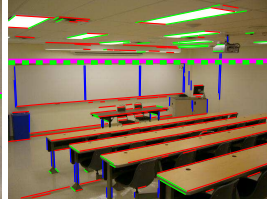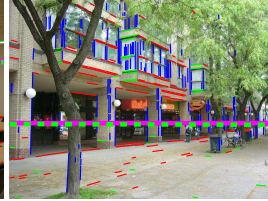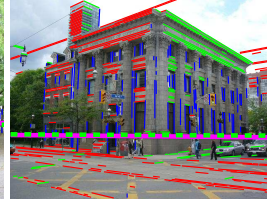YUD: 086, Error: 0.80% YUD: 019, Error: 0.81% YUD: 022, Error: 0.82% YUD: 095, Error: 0.83%

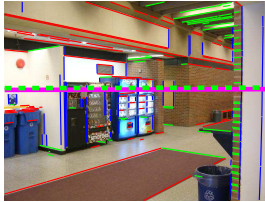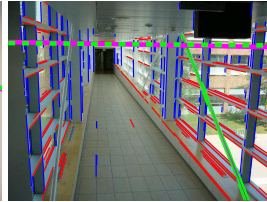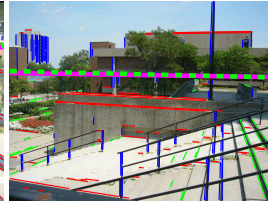YUD: 017, Error: 0.83% YUD: 009, Error: 0.84% YUD: 088, Error: 0.84% YUD: 099, Error: 0.88%
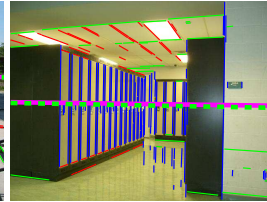
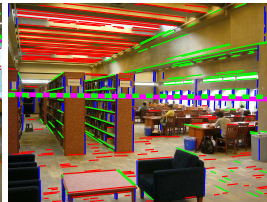YUD: 039, Error: 0.88% YUD: 028, Error: 0.89% YUD: 025, Error: 0.96% YUD: 008, Error: 1.02%
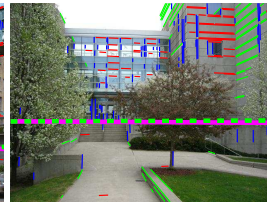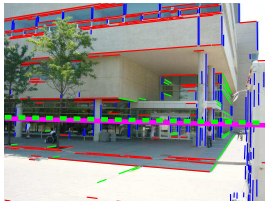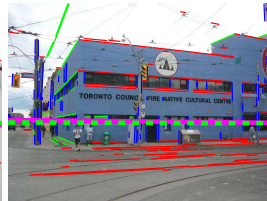
YUD: 029, Error: 1.09% YUD: 037, Error: 1.12% YUD: 097, Error: 1.14% YUD: 041, Error: 1.14%
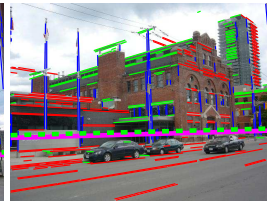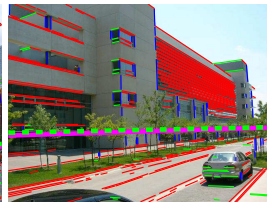
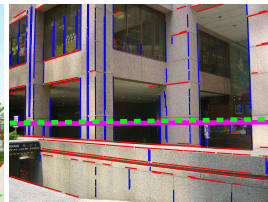YUD: 018, Error: 1.17% YUD: 083, Error: 1.24% YUD: 069, Error: 1.31% YUD: 084, Error: 1.36%
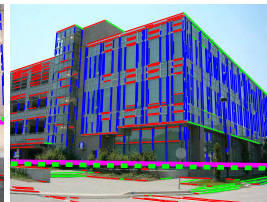
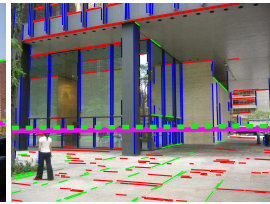YUD: 100, Error: 1.37% YUD: 003, Error: 1.41% YUD: 073, Error: 1.43% YUD: 024, Error: 1.43%
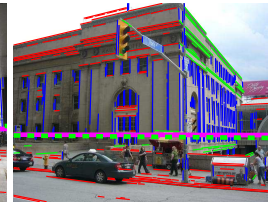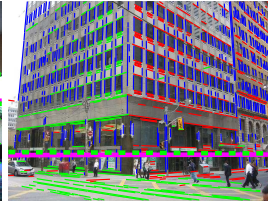
YUD: 005, Error: 1.47% YUD: 031, Error: 1.49% YUD: 071, Error: 1.50% YUD: 074, Error: 1.50%
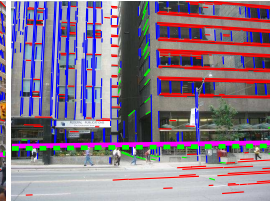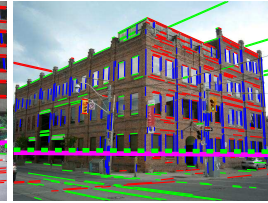
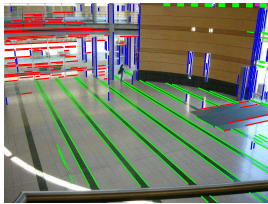YUD: 059, Error: 1.51% YUD: 066, Error: 1.54% YUD: 068, Error: 1.55% YUD: 063, Error: 1.59%
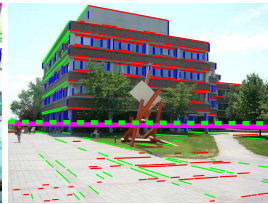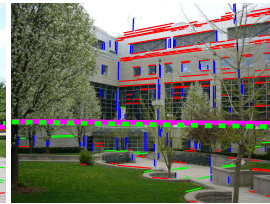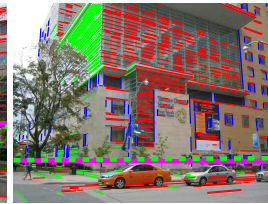
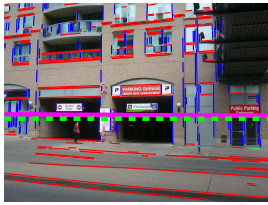YUD: 027, Error: 1.63% YUD: 015, Error: 1.66% YUD: 042, Error: 1.71% YUD: 080, Error: 1.78%
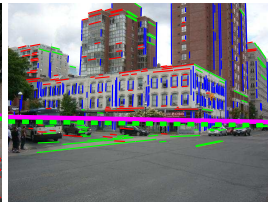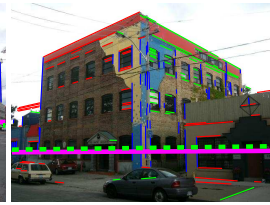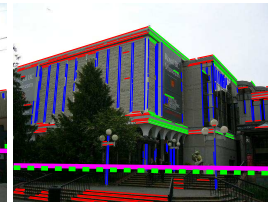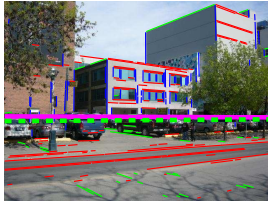
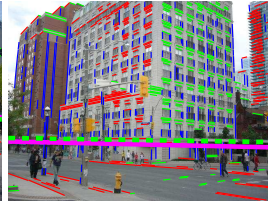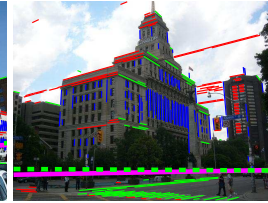YUD: 091, Error: 2.16% YUD: 101, Error: 2.26% YUD: 064, Error: 2.29% YUD: 002, Error: 2.31%

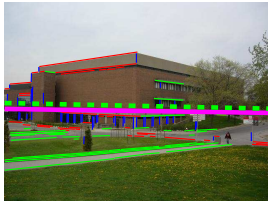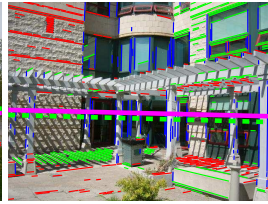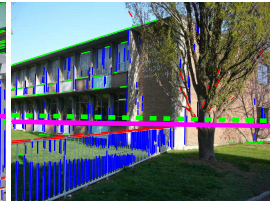YUD: 061, Error: 2.32% YUD: 102, Error: 2.60% YUD: 001, Error: 2.69% YUD: 067, Error: 2.89%
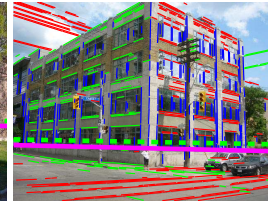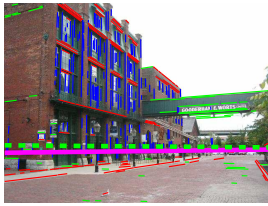
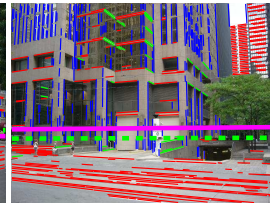YUD: 044, Error: 2.94% YUD: 055, Error: 2.96% YUD: 030, Error: 3.31% YUD: 065, Error: 3.43%
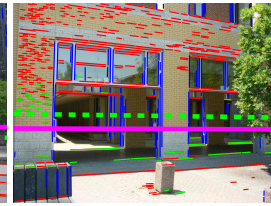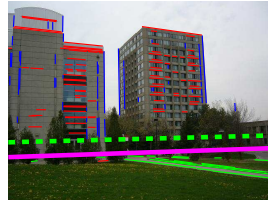
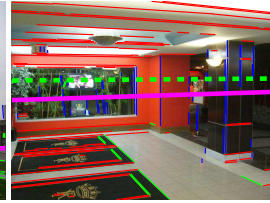YUD: 087, Error: 3.72% YUD: 082, Error: 3.87% YUD: 070, Error: 5.33% YUD: 013, Error: 7.33%

YUD: 045, Error: 7.42% YUD: 075, Error: 9.84%