Supplementary Material for Facial Expression Intensity Estimation Using Ordinal Information

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1. Derivation of ADMM updates

The proposed optimization problem has the following form.

$$\min_{u,z} \frac{1}{2}u^T\Lambda u + \mu^T l(\lfloor z \rfloor_0) \quad \text{s.t. } Au + c = z \tag{1}$$

The general form of ADMM updates are given by

$$u^{k+1} := \arg\min_u L_\rho(u, z^k, v^k) \tag{2}$$

$$z^{k+1} := \arg\min_z L_\rho(u^{k+1}, z, v^k) \tag{3}$$

$$v^{k+1} := v^k + \rho(Au^{k+1} - z^{k+1} + c) \tag{4}$$

where L_ρ is the augmented Lagrangian with multiplier ρ and l is some element-wise function.

In this section, we provide a detailed derivation of updates for the primal variables specified by Eq.(2)-Eq.(3) and dual variable specified by Eq.(4) when using two different functions l.

1.1. Hinge loss case

Choose $l(x) = x$. The augmented Lagrangian is given by

$$L_\rho(u, z, v) = \frac{1}{2}u^T\Lambda u + \mu^T \lfloor z \rfloor_0 + v^T(Au - z + c) + \frac{\rho}{2}||Au - z + c||_2^2 \tag{5}$$

At k^{th} iteration, to update $u \in \mathbb{R}^n$, we solve for u as follows

$$u^{k+1} = \arg\min_u L_\rho(u, z^k, v^k) \tag{6}$$

$$= \arg\min_u \frac{1}{2}u^T(\Lambda + \rho A^T A)u + (v^k + \rho c - \rho z^k)^T Au \tag{7}$$

$$= \arg\min_u \frac{1}{2}u^T(\frac{1}{\rho}\Lambda + A^T A)u + (\frac{1}{\rho}v^k + c - z^k)^T Au \tag{8}$$

$$= [\frac{1}{\rho}\Lambda + A^T A]^{-1}A^T(z^k - \frac{1}{\rho}v^k - c) \tag{9}$$

To update $z \in \mathbb{R}^m$, we solve for z as follows with u^k updated by u^{k+1}

$$z^{k+1} = \arg\min_z L_\rho(u^{k+1}, z, v^k) \tag{10}$$
$$= \arg\min_z \mu^T \lfloor z \rfloor_0 - (v^k)^T z + \frac{\rho}{2}(z - Au^{k+1} - c)^T(z - Au^{k+1} - c) \tag{11}$$
$$= \arg\min_z \frac{1}{2}\mu^T(|z| + z) - (v^k)^T z + \frac{\rho}{2} z^T z - \rho(Au^{k+1} + c)^T z \tag{12}$$
$$= \arg\min_z \frac{1}{2}\mu^T|z| + \frac{\rho}{2} z^T z - \rho(\frac{1}{\rho}v^k + Au^{k+1} + c - \frac{1}{2\rho}\mu)^T z \tag{13}$$
$$= \arg\min_z \frac{1}{2}\mu^T|z| + \frac{\rho}{2}||z - a||_2^2 \tag{14}$$
$$= \arg\min_z \sum_{i=1}^m \frac{\mu_i}{2}|z_i| + \frac{\rho}{2}(z_i - a_i)^2 \tag{15}$$
$$= \arg\min_z \sum_{i=1}^m |z_i| + \frac{\rho}{\mu_i}(z_i - a_i)^2 \tag{16}$$
$$= [S_{\frac{\mu_1}{2\rho}}(a_1), \cdots, S_{\frac{\mu_m}{2\rho}}(a_m)]^T \tag{17}$$

where $a = \frac{1}{\rho}v^k + Au^{k+1} + c - \frac{1}{2\rho}\mu \equiv \{a_i\}$ and $S_\kappa(a_i) = \begin{cases} a_i - \kappa, & \text{if } a_i > \kappa \\ 0, & \text{if } |a_i| \le \kappa \\ a_i + \kappa, & \text{if } a_i < -\kappa \end{cases}$, where $\kappa > 0$ is some constant.

To update dual variable $v \in \mathbb{R}^p$, we use gradient ascent with u^k updated by u^{k+1} and z^k updated by z^{k+1} as follows

$$v^{k+1} = v^k + \rho\frac{\partial L_\rho}{\partial v} \tag{18}$$
$$= v^k + \rho(Au^{k+1} - z^{k+1} + c) \tag{19}$$

where the step size is chosen as ρ, the augmented Lagrangian multiplier.

1.2. Squared hinge loss case

Choose $l(x) = x^2$ where the square is applied on each entry in x. The augmented Lagrangian is given by

$$L_\rho(u, z, v) = \frac{1}{2}u^T\Lambda u + \mu^T(\lfloor z \rfloor_0)^2 + v^T(Au - z + c) + \frac{\rho}{2}||Au - z + c||_2^2 \tag{20}$$

At k^{th} iteration, the update for $u \in \mathbb{R}^n$ is the same as Eq.(9).

To update $z \in \mathbb{R}^m$, we solve for z as follows with u^k updated by u^{k+1}

$$z^{k+1} = \arg\min_z L_\rho(u^{k+1}, z, v^k) \tag{21}$$
$$= \arg\min_z \mu^T(\lfloor z \rfloor_0)^2 - (v^k)^T z + \frac{\rho}{2}(z - Au^{k+1} - c)^T(z - Au^{k+1} - c) \tag{22}$$
$$= \arg\min_z \frac{1}{4}\mu^T(|z| + z)^2 - (v^k)^T z + \frac{\rho}{2} z^T z - \rho(Au^{k+1} + c)^T z \tag{23}$$
$$= \arg\min_z \frac{1}{4}\mu^T(|z| + z)^2 + \frac{\rho}{2} z^T z - \rho(\frac{1}{\rho}v^k + Au^{k+1} + c)^T z \tag{24}$$
$$= \arg\min_z \frac{1}{4}\mu^T(|z| + z)^2 + \frac{\rho}{2}||z - a||_2^2 \tag{25}$$
$$= \arg\min_z \sum_{i=1}^m \frac{\mu_i}{4}(|z_i| + z_i)^2 + \frac{\rho}{2}(z_i - a_i)^2 \tag{26}$$
$$= \begin{cases} \frac{\rho a_i}{\rho + 2\mu_i}, & \text{if } a_i \ge 0 \\ a_i, & \text{if } a_i < 0 \end{cases}, \ i = 1, ..., m \tag{27}$$

where $a = \frac{1}{\rho}v^k + Au^{k+1} + c \equiv \{a_i\}$.

The update for $v \in \mathbb{R}^p$ is the same as Eq.(19).

2. Detailed results

2.1. CK+ dataset results

The following results correspond to the left column of Figure 4 in the paper.

Table 1. Results on CK+ dataset.

PCC								
Method	Anger	Contempt	Disgust	Fear	Happy	Sadness	Surprise	Average
RVR	0.6090	0.5133	0.7450	0.7540	0.7729	0.5652	0.7709	0.6758
SVR	0.6428	0.5785	0.7664	0.7659	0.8007	0.5898	0.7838	0.7040
SVOR	0.5121	0.4732	0.5395	0.2269	0.5623	0.5264	0.4178	0.4655
GPOR	0.6633	0.3700	0.6770	0.7235	0.6767	0.5698	0.7566	0.6338
OR	0.6272	0.5992	0.7735	0.7171	0.8154	0.4997	0.7902	0.6889
Rankboost	0.4612	0.5704	0.6195	0.6910	0.7530	0.5457	0.7015	0.6203
OSVR-L1	**0.6669**	0.6117	0.7680	0.7770	0.8124	**0.6483**	0.8024	0.7267
OSVR-L2	0.6664	**0.6194**	**0.7886**	**0.7780**	**0.8229**	0.6199	**0.8060**	**0.7288**
ICC								
Method	Anger	Contempt	Disgust	Fear	Happy	Sadness	Surprise	Average
RVR	0.6025	0.5104	0.7269	0.7540	0.7478	0.5607	0.7409	0.6633
SVR	0.6400	0.5764	0.7439	0.7657	0.7731	0.5880	0.7518	0.6913
SVOR	0.4369	0.4706	0.4825	0.0740	0.5621	0.3400	0.3993	0.3950
GPOR	0.6602	0.3430	0.6622	0.7103	0.6698	0.5445	0.7509	0.6201
OR	0.2889	0.4832	0.1123	0.6227	0.3721	0.2695	0.3275	0.3537
Rankboost	0.3172	0.5181	0.5139	0.5846	0.6286	0.4090	0.5353	0.5010
OSVR-L1	**0.6641**	**0.6105**	0.5492	**0.7667**	0.7651	0.3800	**0.7722**	0.6440
OSVR-L2	0.6612	0.6039	**0.7861**	0.7627	**0.8179**	**0.6153**	0.7674	**0.7164**
MAE								
Method	Anger	Contempt	Disgust	Fear	Happy	Sadness	Surprise	Average
RVR	2.3211	2.5629	2.1056	1.7120	2.1044	2.3907	2.0696	2.1809
SVR	2.1653	2.4408	2.0388	**1.6439**	1.9891	2.3551	2.0683	2.1002
SVOR	7.1501	11.0095	11.4258	17.0833	3.8575	3.7846	7.9617	8.8961
GPOR	2.1867	3.5972	2.2366	2.0750	2.1381	2.7448	**1.8268**	2.4007
OR	5.2952	6.9092	4.9716	4.2686	4.0911	4.7195	4.9884	5.0348
Rankboost	5.4654	4.5819	4.5996	3.5086	5.3125	4.3438	4.3106	4.5889
OSVR-L1	**2.0840**	2.3349	2.8976	1.8722	2.5816	2.6914	1.9693	2.3473
OSVR-L2	2.1185	**2.0748**	**1.7067**	1.9531	**1.6863**	**2.2861**	2.0436	**1.9813**

2.2. BU-4DFE dataset results

The following results correspond to the right column of Figure 4 in the paper.

Table 2. Results on BU-4DFE dataset.

PCC							
Method	Anger	Disgust	Fear	Happy	Sadness	Surprise	Average
RVR	0.0752	0.1180	0.0480	0.3734	0.0480	0.1832	0.1410
SVR	0.3553	0.5330	0.2784	0.6407	0.2646	0.6165	0.4481
SVOR	0.4311	0.5348	0.3422	0.6414	0.4264	0.4618	0.4729
GPOR	0.4223	0.5769	0.3295	0.5983	0.3709	0.6103	0.4847
OR	0.4831	0.5850	0.3853	**0.6864**	0.3858	**0.6578**	0.5306
Rankboost	0.2964	0.4966	0.2831	0.5508	0.3926	0.4209	0.4067
OSVR-L1	**0.5256**	0.5936	**0.4085**	0.6330	0.4337	0.6416	0.5393
OSVR-L2	0.4890	**0.6103**	0.3900	0.6627	**0.4638**	0.6562	**0.5453**
ICC							
Method	Anger	Disgust	Fear	Happy	Sadness	Surprise	Average
RVR	0.0236	0.0381	0.0179	0.1337	0.0162	0.0642	0.0587
SVR	0.0156	0.0265	0.0883	0.3881	0.0092	0.6118	0.1899
SVOR	0.1324	0.3822	0.2297	0.2677	0.2908	0.4553	0.2930
GPOR	0.4215	0.5764	0.3254	0.5916	0.3676	0.6049	0.4813
OR	0.0015	0.0326	0.0767	0.0049	0.3525	0.0416	0.0850
Rankboost	0.1668	0.2825	0.1461	0.3503	0.2130	0.2753	0.2390
OSVR-L1	0.1989	**0.5873**	**0.3958**	0.6327	**0.4313**	0.6387	0.4808
OSVR-L2	**0.3803**	0.5625	0.3887	**0.6335**	0.4010	**0.6488**	**0.5025**
MAE							
Method	Anger	Disgust	Fear	Happy	Sadness	Surprise	Average
RVR	3.4696	3.3982	3.4374	3.2262	3.4585	3.3187	3.3848
SVR	4.5057	4.4750	3.9729	3.2078	4.5527	**2.0387**	3.7921
SVOR	4.4914	4.1433	9.8462	5.8363	5.5007	9.7525	6.5951
GPOR	2.8724	2.4997	3.3113	2.4529	3.0547	2.4255	2.7694
OR	4.7513	4.6292	5.2261	4.7446	4.6202	4.7105	4.7803
Rankboost	6.2405	5.0756	5.7305	4.2321	5.4463	5.4575	5.3637
OSVR-L1	3.8096	2.1044	**2.5532**	2.0510	2.8388	2.0845	2.5736
OSVR-L2	**2.3003**	**2.0545**	2.7670	**1.8630**	**2.3631**	2.1067	**2.2424**