Efficient Global Point Cloud Alignment Using Bayesian Nonparametrics

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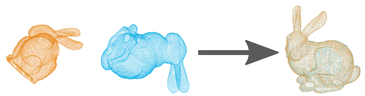
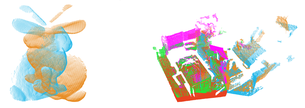
Point Cloud Alignment

Problem
find the 3D rotation/translation that aligns two point clouds

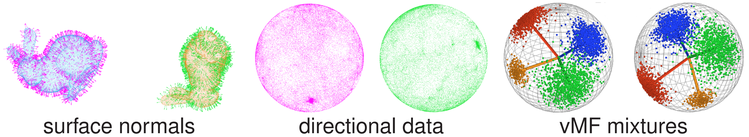


Challenge
6D opt with local optima, partial overlap, noise, outliers



Preprocessing

- decouple rotational/translational alignment using **surface normals**
- use parsimonious **BNP mixture** representation of normals / points



surface normals | directional data | vMF mixtures

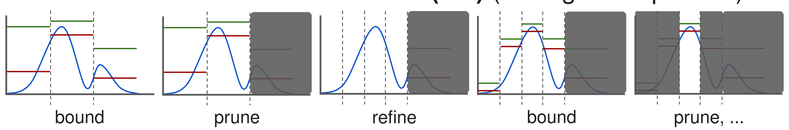
point cloud data | multivariate normal mixtures

Two-Stage Optimization

minimize L^2 norm between mixture densities
1) find rotation using vMF mixtures
2) fix rotation, find translation using normal mixtures

$$\hat{q} = \underset{q\in\mathbb{S}^3}{\operatorname{argmax}} \int_{\mathbb{S}^2} \hat{p}_1(n)\hat{p}_2(q\circ n)\mathrm{d}n \qquad \hat{t} = \underset{t\in\mathbb{R}^3}{\operatorname{argmax}} \int_{\mathbb{R}^3} \hat{p}_1(x)\hat{p}_2(\hat{q}\circ x + t)\mathrm{d}x$$

solve each with **branch & bound (BB)** (finds global optimum)

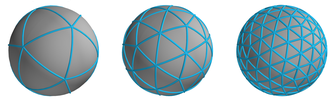


bound | prune | refine | bound | prune, ...

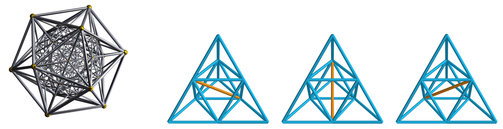
BB needs:
1) a **refinable tessellation** of the space
2) a **upper/lower bounds** on the optimum in each cell

Rotational Alignment

Novel Refinable Tessellation of $\mathbb{S}^3$
similar to the triangular tessellation of $\mathbb{S}^2$ starting with the projected icosahedron...



...except uses 4D tetrahedra and starts with the *600-cell*

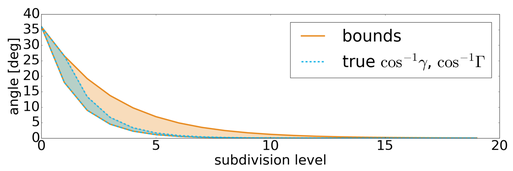


Refinement: each tetrahedron refines to 8 tetrahedra, and we select the internal edge that minimizes distortion

Bounds: solve a few small eigenvalue problems for upper, evaluate the objective anywhere in the cell for lower

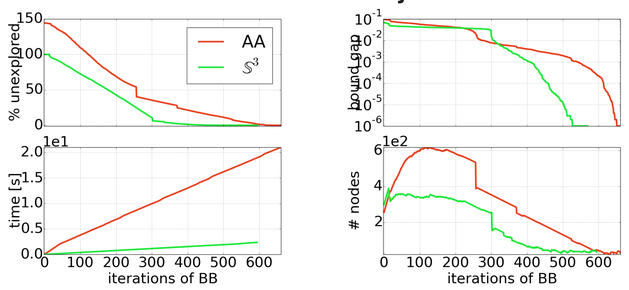
Theorem: For angular precision ϵ, need N refinements:

$$N = \max\left\{0, \left\lceil \log_2 \frac{(\cos\gamma_0)^{-1} - 1}{(\cos\epsilon/2)^{-1} - 1} \right\rceil\right\} \qquad \gamma_0 = 36°.$$



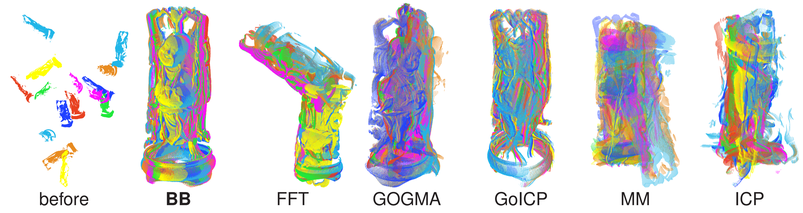
Evaluation of $\mathbb{S}^3$ Tessellation

Stanford Bunny



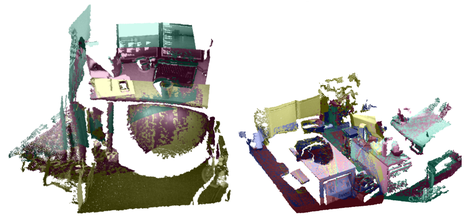
Experiments

Happy Buddha: severe partial overlap

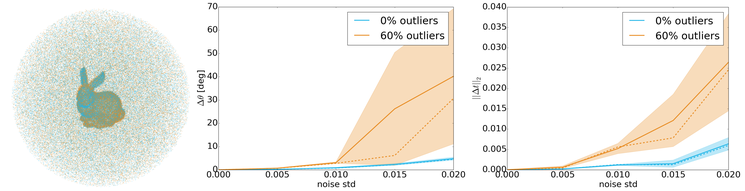


before | **BB** | FFT | GOGMA | GoICP | MM | ICP

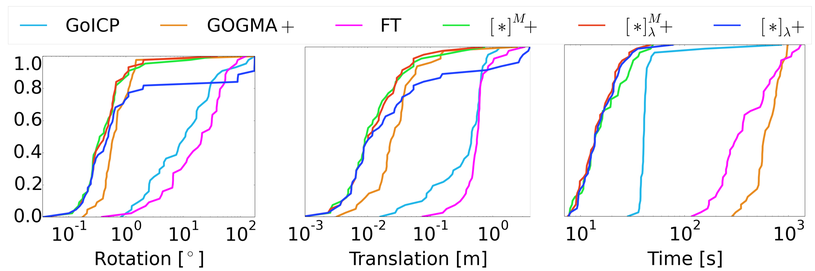
Office: partial overlap / noise



Noisy Bunny: severe noise / outliers (60%)



Apartment: rotational symmetry, outliers



BB is 1-2 orders of magnitude faster with state-of-the-art performance

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