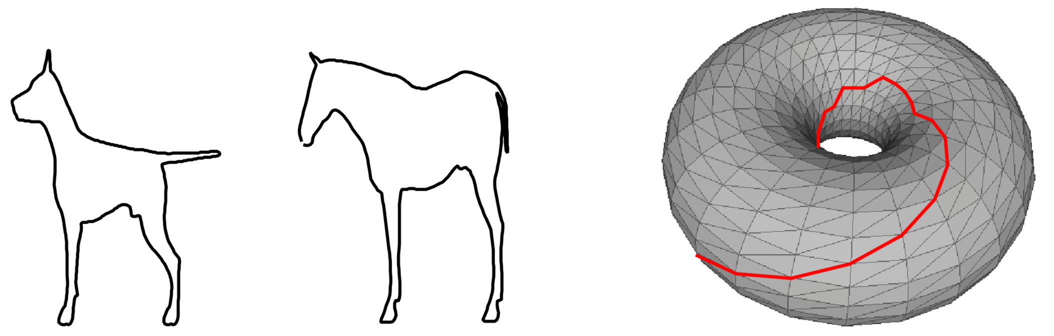
Product Manifold Filter: Non-Rigid Shape Correspondence via Kernel Density Estimation in the Product Space

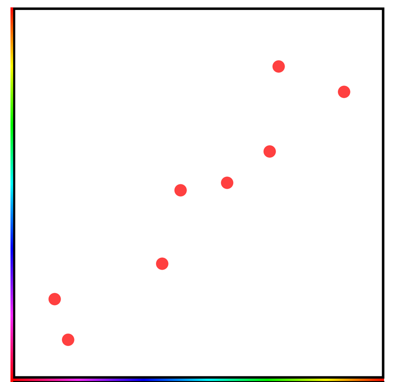
Matthias Vestner[1], Roee Litman[2], Emanuele Rodolá[3], Alex Bronstein[2,4], and Daniel Cremers[1]

[1]Technical University of Munich [2]Tel Aviv University [3]USI Lugano [4]Technion, Israel Institute of Technology

Method & 2D Illustration

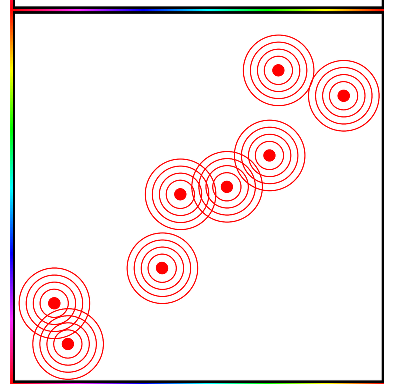


We want to reconstruct the graph $\Pi = \{(x, \pi(x)), x \in \mathcal{X}\} \subset \mathcal{X} \times \mathcal{Y}$ of the bijective correspondence $\pi : \mathcal{X} \to \mathcal{Y}$



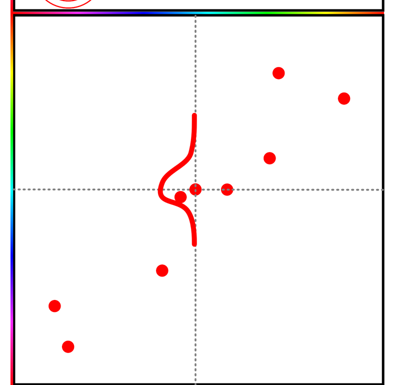
Input:
Possibly sparse and noisy sampling

$$\{(\xi_k, \eta_k)\}_{k=1}^{m}$$



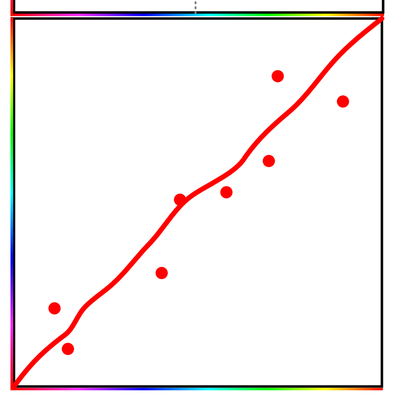
Kernel density estimation
$f(x, y) = \sum K(d_\mathcal{X}(x, \xi_k))K(d_\mathcal{Y}(y, \eta_k))$
with geodesic Gaussian kernels

$$K(d) = \exp(-\frac{d^2}{2\sigma^2})$$



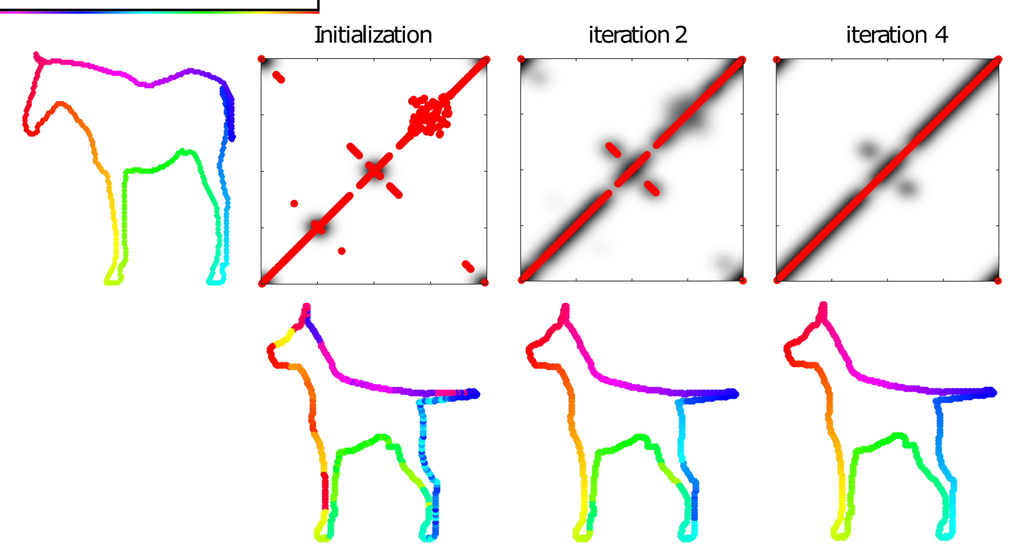
Per point estimation of $\pi(x)$:

$$\hat{\pi}(x) = \underset{y}{\operatorname{argmax}}\, f(x, y)$$

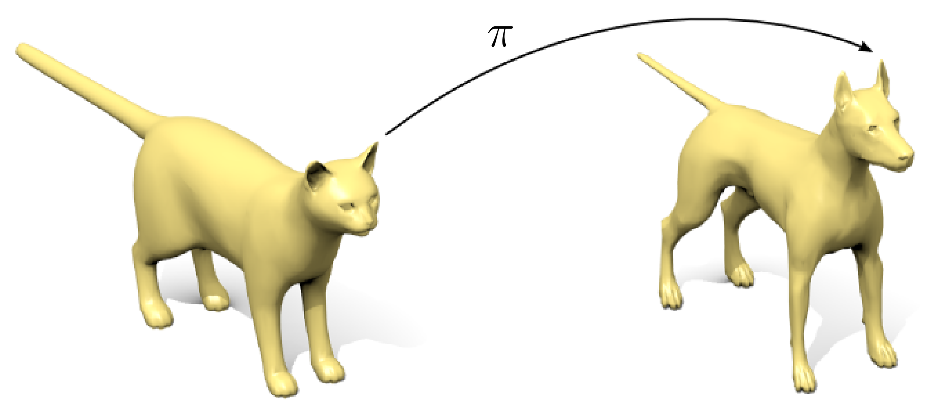


Estimator of the bijective map π:

$$\hat{\pi} = \underset{\pi:\mathcal{X}\xrightarrow{1:1}\mathcal{Y}}{\operatorname{argmax}} \int_\mathcal{X} f(x, \pi(x))dx$$



Contribution



- **guaranteed bijection**
- **no isometry assumption**
- **applicable in wide range of pipelines**

Discretization

The discrete version of the kernel density estimation f is given as a payoff matrix

$$\mathbf{F} = \mathbf{K}_\mathcal{X}\mathbf{K}_\mathcal{Y}^T \in \mathbb{R}^{n \times n}$$

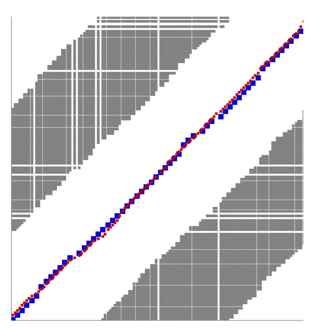
with

$$(\mathbf{K}_\mathcal{X})_{ik} = K(d_\mathcal{X}(x_i, \xi_k)) \in \mathbb{R}^{n \times m}$$
$$(\mathbf{K}_\mathcal{Y})_{ik} = K(d_\mathcal{Y}(y_i, \eta_k)) \in \mathbb{R}^{n \times m}.$$

Finding the estimator $\hat{\pi}$ becomes a linear assignment problem (LAP)

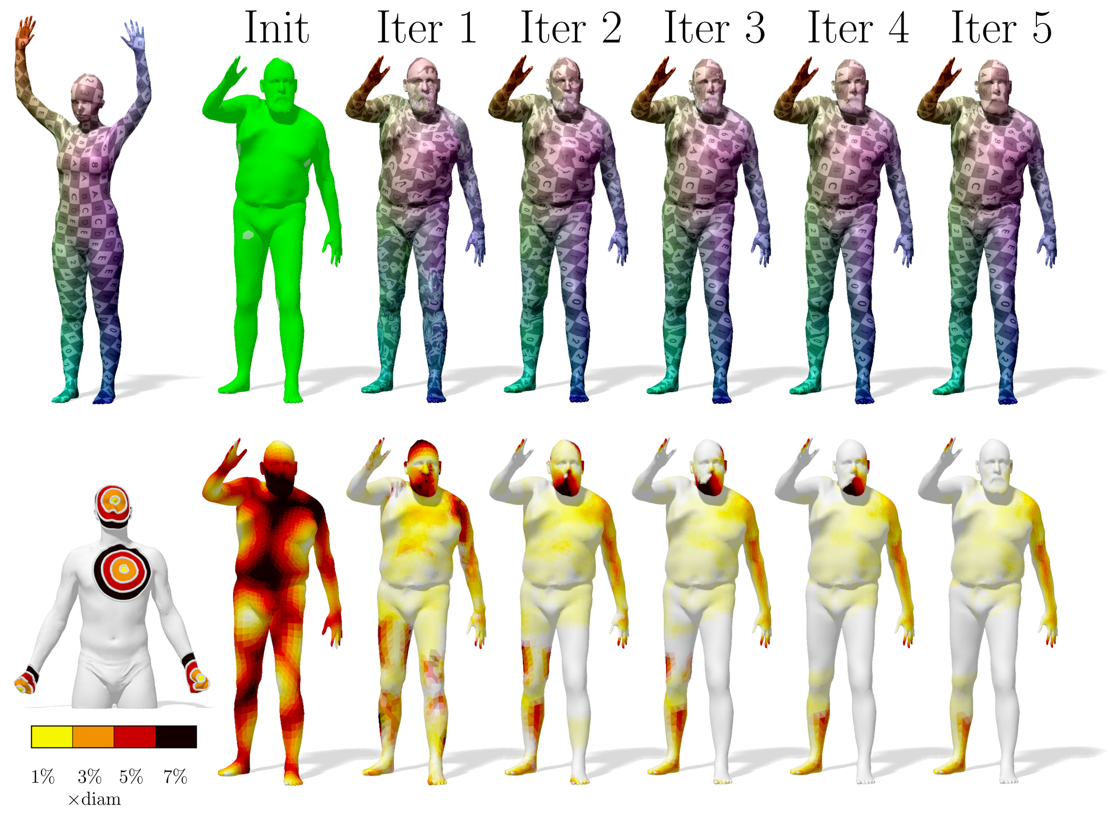
$$\hat{\mathbf{\Pi}} = \arg\max_{\mathbf{\Pi}} \langle \mathbf{\Pi}, \mathbf{F} \rangle$$

High resolution, Multiscale

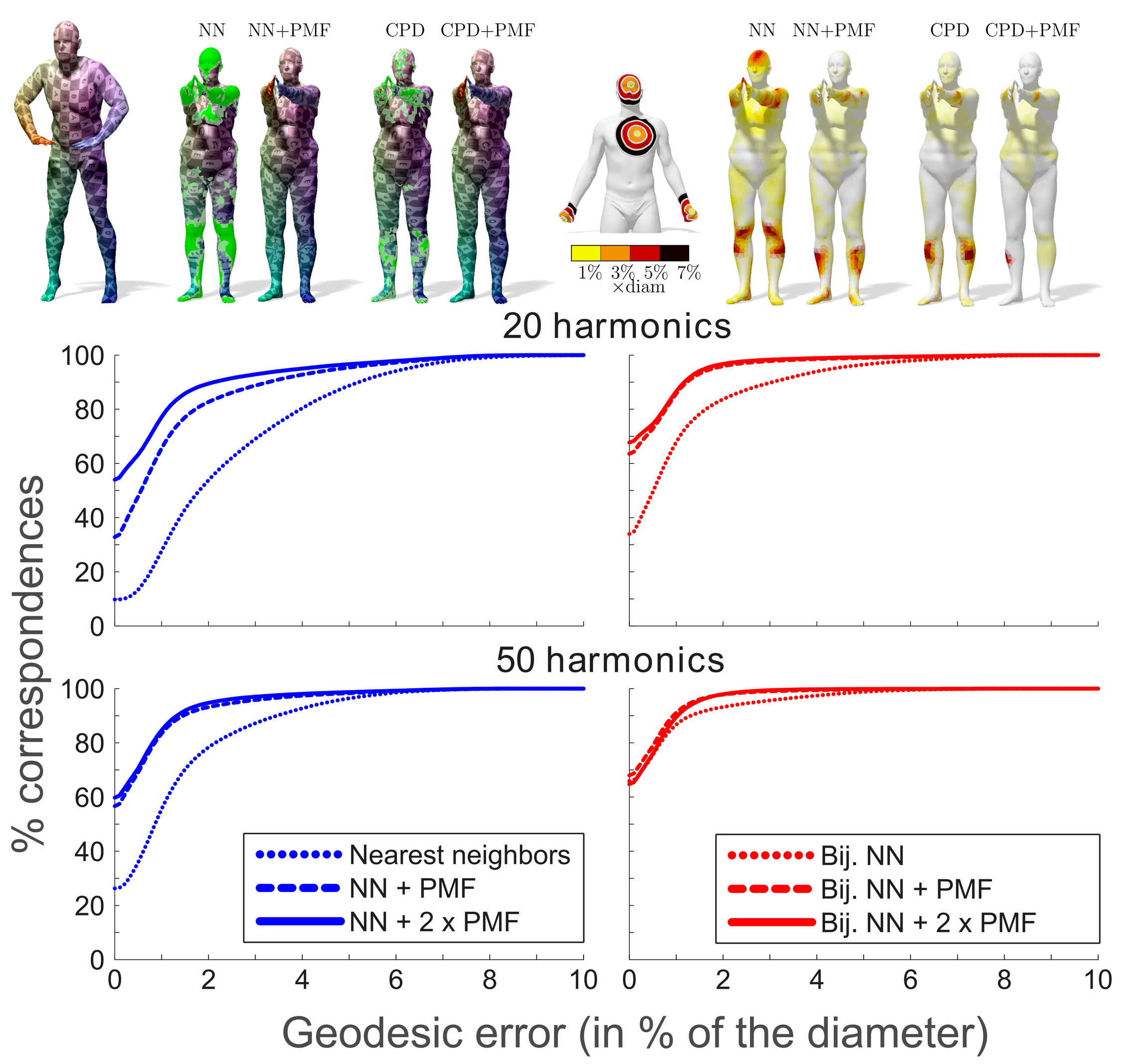


- correspondence on the coarse scale (blue) constrains the feasible correspondences on the finer scale (red)
- sparse payoff matrix

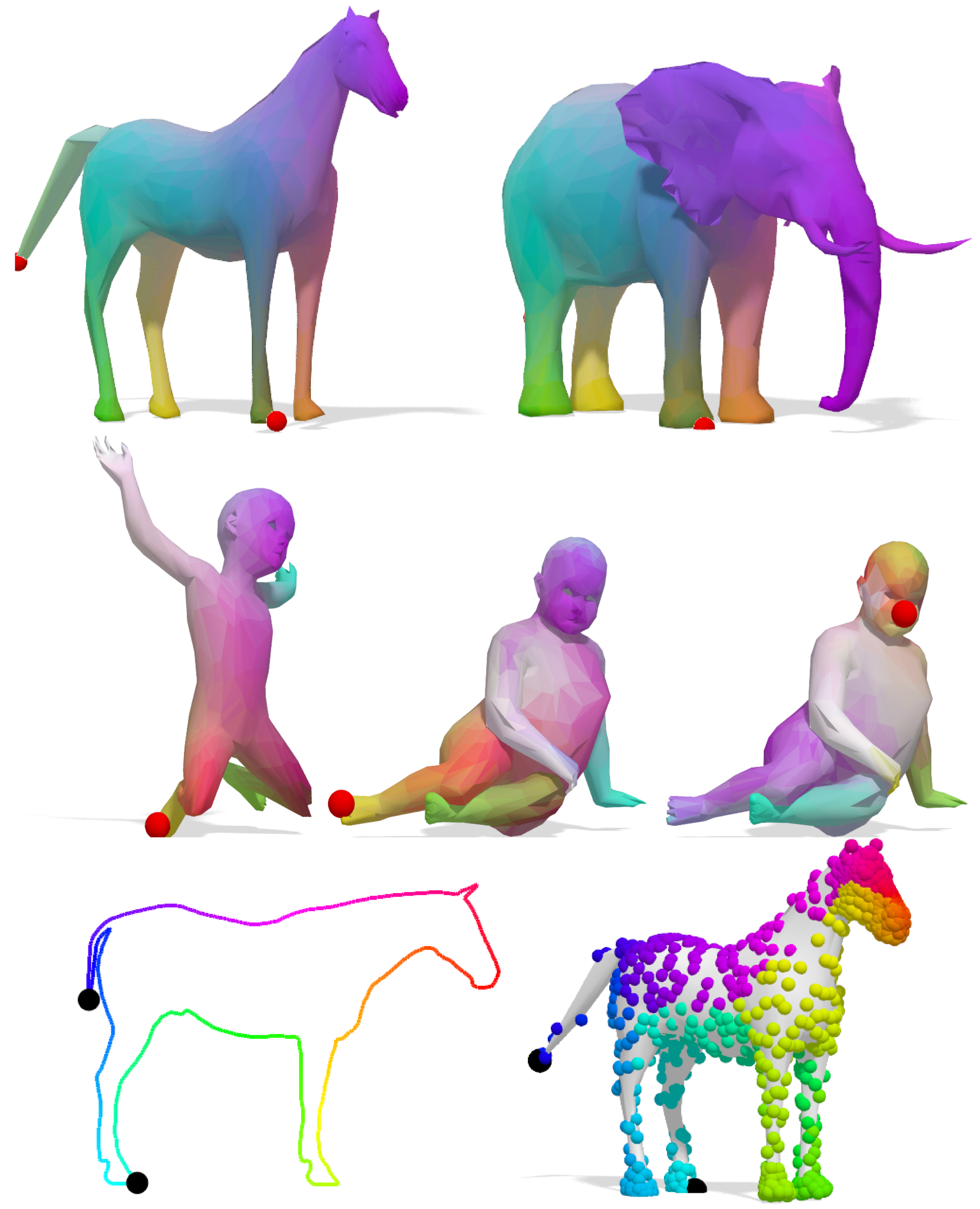
Recovery from sparse correspondence



Recovery from functional map / noisy input



Non isometric deformation



References

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[5] Bogo, Federica, et al. "FAUST: Dataset and evaluation for 3D mesh registration." (CVPR 2014)