Simultaneous Visual Data Completion and Denoising Based on Tensor Rank and Total Variation Minimization and Its Primal-Dual Splitting Algorithm

T. Yokota and H. Hontani (Nitech)

Nagoya Institute of Technology

科研費 KAKENHI

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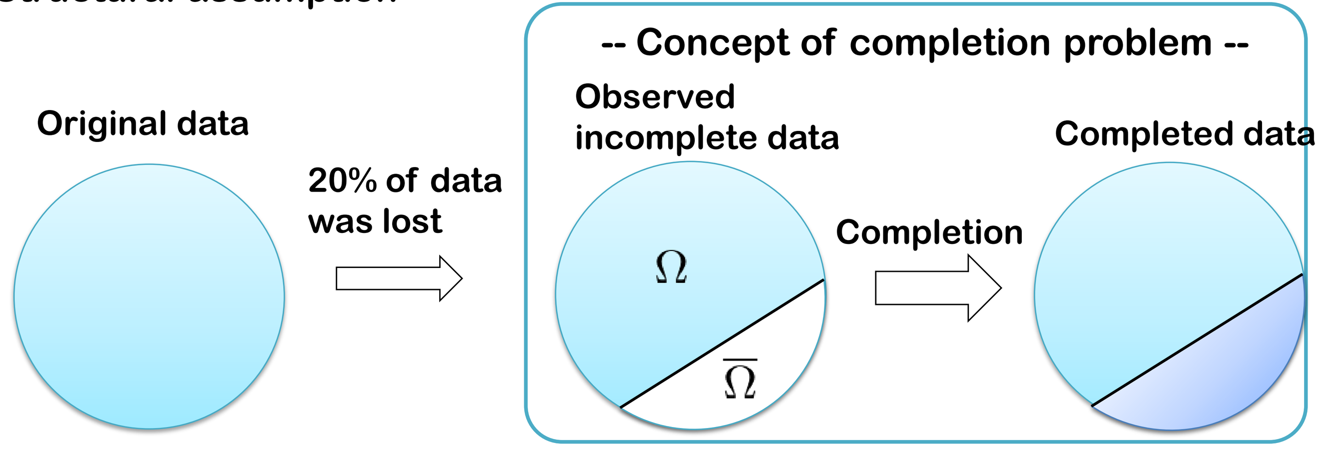
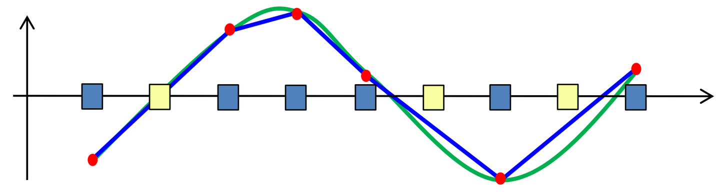
Introduction (data completion)

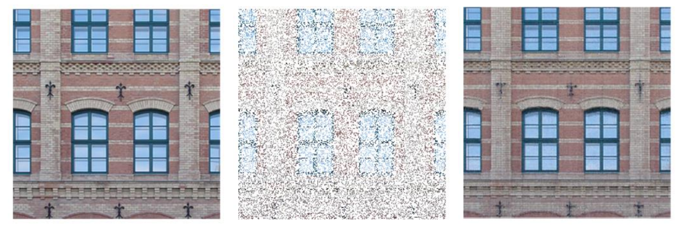
- Completion is a procedure to recover missing values by using
  - ✓ Available parts of data
  - ✓ Structural assumption



- Ex.) Vector completion
  - ✓ Linear interpolation
  - ✓ Polynomial interpolation

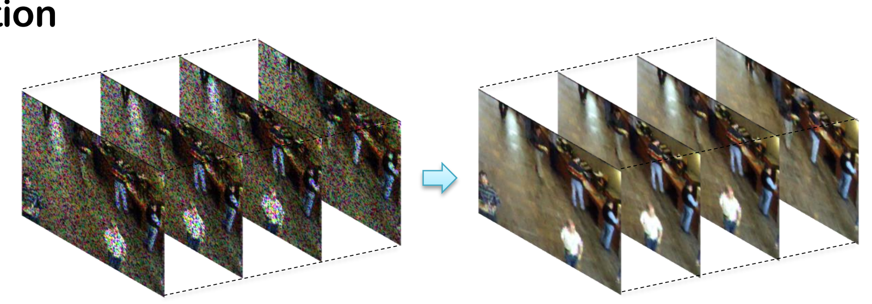


- Ex.) Matrix completion
  - ✓ Low-rank matrix completion
  - ✓ Bilinear interpolation



Original Missing Recovered

- Ex.) Tensor completion
  - ✓ Low-rank tensor completion
  - ✓ Trilinear interpolation
  - ✓ Tensor decomposition



Simultaneous Tensor Completion and Denoising

- If given incomplete data is <u>with noise</u>, ordinary completion techniques are not so useful (or can not be applied).
  - ✓ N-th order tensor: $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$
  - ✓ Function: $f(\mathcal{X}) : \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N} \rightarrow \mathbb{R}$

Regularization function (e.g., nuclear-norm, TV-norm, L1-norm etc)

(only) Completion problem

$$\underset{\mathcal{X}}{\text{minimize}}\ f(\mathcal{X}),\ \text{s.t.} P_\Omega(\mathcal{X}) = P_\Omega(\mathcal{X}_{\text{observed}})$$

Support set projection (missing elements to be zero)

Completion &Denoising problem

$$\underset{\mathcal{X}}{\text{minimize}}\ f(\mathcal{X}),\ \text{s.t.} ||P_\Omega(\mathcal{X} - \mathcal{X}_{\text{observed}})||_F^2 \leq \delta$$

Noise threshold

Noise threshold δ VS Lagrange parameter μ

- δ: <u>can be decided from signal to noise ratio</u> of data, and it does not depend regularizers.
- μ: is <u>difficult to decide</u> since it depends regularizers.
- Noise constraint form is appropriate in practice!! But optimization is little bit complicated.

$$\underset{\mathcal{X}}{\text{minimize}}\ f(\mathcal{X}),\ \text{s.t.} ||P_\Omega(\mathcal{X} - \mathcal{X}_{\text{observed}})||_F^2 \leq \delta$$

Convertible, but corresponding values of λ and δ are difficult to know

$$\underset{\mathcal{X}}{\text{minimize}}\ f(\mathcal{X}) + \frac{\mu}{2}||P_\Omega(\mathcal{X} - \mathcal{X}_{\text{observed}})||_F^2$$

Proposed Method

- We propose a direct solution method for Low-rank and TV regularizations with noise inequality and box constraints.

Tensor TV norm — Tensor nuclear norm

$$\underset{\mathcal{X}}{\text{minimize}}\ \alpha f_{\text{TV}}(\mathcal{X}) + \beta f_{\text{LR}}(\mathcal{X}),$$
$$\text{s.t.}\quad v_{\min} \leq \mathcal{X} \leq v_{\max},$$
$$||P_\Omega(\mathcal{X}_{\text{observed}} - \mathcal{X})||_F^2 \leq \delta$$

$$\alpha \geq 0,\ \beta \geq 0,\ \alpha + \beta = 1$$

- Tensor TV norm

$$f_{\text{TV}}(\mathcal{X}) := \sum_{i_1, i_2, \ldots, i_N} \sqrt{\sum_{n=1}^N w_n (\nabla_n x_{i_1, i_2, \ldots, i_N})^2}$$

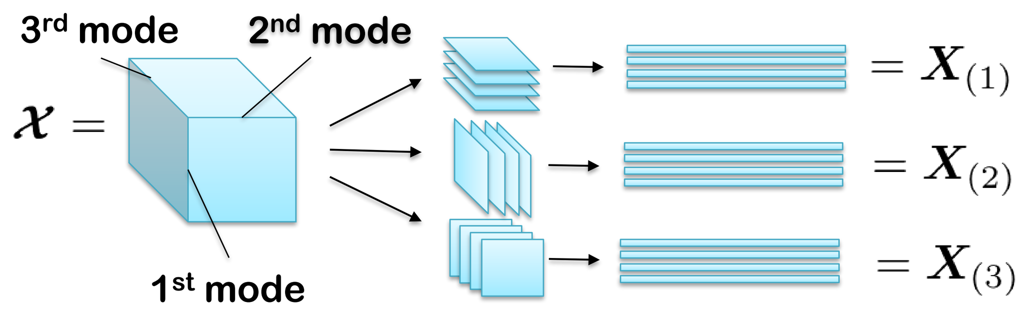
Differential with respect to n-th axis

- Tensor nuclear norm

$$f_{\text{LR}}(\mathcal{X}) := \sum_{n=1}^N \lambda_n ||\mathbf{X}_{(n)}||_*$$

$||\cdot||_*$: sum of all singular values of matrix

Definition of mode matrix unfolding



Convex Optimization

- Consider unconstrained form

Indicator functions

$$\underset{\mathcal{X}}{\text{minimize}}\ \alpha f_{\text{TV}}(\mathcal{X}) + \beta f_{\text{LR}}(\mathcal{X}) + i_{\mathcal{D}}(\mathcal{X}) + i_\delta(\mathcal{X})$$

$$i_{\mathcal{D}}(\mathcal{X}) = \begin{cases} 0 & v_{\min} \leq \mathcal{X} \leq v_{\max} \\ \infty & \text{otherwise} \end{cases}$$

$$i_\delta(\mathcal{X}) = \begin{cases} 0 & ||P_\Omega(\mathcal{X}_{\text{observed}} - \mathcal{X})||_F^2 \leq \delta \\ \infty & \text{otherwise} \end{cases}$$

- ✓ Variable splitting

$$\underset{\mathcal{X}}{\text{minimize}}\ i_{\mathcal{D}}(\mathcal{U}) + i_\delta(\mathcal{X})$$
$$+ \alpha||\mathbf{Y}||_{2,1} + \beta \sum_{n=1}^N \lambda_n ||\mathbf{Z}_{(n)}^{(n)}||_*,$$
$$\text{s.t.}\ \mathcal{U} = \mathcal{X},\ \mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_N],$$
$$\mathbf{y}_n = \sqrt{w_n}\mathbf{D}_n \mathbf{x},\ \mathcal{Z}^{(n)} = \mathcal{X}\ (\forall n)$$

$$||\mathbf{Y}||_{2,1} = \sum_{i=1}^{I_1 I_2 \cdots I_N} \sqrt{\sum_{n=1}^N y_{in}}$$

$\mathbf{x}$: vector expression of $\mathcal{X}$

- Primal-dual splitting algorithm

Primal step: update $\mathcal{X}^k$

Dual step (parallelizable): update $\mathcal{U}^k$, update $\mathbf{Y}^k$, update $\mathcal{Z}^{(n)k}$ $(\forall n)$

set step-size (γ_1, γ_2)

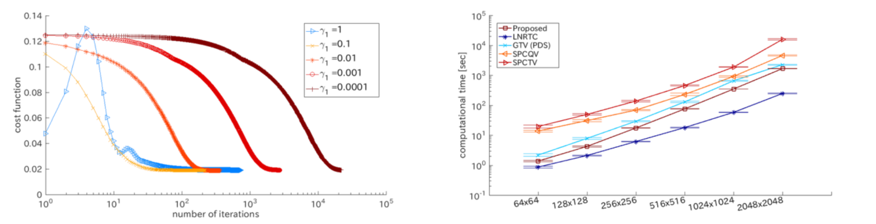
$$\mathbf{x}^{k+1} = \text{prox}_{i_\delta}\left[\mathbf{x}^k - \gamma_1\left(\mathbf{u}^k + \sum_n \mathbf{z}^{(n)k} + \sum_n \sqrt{w_n}\mathbf{D}_n^T \mathbf{y}_n^k\right)\right]$$
$$\tilde{\mathbf{x}} \leftarrow 2\mathbf{x}^{k+1} - \mathbf{x}$$
$$\tilde{\mathbf{u}} \leftarrow \mathbf{u}^k + \gamma_2 \tilde{\mathbf{x}}$$
$$\mathbf{u}^{k+1} = \tilde{\mathbf{u}} - \gamma_2 \text{prox}_{i_{\mathcal{D}}}[\tilde{\mathbf{u}}/\gamma_2]$$
$$\widetilde{\mathbf{Y}} \leftarrow \mathbf{Y}^k + \gamma_2[\sqrt{w_1}\mathbf{D}_1\tilde{\mathbf{x}}, \ldots, \sqrt{w_N}\mathbf{D}_N\tilde{\mathbf{x}}]$$
$$\mathbf{Y}^{k+1} = \widetilde{\mathbf{Y}} - \gamma_2 \text{prox}_{\frac{\alpha}{\gamma_2}||\cdot||_{2,1}}[\widetilde{\mathbf{Y}}/\gamma_2]$$
$$\widetilde{\mathbf{Z}}^{(n)} \leftarrow \mathbf{Z}_{(n)}^{(n)k} + \gamma_2 \widetilde{\mathbf{X}}_{(n)}\ (\forall n)$$
$$\mathbf{Z}_{(n)}^{(n)k+1} = \widetilde{\mathbf{Z}}^{(n)} - \gamma_2 \text{prox}_{\frac{\beta\lambda_n}{\gamma_2}||\cdot||_*}[\widetilde{\mathbf{Z}}^{(n)}/\gamma_2]$$

$k \leftarrow k+1$

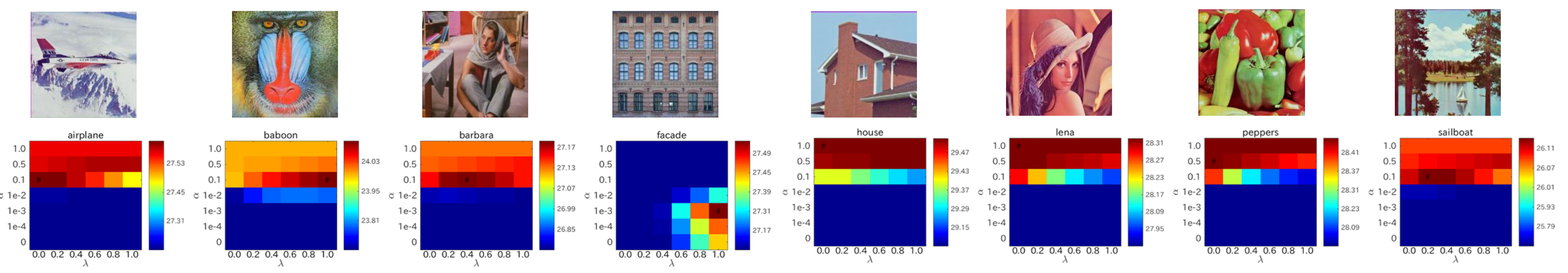
Definition of proximal map $\quad \text{prox}_{\lambda g}[\mathbf{z}] := \underset{\mathbf{x}}{\text{argmin}}\ \lambda g(\mathbf{x}) + \frac{1}{2}||\mathbf{z} - \mathbf{x}||_2^2$

Experimental Results

- Conv. behavior & times ➔



- Various parameter settings
  - ✓ Weight for TV & LR regularizations: $\alpha,\ \beta = 1 - \alpha$
  - ✓ Weights in multi-mode low-rank regularizations: (y-axis)$\lambda_1 = \lambda/2$, (x-axis)$\lambda_2 = \lambda/2$, and (c-axis)$\lambda_3 = 1 - \lambda$



- MR images [256*256*24]

PSNRs were compared with

<u>Convex approaches</u>

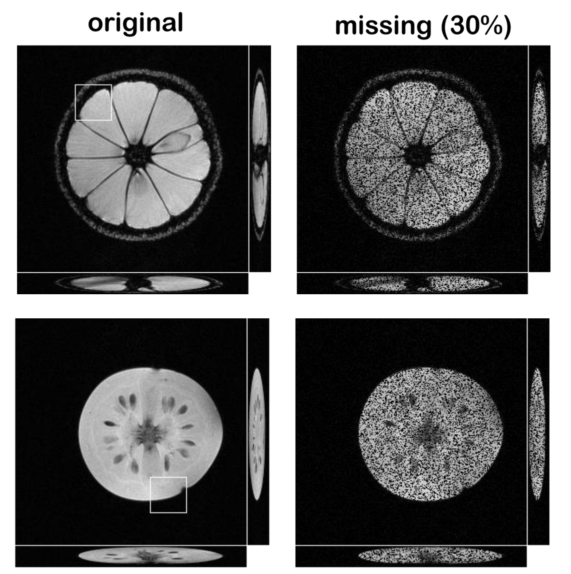
[1] GTV: Generalized total variation regularization (Guo *et al.*, *CVPR*, 2015.)

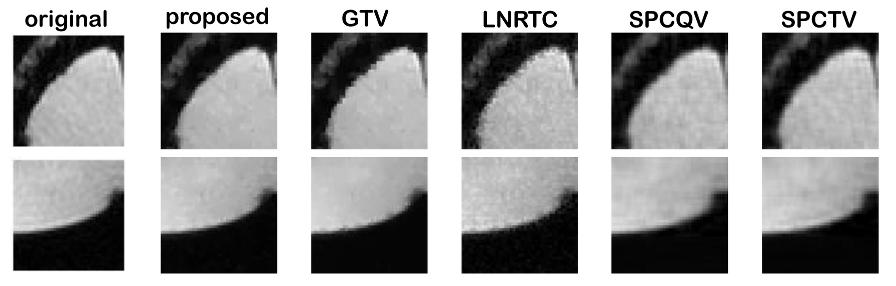
[2] LNRTC: low-n-rank tensor completion (Gandy *et al.*, *Inverse Problem*, 2011.)

<u>Non-convex approaches</u>

[3] SPCQV: smooth PARAFAC tensor completion with quadratic variation regularization (Yokota *et al.*, *IEEE-TSP*, 2016.)

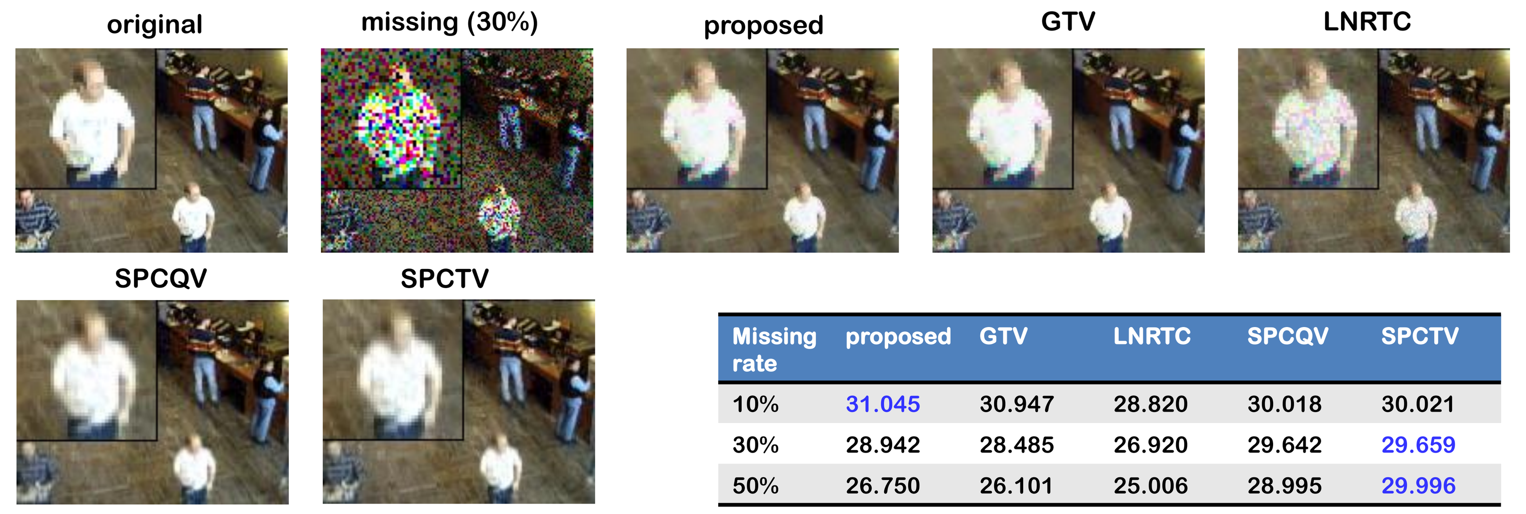
[4] SPCTV: smooth PARAFAC tensor completion with total variation regularization (Yokota *et al.*, *IEEE-TSP*, 2016.)





	Missing rate	proposed	GTV	LNRTC	SPCQV	SPCTV
citrus	10%	25.646	25.186	23.852	23.743	23.706
citrus	30%	23.410	22.920	20.948	22.251	22.115
citrus	50%	20.919	20.644	18.112	20.459	20.162
tomato	10%	27.980	27.865	26.231	24.896	24.890
tomato	30%	27.187	26.782	24.516	24.492	24.460
tomato	50%	26.014	25.429	22.785	23.825	23.717

- Color movie (4d-tensor: 120*160*3*100)



Missing rate	proposed	GTV	LNRTC	SPCQV	SPCTV
10%	31.045	30.947	28.820	30.018	30.021
30%	28.942	28.485	26.920	29.642	29.659
50%	26.750	26.101	25.006	28.995	29.996

- Conclusions
  - ✓ Convex optimization based visual data recovery is proposed.
  - ✓ Convex approach is fast & efficient, but non-convex approach is more accurate for highly missing cases.